MINIMUM PROBABILITY OF ERROR FOR ASYNCHRONOUS MULTIPLE ACCESS COMMUNICATION SYSTEMS

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ABSTRACT

Consider an ideal white gaussian channel shared by K users that transmit independent data streams by modulating antipodally a set of assigned signal waveforms without maintaining relative synchronism among them. This paper investigates the probability of error achievable by an optimum K-user coherent detector. It is shown that unless signals with poor cross-correlation properties are used or the interfering users are comparatively weak, the minimum error probabilities have the same asymptotic behavior as in a single-user communication system, i.e. there is no performance degradation due to the presence of other users. As illustrated by several examples, this implies that in the SNR region of interest, the error probability of the conventional detector is not necessarily close to the minimum even if signals with low cross-correlations are used.

I. INTRODUCTION

Consider a digital communications system in which the channel is shared by K users that modulate antipodally a set of assigned signal waveforms, without carrier phase or bit-interval synchronism among them. The coherent receiver commonly employed in this multiple access communication system [12] consists of a set of matched filters synchronized to the signal of each user and followed by a zero-threshold (Fig. 1), i.e., assuming that the channel introduces additive white gaussian noise, the conventional receiver can be viewed as a bank of optimum detectors for single user communication. However, since in general the input to every threshold has an additive component of multiple access interference (cross-correlation with the signals of the other users), the conventional receiver is optimum for the detection of a given user only asymptotically as the SNR's of the other users go to zero. When signals of large bandwidth are allowed (e.g., in Direct-sequence Spread-Spectrum), the designer can select a signal set with very low cross-correlations for all possible delays. In these circumstances, the performance of the conventional receiver has been thoroughly analyzed ([8], [13] and the references therein) and has been shown to be very acceptable for signal sets having good correlation properties.

The goal of the present work is to obtain the <u>minimum</u> probability of error achievable for the usual asynchronous multiple-access digital communications model; we will derive and analyze an optimum K-user coherent detector (the <u>a priori</u> unknown relative delay

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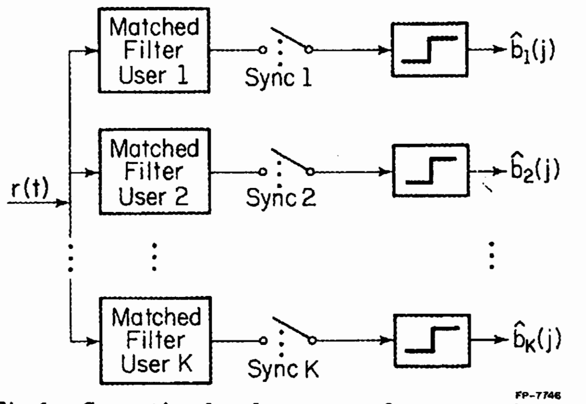

Fig 1. Conventional detector for asynchronous multiple-access communications.

and carrier phase of each active user are assumed to be acquired) which bases every bit decision on the whole observation interval. Under these conditions it is straightforward to show that the whole sequence of outputs of the bank of matched filters for all users is a sufficient statistic for the detection of the transmitted information sequence. The bank of matched filters is followed by a decision device (Fig. 2) that depends on the optimality criterion employed, the major criteria being minimum bit error rate and maximum-<u>a-posteriori</u> sequence detection. Although the performance measure of interest is the set of error probabilities for each user, MAP sequence detection may be preferable since it gives the most probable transmitted information sequence given the received data or equivalently it selects among the possible noise realizations the one with minimum power. Also, while for high SNR both receivers achieve the same error probabilities, the minimum BER detector is considerably more complex ([10], [17]) than the MAP sequence detector due to the inherently nongaussian (composite hypotheses) character of the minimum error probability problem.

As is shown in Section II, the decision device for MAP sequence detection can be implemented by a Viterbi algorithm with 2^K states and a K-dimensional version of the branch metric employed in the maximum-likelihood sequence detection of PAM communications through linear channels that introduce intersymbol interference of approximately finite length. The main result of the minimum error probability analysis of Section III is that when signal sets with good cross-correlation properties are employed the optimum probability of error of every user has the same asymptotic behavior (increasing SNR) as that achievable in single-user communication. Since this asymptotic

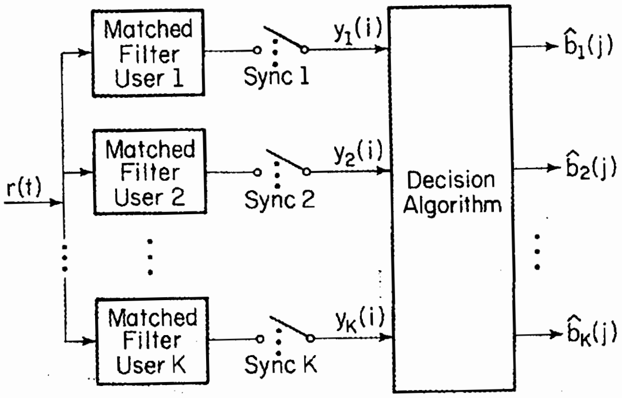

Fig 2. Optimum sequence detector for asynchronous multiple-access communications.

error probability performance is, in most cases of interest and unless the system operates in the low SNR region, appreciably superior to that exhibited by the conventional receiver, the optimum receiver offers important performance advantages. Furthermore, even if signal waveforms with poor cross-correlations are used, the optimum sequence receiver provides multiaccess capability and it is shown that except for trivial cases, and in contrast with the conventional receiver, any pre-specified probability of error is achievable with high enough SNR. Finally, in Section IV the computation of the average and worst-case of the error probability bounds found in Section III is illustrated numerically.

II. OPTIMUM SEQUENCE DETECTOR

In the usual asynchronous K-user data communications model (cf. [12]), the input to the receiver is

$$r(t) = S(t,\underline{b}) + n(t) \qquad -MT \le t < (M+2)T \tag{2.1}$$

where $n(t)$ is zero-mean white gaussian noise (double-sided spectral height of $N_0/2$), T is the bit interval duration (assumed to be the same for all users) and the received information-bearing signal (with binary antipodal modulation) is

$$S(t,\underline{b}) = \sum_{i=-M}^{M} \sum_{k=1}^{K} b_k(i)\, s_k(t - iT - \tau_k) \tag{2.2}$$

where $\underline{b} = \{b(i) = [b_1(i), \ldots, b_K(i)]^T,\ i = -M, \ldots, M\}$ and $b_k(i) \in \{-1,1\}$, $s_k(t)$ (= 0 outside $[0,T]$) and $\tau_k \in [0,T)$ are the i^{th} bit, the signal waveform and the delay (modulo T with respect to an arbitrary reference) respectively of the k^{th} user.

The MAP bit-sequence detection problem for the model (2.1-2) is to solve for the bit-sequence that maximizes the *a posteriori* probability $P[\underline{b}|r(t), -MT \le t < (M+2)T]$. Because the noise is white and gaussian and all sequences $\underline{b}$ are equiprobable, this amounts [16] to finding the sequence that minimizes $\|r - S(\underline{b})\|$, the Euclidean distance between the transmitted and received signals, or equivalently that maximizes,

$$\Omega(\underline{b}) = 2\,\langle r, S(\underline{b})\rangle - \|S(\underline{b})\|^2. \tag{2.3}$$

Denote by $y_k(i)$ the output of a matched filter for the i^{th} bit of the k^{th} user; i.e.,

$$y_k(i) = \int_{\tau_k+iT}^{\tau_k+(i+1)T} r(t)\, s_k(t-iT-\tau_k)\, dt, \tag{2.4}$$

and the K x K correlation matrix $H(i)$ whose k-row j-column element is

$$H_{kj}(i) = \int_0^T s_k(t)\, s_j(t+iT+\tau_k-\tau_j)\, dt. \tag{2.5}$$

Since $s_k(t) = 0$ for $t \notin [0,T]$ and $k = 1, \ldots, K$, $H(i)$ has the properties:

$$H(i) = \underline{0} \quad \text{for } |i| > 1, \tag{2.6a}$$

$$H(i) = H^T(-i) \tag{2.6b}$$

and with an adequate numbering of the users, $H(1)$ is a triangular matrix with zero diagonal.

With the above definitions, we can express both terms of the right-hand side of (2.3) in the form,

$$\langle r, S(\underline{b})\rangle = \sum_{i=-M}^{M} b^T(i)\, y(i), \tag{2.7}$$

and

$$\|S(\underline{b})\|^2 = \sum_{i=-M}^{M} \sum_{j=-M}^{M} b^T(i)\, H(i-j)\, b(j), \tag{2.8}$$

with $\underline{y} = \{y(i) = [y_1(i), \ldots, y_K(i)]^T,\ i = -M, \ldots, M\}$. Even though as explained in Section I, $y_k(i)$ is not a sufficient statistic for the detection of $b_k(i)$, (2.3) and (2.7) imply that the whole sequence of outputs of the bank of K matched filters, $\underline{y}$, is a sufficient statistic for the decision of the most likely sequence $\underline{b}$. Therefore, we can proceed to obtain the decision algorithm that selects the most likely $\underline{b}$ upon observation of $\underline{y}$ (Fig. 2). The brute-force method of evaluating (2.3) for each of the $2^{K(2M+1)}$ possible sequences $\underline{b}$ has no practical significance due to the exponential dependence on the length of the observations. The key to the reduction of computational effort is the use of properties (2.6a-b) of the matrix $H(i)$, which imply that (2.8) can be written in the form

$$\|S(\underline{b})\|^2 = \sum_{i=-M}^{M} b^T(i)\,[H(0)b(i) + 2H(1)b(i-1)] \tag{2.9}$$

with $b(-M-1) = \underline{0}$. Hence (see [5], [19]) the sequence that maximizes (2.3) can be computed by a Viterbi algorithm with state space $S = \{-1,1\}^K$ and branch metric given by

$$\lambda_i(x_i, x_{i-1}) = x_i^T[2y(i) - H(0)x_i - 2H(1)x_{i-1}] \tag{2.10}$$

where $x_i \in S$. Consequently, the decision rule that gives the MAP sequence can be implemented by a front end of matched filters (as in the conventional receiver) followed by a recursive shortest-path decision algorithm whose computational complexity is equivalent to the ML decoding of a convolutional code of rate K/K and constraint length equal to 2. Note that assuming the use of a bank of conventional receivers for single user communications with observations corrupted by independent noise processes, Schneider [14] arrived at a discrete-time model with additive white noise and conjectured the use of a Viterbi algorithm with 2^{2K} states.

Although the decision delay is unbounded because an optimum decision cannot be made until all states share a common subpath, a well-known advantage of the Viterbi algorithm is that little degradation of performance occurs when the algorithm uses an adequately chosen fixed decision lag. On the other hand, for large numbers of users the dimensionality of the state space becomes infeasible and hence the designer must resort to suboptimal implementations based either on the Viterbi algorithm (e.g. [2],[6]) or on other schemes such as sequential decoding (e.g., [9], [19]).

III. ERROR PROBABILITY ANALYSIS

III.1 Upper and lower bounds

This section is devoted to the analysis of the minimum bit error probability for the asynchronous multiple-access model (2.1-2) as a function of the received set of delays and carrier phases. Specifically, we analyze the probability, P_k, that the optimum *a posteriori* bit-decision corresponding to the 0^{th} bit of the k^{th} user is erroneous; i.e. $P_k = P[b_k(0) \neq \hat{b}_k(0)]$, where

$$\hat{b}_k(i) = \arg\max_{b \in \{-1,1\}} P[b_k(i) = b \mid r(t), -MT \leq t < (M+2)T] \quad (3.1)$$

Define the set of nonzero error sequences $E = \{\underline{\varepsilon} = \{\varepsilon(i) \in \{-1,0,1\}^K,\ i = -M, \ldots, M;\ \text{s.t. } \varepsilon(j) \neq \underline{0} \text{ for some } j\}\}$. An error sequence is called decomposable if there exists a pair of sequences $\underline{\varepsilon}' \in E$, $\underline{\varepsilon}'' \in E$, such that[1] $\underline{\varepsilon}' < \underline{\varepsilon}$, $\underline{\varepsilon}'' < \underline{\varepsilon}$, $\underline{\varepsilon}' + \underline{\varepsilon}'' = \underline{\varepsilon}$, and $\|S(\underline{\varepsilon})\|^2 = \|S(\underline{\varepsilon}')\|^2 + \|S(\underline{\varepsilon}'')\|^2$. Define the following subsets of error sequences

$$F_k = \{\underline{\varepsilon} \in E,\ \text{s.t. } \varepsilon_k(0) \neq 0 \text{ and } \underline{\varepsilon} \text{ is not decomposable}\}$$

$$A_k(\underline{b}) = \{\underline{\varepsilon} \in E,\ \text{s.t. } b(i) - 2\varepsilon(i) \in \{-1,1\}^K,\ i = -M, \ldots, M, \text{ and } \varepsilon_k(0) = b_k(0)\},$$

i.e., $A_k(\underline{b})$ is the set of error sequences that affect the 0^{th} bit of the k^{th} user and that are admissible given that $\underline{b}$ is transmitted. Define the functions

$$w(\underline{\varepsilon}) = \sum_{i=-M}^{M} \sum_{k=1}^{K} |\varepsilon_k(i)|,$$

$$I_k(\underline{b}) = \Big[\sum_{\substack{i=1 \\ i \neq k}}^{K} b_i(0) H_{ik}(0) + b_i(1) H_{ik}(-1) + b_i(-1) H_{ik}(1)\Big] / H_{kk}(0)$$

and

$$d_k(\underline{b}) = \min_{\underline{\varepsilon} \in A_k(\underline{b})} \|S(\underline{\varepsilon})\|$$

The minimum energy of any error sequence such that $\varepsilon_k(0) \neq 0$, will be denoted by $d^2_{k,min}$, i.e.,

$$d_{k,min} = \min_{\underline{b}} d_k(\underline{b}) = \min_{\underline{\varepsilon} \in F_k} \|S(\underline{\varepsilon})\|,$$

which corresponds to one half of the minimum RMS of the difference between the signals of any pair of transmitted sequences that differ in the 0^{th} bit of the k^{th} user.

[1]We denote $\underline{\varepsilon}' < \underline{\varepsilon}$ if $|\varepsilon'_j(i)| \leq |\varepsilon_j(i)|$ for all $1 \leq j \leq K$ and $-M \leq i \leq M$.

Proposition 1[2]

The minimum error probability of the k^{th} user is upper bounded by

$$P_k \leq \sum_{\underline{\varepsilon} \in F_k} 2^{-w(\underline{\varepsilon})} Q(\|S(\underline{\varepsilon})\| \sqrt{2/N_0}) \quad (3.2)$$

and by

$$P_k \leq E_{\underline{b}}[Q(\sqrt{2H_{kk}(0)/N_0}(1 + I_k(\underline{b})))] \quad (3.3)$$

where the expectation is over the ensemble of equally likely transmitted sequences $\underline{b}$.

Proposition 2

The minimum error probability of the k^{th} user is lower bounded by

$$P_k \geq P[d_k(\underline{b}) = d_{k,min}]\ Q(d_{k,min}\sqrt{2/N_0}), \quad (3.4)$$

and by

$$P_k \geq Q(\sqrt{2\, H_{kk}(0)/N_0}). \quad (3.5)$$

The bounds (3.3) and (3.5) are, respectively, the k^{th} user error probability achieved by the conventional receiver (Fig.1) and the k^{th} user optimum error probability if no other user were active. It is easy to see that they are tight in the low SNR region. On the other hand, the bounds (3.2) —an upper bound to the k^{th} user error probability of the optimum sequence detector of Section II— and (3.4) are tight in the high SNR region as is demonstrated in the following subsection.

III.2 Asymptotic Probability of Error

Because of the asymptotic behavior of the Q-function ($\lim_{\sigma \to 0} Q(x_1/\sigma)/Q(x_2/\sigma) = 0$ if $x_1 > x_2$), the only sequences that contribute appreciably to the upper bound (3.2) when $N_0 \to 0$, are those with minimum energy. Therefore (3.2) and (3.4) result in

$$C^L_k Q(d_{k,min}\ 2/N_0) \leq P_k \leq C^U_k Q(d_{k,min}\ 2/N_0), \quad (3.6)$$

where

$$C^L_k = P[d_k(\underline{b}) = d_{k,min}] = P[\bigcup_{\substack{\underline{\varepsilon} \in F_k \text{ s.t.} \\ \|S(\underline{\varepsilon})\| = d_{k,min}}} \{\underline{\varepsilon} \in A_k(\underline{b})\}], \quad (3.7)$$

and

$$C^U_k = \sum_{\substack{\underline{\varepsilon} \in F_k \text{ s.t.} \\ \|S(\underline{\varepsilon})\| = d_{k,min}}} 2^{-w(\underline{\varepsilon})} = \sum_{\substack{\underline{\varepsilon} \in F_k \text{ s.t.} \\ \|S(\underline{\varepsilon})\| = d_{k,min}}} P[\underline{\varepsilon} \in A_k(\underline{b})] \quad (3.8)$$

Since the upper and lower asymptotic bounds of (3.6) differ by a multiplicative constant independent of N_0, the k^{th} user error probability for high signal to noise ratios is equivalent to that of an antipodal single-user system with bit-energy equal to $d^2_{k,min}$. This suggests that a measure of the degradation in

[2]Proofs are omitted in the Conference Record because of space limitations.

optimum performance (for high SNR) caused by the presence of the rest of the users can be given by the k^{th} <u>user asymptotic efficiency</u>, $\eta_k \in [0,1]$, defined as the ratio between the effective and actual SNR's; i.e.,

$$\eta_k = d^2_{k,min}/H_{kk}(0) \tag{3.9}$$

Analogously, the asymptotic efficiency for the conventional receiver can be obtained straightforwardly from the right-hand side of (3.3)[3],

$$\eta^c_k = \max{}^2\{0,\ Z_k/H_{kk}(0)\}, \tag{3.10}$$

with

$$Z_q = H_{qq}(0) - \sum_{\substack{n=1 \\ n\neq q}}^{K} \sum_{r=-1}^{1} |H_{qn}(r)|, \quad 1 \le q \le K, \tag{3.11}$$

where the asymptotic efficiency has been taken to be zero in the case in which for some $\underline{b}$, $I_k(\underline{b}) \le -1$ (nonzero limit of probability of error).

This phenomenon of multiple-access limitation of the achievable error probability by the conventional receiver can occur even if signal sets with good cross-correlation properties are employed, provided that the number and relative power of the interfering users is large enough. This behavior is in sharp contrast with that of the optimum receiver. For $d_{k,min} = 0$ it is necessary that two different information sequences result in the same received noiseless signal, and this can only happen for signal constellations with poor cross-correlations. Moreover, even if for some set of delays, carrier phases and received energies $d_{k,min} = 0$, this occurs with probability zero if, as in the usual asynchronous model, the <u>a priori</u> unknown delays and phases are uniformly distributed and nontrivial signals are employed. This fact is a corollary of the following result.

<u>Proposition 3</u>

Suppose that in a probability space (Ω,F,P)

i) τ_k is a continuous random variable,

ii) $\{\int_0^T s_i^2(t)\,dt,\ \tau_i;\ i = 1,\dots,K\}$ are independent random variables,

iii) $\int_0^T s_k^2(t)\,dt \neq 0$ a.s. P

Then, $d_{k,min} \neq 0$ a.s. P

Also of interest is to find out under what conditions on the signal cross-correlations there is no degradation in the effective SNR of a particular user due to the existence of other active users. It can be seen from (3.10) that for the conventional receiver this is only possible when the cross-correlations with the rest of the users are zero; i.e., with zero probability. Fortunately, the behavior of the optimum error probability is different. In order that $\eta_k \neq 1$ there must exist an error sequence with $\varepsilon_k(0)$ and at least another component different from zero, whose energy $\|S(\underline{\varepsilon})\|^2$ is smaller than $H_{kk}(0)$. From the inspection of (2.8) or (2.9) one can argue in coarse terms that this can occur only if the cross-correlations are at least comparable to the energies of the rest of the signals. A simple sufficient condition for $\eta_k = 1$ (see (2.9)) that formalizes this reasoning is that for all x and $y \in \{-1,0,1\}^K$

$$x^T[H(0)x + 2H(1)y] \ge \begin{cases} 0 & \text{if } x_k = 0 \\ H_{kk}(0) & \text{if } |x_k| = 1. \end{cases}$$

The following result provides another sufficient condition for an asymptotic efficiency equal to one which is stronger than the condition for $\eta^c_k \neq 0$; i.e. $I_k(\underline{b}) > -1$. (Counterexamples can be found in which $\eta^c_k \neq 0$ and $\eta_k \neq 1$).

<u>Proposition 4</u>

Suppose that

i) For every $q \neq k$

$$Z_q \ge \max\{|H_{qk}(-1)| + |H_{qk}(1)|,\ |H_{qk}(0)|\}$$

ii) $$H_{kk}(0) - \sum_{\substack{q=1 \\ q\neq k}}^{K} \min\{Z_q - \sum_{r=-1}^{1} |H_{qk}(r)|,\ 0\} \ge 0.$$

Then $\eta_k = 1$.

<u>Corollary</u>

If for every $q \neq k$, $Z_q \ge \sum_{r=-1}^{1} |H_{qk}(r)|$, then $\eta_k = 1$.

One instance in which it is possible to obtain an analytical solution for the minimum error energy problem is the case of two users:

<u>Proposition 5</u>

If $K = 2$ then

$$\eta_k = 1 - \max\{0,\ 2|H_{12}(0)| - H_{ii}(0),\ 2|H_{12}(1)| - H_{ii}(0),\ 2|H_{12}(0)| + 2|H_{12}(1)| - 2H_{ii}(0)\}/H_{kk}(0) \tag{3.12}$$

where $i \neq k$.

IV. <u>NUMERICAL EXAMPLES</u>

Two pairs of lower and upper bounds to the k^{th} user minimum error probability have been presented in Section III and they have been shown to be tight asymptotically; nonetheless, it remains to ascertain the SNR level for which such asymptotic approximation is sufficiently accurate. In the sequel, this question is illustrated by several examples of the computation of averages and extreme cases of the above bounds. Moreover, the explicit expressions of the asymptotic efficiency found in the case of two users will be employed to investigate the relative effects of the near-far problem on the conventional and optimum detectors.

The computation of the upper bound to the error probability of the optimum sequence detector is carried out by generating a subset of F_k that achieves a sufficient degree of approximation to the series of the right-hand side of (3.2). This subset of F_k is obtained via a 9-ary tree in which the root represents the error sequence whose only nonzero element is $\varepsilon_k(0) = 1$ and in which the sons of every node are its indecomposable augmented sequences (from the left and right with an element of the set $\{-1,0,1\}$) whose relative contribution to the partial sum is greater than a convergence parameter. For a given accuracy, the speed

[3]Note the discrepancy between η^c_k and previously proposed measures of SNR degradation based on the gaussian noise modeling of the multiple access interference [20].

of convergence of this procedure depends on the maximum value of N_0 for which the bound is computed and on the magnitude of the signal cross-correlations.

Denoting by $\|S(\underline{\varepsilon},\underline{\tau})\|^2$ the explicit dependence of the error energies on the set of delays, it is easy to see that if the signal waveforms are constant on the intervals $[nT_c,(n+1)T_c)$ where $n=0,\ldots,N-1$ and $NT_c=T$, then $\|S(\underline{\varepsilon},\underline{\tau})\|^2$ is affine in $\underline{\tau}$ in every cube $[n_1T_c,(n_1+1)T_c)\times\ldots\times[n_KT_c,(n_K+1)T_c)$. This property and the fact that the function $Q(x)$ is convex imply that the worst-case of the optimum sequence upper bound occurs for $\underline{\tau}=(n_1T_c,\ldots,n_KT_c)$ and that an upper bound to the average k^{th} user probability of error is given by

$$T^{1-K}\underbrace{\int_{\tau_1=0}^{T}\cdots\int_{\tau_K=0}^{T}}_{i\neq k}\sum_{\underline{\varepsilon}\in F_k}2^{-w(\underline{\varepsilon})}Q(\|S(\underline{\varepsilon},\underline{\tau})\|\,2/N_0)\,d\underline{\tau}$$

$$\leq N^{1-K}\sum_{\substack{\tau_i=n_iT_c\\ n_i=0,\ldots,N-1}}\sum_{\underline{\varepsilon}\in F_k}2^{-w(\underline{\varepsilon})}Q(\|S(\underline{\varepsilon},\underline{\tau})\|\,2/N_0). \qquad (4.1)$$

Our first example is a baseband asynchronous system with two equal-energy users that employ a simple set of signal waveforms (Fig.3). In this figure the upper bounds to the best and worst cases of the optimum detector are indistinguishable from each other, and for SNR higher than about 6 dB, from the single user lower bound (which is also the minimum energy lower bound since $\eta_1=1$). Note also that the maximum interference coefficient is $I_1=1/3$ for all delays and the performance of the conventional receiver varies very slightly with the relative delay.

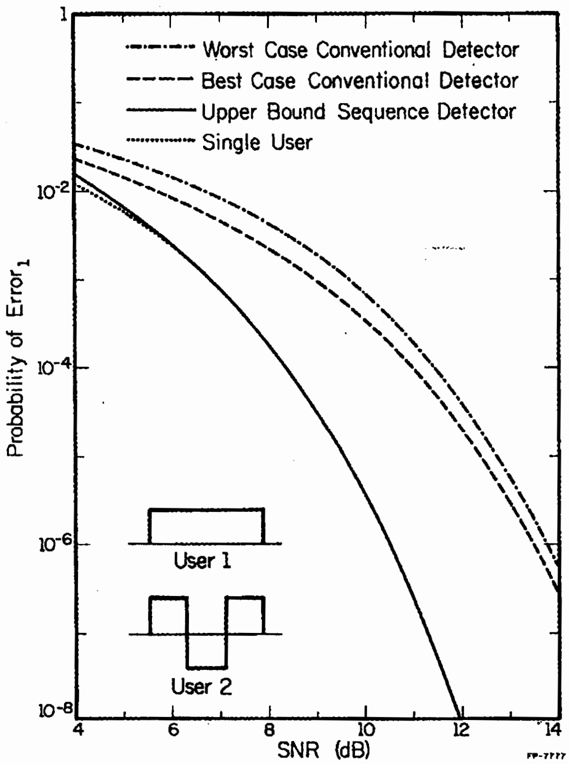

Fig 3. Best and worst-cases of the error probability of user 1 achieved by conventional and optimum detectors.

In the next examples, we employ a set of spread-spectrum signals: three maximal-length signature sequences of length 31 generated to maximize a

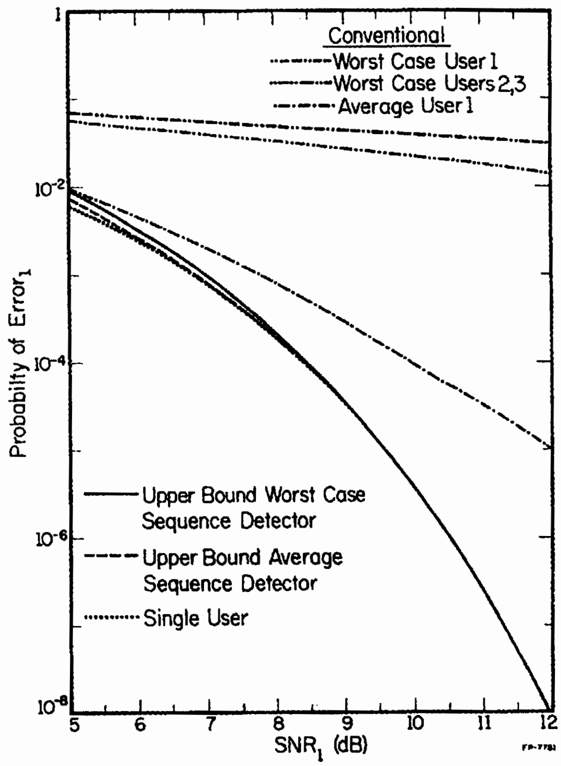

Fig 4. Worst-case and average error probabilities achieved by conventional and optimum detectors with 3 active users employing max-SNR m-sequences of length 31.

signal-to-multiple-access interference functional [7,Table 5]. The average probability of error of the conventional receiver for equal-energy users employing this signal set has been thoroughly studied previously ([1],[8],[13]) and in Fig.4 we reproduce (from [8,Fig.2]) the average error probability of user 1 achieved by the coherent conventional detector. Also shown in Fig.4 are the worst-cases of the conventional detector (recall that the worst possible set of carrier phases corresponds to the baseband case) and upper bounds to the baseband worst-case and average minimum error probabilities for user 1. From the observation of Fig.4 we can conclude that for error probabilities of 10^{-2} the average performance of the conventional detector is fairly close to the single user lower bound, but the worst-case error probability is notably poor for the whole SNR range considered in the figure. Note, however, that since the signal set has good cross-correlations for most of the relative delays, error probabilities close to the worst-case curve will occur with small probability. The worst-case and, especially, the average upper bounds on the optimum sequence detector performance are remarkably close to the single user lower bound, and show that the minimum error probability not only has a low average (around one order of magnitude better than that of the conventional detector, at 9 dB) but its dependence on the delays is negligible for most practical purposes.

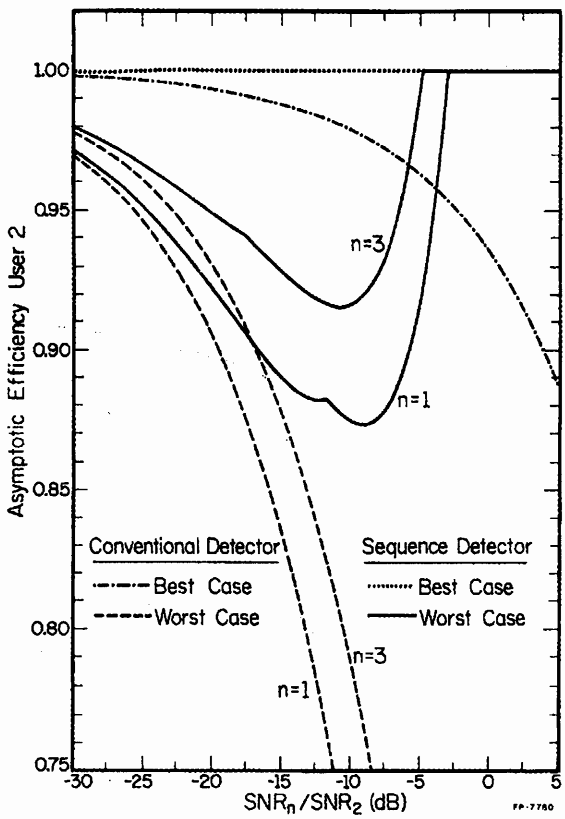

Fig 5. Asymptotic efficiency as a function of the relative energy of the interfering user. K=2, N = 31.

The next example investigates the near-far problem (i.e.,the effects of unequal received energies) for two users that employ a subset of the previous set of maximal-length signature sequences. The conventional and optimum asymptotic efficiency of user 2 obtained via (3.10) and (3.11) is shown in Fig. 5 as a function of the relative energy of the interfering user. As should be expected, the conventional detector approaches the optimum efficiency as the interfering user becomes weaker, and decreases monotonically with the energy ratio until it reaches zero (multiple-access limitation) for relative energies in the intervals (+6.3,+30 dB) -users 2 and 1- and (+9,+30 dB) -users 2 and 3-.

By means of (3.11) it is easy to show that in the case of two users the optimum asymptotic efficiency is equal to one if and only if the relative energy of the interfering user is <u>greater</u> than $4\alpha^2$, where $\alpha=\max\{|H_{12}(0)|,|H_{12}(1)|\}/(H_{11}(0)H_{22}(0))^{1/2}$. Furthermore if the relative energy of the interfering user is less than $4\alpha^2$ then the lowest possible efficiency is $1-\max\{\alpha^2,\beta^2/2\}$, where $\beta=(|H_{12}(0)|+|H_{12}(1)|)/(H_{11}(0)H_{22}(0))^{1/2}$ (cf. Fig. 5). The intuitive reason for this behavior is that noise, not the randomness of the information of the interfering users, is the primary source of the errors committed in the optimum detection of the user of interest, if the interfering users are sufficiently powerful. When users 1 and 2 are active, the worst-case optimum asymptotic efficiency of Fig.5 corresponds to α=11/31, β=15/31 and shows clearly that the minimum energy is achieved by three different error sequences depending on the relative energy of the interfering user. Bounds on the error probabilities corresponding to this example (worst-case relative delay between users 1 and 2) are calculated in Fig.6, for three relative energies,namely, SNR_2/SNR_1=-10 dB, -5 dB, 0 dB, wherein the optimum asymptotic efficiencies are 0.88,0.92 and 1.0 respectively (Fig.5). It is interesting to observe in the graphics corresponding

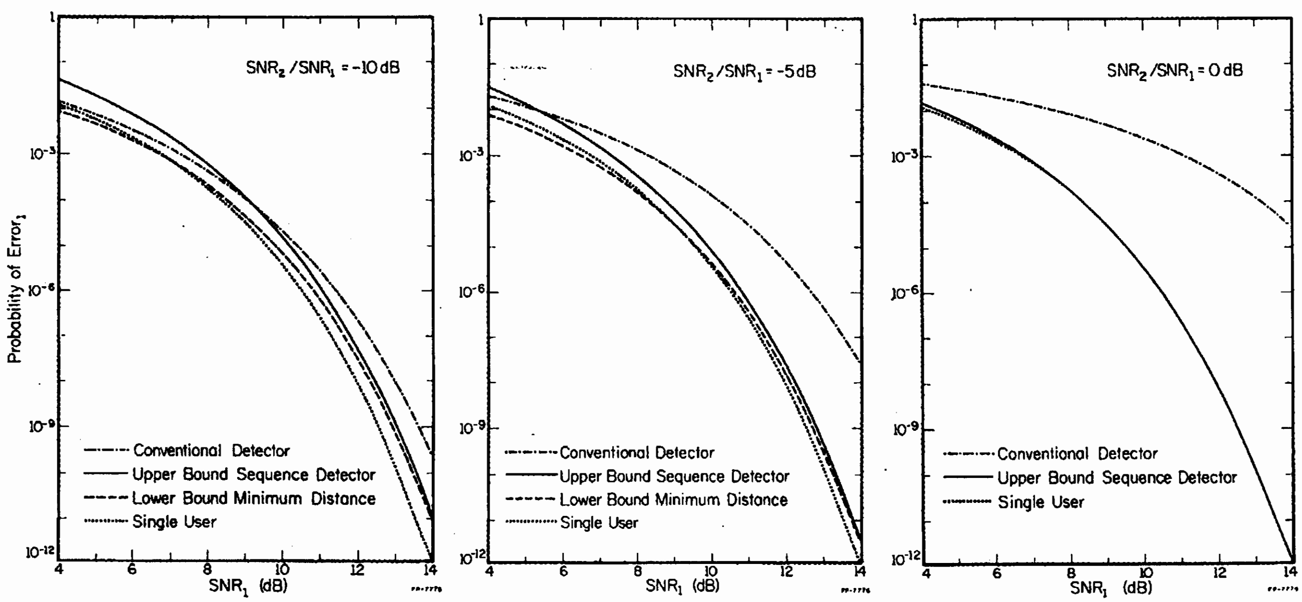

Fig 6. Bounds on the minimum error probability of user 1. Worst-case delays and 2 active users.

to SNR_2/SNR_1=-10 dB, -5 dB that all four bounds derived in Section III play a role in some SNR interval, in particular, the error probability of the conventional detector is lower than the upper bound to the optimum sequence detector for small SNR. It is apparent in Fig. 6 the opposite effect of an increase in the energy of the interfering users on the minimum and conventional probabilities of error: while the optimum sequence detector bounds become tighter and closer to the single user lower bound, the conventional error probability grows rapidly until it becomes multiple-access limited (for SNR_2/SNR_1=6.3 dB).

V. CONCLUDING REMARKS

By invoking central limit arguments, it has been generally conjectured that the error probability of the conventional receiver is quasi-optimal if spread-spectrum signals are employed. The main reason the results of this paper do not support this conjecture is not the inaccuracy of the gaussian approximation for a finite number of users with nonzero power. Rather, it is the fact that every decision of the conventional receiver is constrained to be based only on the time interval corresponding to one bit of one user, which accounts for the loss of optimality in situations where the background noise is not dominant.

It has been shown that in the high SNR region, the minimum bit error rate is closely approximated by the error probability of the optimum sequence detector. For an assessment of the feasibility of this receiver, two aspects deserve particular attention. First, the evident drawback of the exponential complexity of the decision algorithm in the total number of users is toned down by the comparatively small performance gains attained by not neglecting weak users with other destinations, and by the results of less demanding suboptimal algorithms for related shortest path problems. Second, as each user becomes active the sequence detector is required to synchronize to its bit-epoch, carrier phase and information (recall that the transmission is assumed to start at a finite time instant), and needs to determine the cross-correlations $\{H_{ik}(r),\ i=1,\ldots,K,\ r=0,1\}$, which are functions of the actual relative amplitudes, delays and phases.

The results presented here open the possibility of a tradeoff between the complexities of the receiver and the signal constellation in order to achieve a fixed level of performance; the actual compromise being dictated by the relative received powers at the various destinations. Such a tradeoff is likely to favor an optimum sequence receiver that takes into account only those unwanted users that are not comparatively weak at its location. On the other hand, good asymptotic efficiencies for all users require highly complex signals when their energies are very dissimilar; however if unit asymptotic efficiencies are achieved for most of the signal delays, a further improvement of the cross-correlation properties has a negligible effect on the minimum error probabilities in the SNR region of interest. The utilization of signal constellations with good asymptotic efficiencies has the additional advantage of reducing the dependence of the minimum error probabilities on the actual received signal delays to inappreciable levels (which are attained by the conventional receiver only for signals of impractical complexity).

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