Multi-Cell Uplink Spectral Efficiency of Randomly Spread DS-CDMA in Rayleigh Fading Channels

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Abstract

A simple multi-cell Rayleigh fading uplink communication model is suggested and analyzed for optimally coded randomly spread DS-CDMA with multiuser detection. The model adheres to Wyner's (1994) infinite linear cell-array model, according to which only adjacent-cell interference is present, and characterized by a single *parameter* $0 \leq \alpha \leq 1$. *The discussion is confined to asymptotic analysis where both the number of users per cell and the processing gain go to infinity, while their ratio goes to some finite constant. The spectral efficiency of various multiuser detection strategies is evaluated assuming single cell-site processing, and* equal transmit powers *for all users in all cells. Comparative results demonstrate how performance is affected by the introduction of inter-cell interference (with and without fading), and what is the penalty associated with the randomly spread coded DS-CDMA strategy.*

1. Introduction

Information theoretic analyses of direct sequence code division multiple access (DS-CDMA) systems have gained much attention in recent years, due to the rapid development of commercial cellular systems employing this multi-access strategy. Results for a *single-cell* DS-CDMA system, were recently presented in [1] – [4] (see also references therein). These works explicitly relate to CDMA systems with *random* spreading sequences, and the limiting scenario is examined, where both the number of users and the processing gain go to infinity, while their ratio goes to some *finite* constant. Assuming equal received powers and no fading, expressions are presented in [1] for spectral efficiencies of the optimum detector, the matched-filter detector, the decorrelator, and the linear minimum mean squared error (MMSE) detector. The impact of frequency-flat fading on these receivers is analyzed in [2]. Signal to interference plus noise ratios (SIRs) at the output of the above mentioned linear detectors are presented in [3], and an extension of these results to fading channels with multi-antenna reception can be found in [5].

This paper addresses *multi-cell* systems using the attractive cellular model suggested by Wyner in [6]. This simple model allows for analytical tractability on the one hand, while giving insight to practical systems on the other. Accordingly, the system's cells compose an infinite *linear* array, where the received signal at each cell-site is the sum of the Rayleigh faded signals received from intra-cell users, plus a factor α ($0 \leq \alpha \leq 1$) times the sum of the faded signals generated by users in the two adjacent cells. Non-adjacent cell users are assumed to produce no interference. The received signal is embedded in ambient Gaussian noise. The multi-cell effect on performance is thus specified by a *single* parameter (α). The same model is also used by the authors in [7], which is devoted to non-fading channels. In [8], a *multiple-cell* receiver as in [6] is considered, and it is demonstrated that fading may turn beneficial even with the optimally coded setting, where no spreading is imposed.

Flat-fading channels are assumed, as in [2], [8] and [9], and the spectral efficiency obtained by employing *optimally* coded (in the information-theoretic sense) *randomly* spread DS-CDMA with multiuser detection is analyzed. As in [1] – [3], the limiting case is considered in which, denoting by K the number of intra-cell users (assumed constant and equal in all cells), and by N the spreading factor (processing gain), $K, N \to \infty$, while $\frac{K}{N} \to \beta < \infty$. The factor β is commonly referred to as the "system load". Assuming *single cell-site processing*, four types of multiuser detection strategies are considered (as in [7]): 1) The *"conventional" matched-filter detector* that treats *all* interference (either intra-cell or inter-cell) as additive white Gaussian noise; 2) A *single-cell optimum (SCO)* detector that "optimally" detects the transmissions of *intra-cell* users, while treating *inter-cell* interference as additive white Gaussian noise; 3) The *linear MMSE detector* that knows the signature sequences of all interfering users (both intra-cell and in adjacent cells) and mitigates their interference by means of a linear MMSE filter; 4) A detector that employs *MMSE based successive interference cancellation (MMSE-SC)* to decode transmissions of intra-cell users, while *inter-cell* interference is mitigated by means of a linear MMSE filter (the MMSE-SC detector is, in fact, optimum in terms of spectral efficiency in the setting considered in this paper, as briefly explained in Subsection 3.4). To distinguish between this detector and a detector that performs MMSE based successive interference cancellation over *all* received transmissions, the latter shall be referred to as the *full MMSE-SC detector* (this detector is equivalent to a decision-feedback receiver). It is emphasized that neither the linear MMSE detector, nor the MMSE-SC detector, try to *decode* the transmissions of adjacent cell users (which might be prohibitive if α is small). In fact, the cell-site detector may actually be ignorant regarding codebooks or code-mask sequences employed in other cells, but is aware, as usually is the case in practice, of the signature sequences of all users in adjacent cells. In addition to the above, it is also assumed that all detectors are provided with the required knowledge regarding the received powers of the interfering signals. The use of adaptive MMSE detection may be attractive in this respect, as it makes no distinction between intra-cell and adjacent-cell interference.

Identifying the spectral efficiency as the fundamental measure of system performance for coded systems (see [1]), the spectral efficiency of all four detection strategies is obtained, and comparatively examined assuming a *constant* (fading independent) transmit power. The results are then compared to analogous results without fading (appropriately reproduced from [7]). Finally, the penalty in system performance due to random spreading is also examined, by comparison (following [9]) to the spectral efficiency of a few corresponding detectors, in the setting in which *all* bandwidth is available for coding (as opposed to bandwidth expansion by DS-spreading).

The structure of this paper is as follows. Section 2 describes the system model. Section 3 presents the spectral efficiencies of the four multiuser detection strategies. Section 4 includes a summary of the obtained results. Finally, Section 5 ends the paper with some concluding remarks.

2. Multi-Cell System Model

Following [6] and [9], the uplink of a fully synchronous cellular CDMA system is considered, whose cells are ordered in an *infinite* linear array. Using the standard discrete time equivalent channel representation, the signal vector received at an arbitrary cell site, at the discrete time related to the transmission of the ith symbol, is given by

$$\boldsymbol{y}_i = \boldsymbol{S}_i \boldsymbol{H}_i \boldsymbol{x}_i + \alpha \boldsymbol{S}_i^- \boldsymbol{H}_i^- \boldsymbol{x}_i^- + \alpha \boldsymbol{S}_i^+ \boldsymbol{H}_i^+ \boldsymbol{x}_i^+ + \boldsymbol{n}_i \,. \quad \text{(2-1)}$$

The vector $\boldsymbol{x}_i = [x_{1,i}, \ldots, x_{K,i}]^T$ in (2-1) comprises the K code symbols transmitted by intra-cell users at the ith discrete time. The vectors $\boldsymbol{x}_i^\pm = \left[x_{1,i}^\pm, \ldots, x_{K,i}^\pm\right]^T$ denote the vectors of code symbols originated from users operating in adjacent cells. These symbols are assumed to be i.i.d., proper complex Gaussian (which conforms with the capacity achieving statistics), with $E\{x_{k,j}\} = 0$ and $E\{|x_{k,j}|^2\} = \bar{P}\ \forall k, j$, where $\bar{P}$ is the equal transmit power of all users. This model is justified by assuming that the codebooks of all users are chosen randomly, governed by an underlying i.i.d. Gaussian distribution per symbol, and *independently* for each message transmission (see [9]).

The matrices $\boldsymbol{S}_i$ and $\boldsymbol{S}_i^\pm$ are $N \times K$ matrices, whose columns are the N-chip long spreading (signature) sequences of the K users in the considered cell and in its adjacent cells, respectively. The entries of the above matrices are treated as i.i.d. zero mean random variables, with variance $1/N$. The vector $\boldsymbol{n}_i$ represents a zero mean white proper complex Gaussian noise vector, with $E\{\boldsymbol{n}_i \boldsymbol{n}_i^\dagger\} = \boldsymbol{I},\ \forall i$. Without loss of generality all received powers are thus normalized with respect to the noise spectral level, and represent in fact the signal to noise ratios (SNRs) at the input to the multiuser detectors.

Finally, $\boldsymbol{H}_i \triangleq \text{diag}\ (h_{1,i}, \ldots, h_{K,i})$ and $\boldsymbol{H}_i^\pm \triangleq \text{diag}\ (h_{1,i}^\pm, \ldots, h_{K,i}^\pm)$, where $\{h_{k,i}\}_{k=1}^K$ and $\left\{h_{k,i}^\pm\right\}_{k=1}^K$ designate the i.i.d., zero mean, complex Gaussian channel fading gains associated with the signals of the different users, at the ith discrete time. It shall be assumed henceforth that as the system size becomes large ($N, K \to \infty$, $\frac{K}{N} \to \beta < \infty$), the empirical distribution of the channel fading gains converges a.s. to a distribution $\mathcal{H}$. For regularity reasons it is assumed that $E_{\mathcal{H}}\left\{|h|^2\right\} = 1$ (where h denotes some arbitrary fading gain), and the fading power is henceforth denoted by $\nu \triangleq |h|^2$.

3. Spectral Efficiency of the Multiuser Detectors

The per cell *spectral efficiency* (see [1], [2]), or throughput, is defined as the total number of bits/sec/Hz that can be transmitted arbitrarily reliably in each cell. The spectral efficiency of a linear detector is conveniently expressed in terms its *multiuser efficiency* (see [10]), defined as the ratio between the detector's output SIR and the SNR (note that multiuser efficiency may depend, as is the case with the MMSE based detectors, on the presence of fading, see [2] and equation (3-6)). Denoting the multiuser efficiency by η, and following central limit results showing that the interference at the output of each of the two linear detectors, i.e., the matched-filter detector and the linear MMSE detector, is well approximated by a Gaussian noise (see [1], [7] and references therein for justification of this Gaussian approximation), the spectral efficiency of these detectors is given by (the logarithms in all expressions may be taken of arbitrary basis, however all numerical results in this paper correspond to base-2 logarithms)

$$\tilde{C} = \beta E_\nu \left\{ \log\left(1 + \nu\eta\bar{P}\right) \right\} , \tag{3-1}$$

which for Rayleigh fading can be expressed in terms of the exponential integral function $E_1(x) \triangleq \int_x^\infty \frac{e^{-t}}{t}\, dt$, $(t > 0)$, yielding

$$\tilde{C} = \beta \log e \, e^{\frac{1}{\bar{P}\eta}} E_1\left(\frac{1}{\bar{P}\eta}\right) . \tag{3-2}$$

$E_\nu\{\cdot\}$ in (3-1) denotes the expectation with respect to the fading power ν. The spectral efficiency of the two non-linear detectors, i.e., the SCO detector and the MMSE-SC detector, is most conveniently evaluated using inter-relations between the spectral efficiency of the optimum multiuser detector and that of the *linear* MMSE detector, as described below.

Due to space limitations, the following subsections only briefly outline how the spectral efficiency in Rayleigh-fading channels of all four multiuser detectors is evaluated. The interested reader is referred to [11] for more details on the derivation of the above results, and to [7] for the corresponding results in non-fading channels. The notation $(\cdot)_{\text{mf}}$, $(\cdot)_{\text{sco}}$, $(\cdot)_{\text{ms}}$, and $(\cdot)_{\text{m-sc}}$ is used to designate entries related to the matched-filter detector, the SCO detector, the linear MMSE detector, and the MMSE-SC detector, respectively. It is noted that when different systems are to be compared (with possibly different spreading gains and data rates), it is useful to express the spectral efficiency in terms of $\frac{E_b}{N_0}$, which is done through the relation $\bar{P} = \frac{1}{\beta}\, \tilde{C}\, \frac{E_b}{N_0}$ (see [2]). However, for simplicity of notations, equations are expressed in terms of the received power (which is in fact the SNR, following the normalization with respect to the noise spectral level).

3.1 The Matched-Filter Detector

In the limiting scenario considered here, the following result of [3] on the convergence of the multiuser efficiency of the matched-filter detector in a *single-cell* system holds.

Lemma 3.1 (Tse-Hanly [3]) *Let the empirical distribution of the received powers of all users converge a.s. as $K, N \to \infty$, $\frac{K}{N} \to \beta < \infty$, to some non-random limit $\mathcal{H}(P)$. Then, the multiuser efficiency of the matched-filter detector converges in probability to a non-random limit η_{mf}, equal for all users, given by*

$$\eta_{\text{mf}} = \frac{1}{1 + \beta E_{\mathcal{H}}\{P\}} , \tag{3-3}$$

where $E_{\mathcal{H}}\{\cdot\}$ represents expectation with respect to $\mathcal{H}(P)$.

Turning to the particular multi-cell model considered here, the matched-filter detector effectively operates in an *equivalent single-cell* system of $3K$ users, one third of which (the intra-cell users) are received at powers $\{\nu\bar{P}\}$, while the remaining two thirds (the adjacent-cell users) are received at powers $\{\alpha^2\nu\bar{P}\}$. Hence, using (3-2) and applying Lemma 3.1, the spectral efficiency of the matched-filter detector is given by:

$$\begin{aligned}\tilde{C}_{\text{mf}} &= \beta E_\nu \left\{ \log\left(1 + \frac{\bar{P}\nu}{1 + \beta\left(1 + 2\alpha^2\right)\bar{P}}\right) \right\} \\ &= \beta \log e \, e^{\left[\frac{1}{\bar{P}} + \beta(1+2\alpha^2)\right]} E_1\left[\frac{1}{\bar{P}} + \beta(1 + 2\alpha^2)\right] .\end{aligned} \tag{3-4}$$

3.2 The Linear MMSE detector

Following is a result from [3], as it is formulated in [2], that applies to single-cell systems and flat fading channels.

Lemma 3.2 (Tse-Hanly [3]) *Let the empirical distribution of the received powers of all users converge a.s. as $K, N \to \infty$, $\frac{K}{N} \to \beta < \infty$, to some non-random limit $\mathcal{H}(P)$. Then, the multiuser efficiency of the linear MMSE detector converges a.s. to a non-random limit η_{ms}, equal for all users, given by the unique positive solution to the implicit equation*

$$\eta_{\text{ms}} + \beta E_{\mathcal{H}}\left\{\left[\frac{\eta_{\text{ms}} P}{1 + \eta_{\text{ms}} P}\right]\right\} = 1 . \tag{3-5}$$

Using the single-cell $3K$-user system interpretation as in Subsection 3.1 above, the multiuser efficiency of the linear MMSE detector (in the multi-cell setting) is given by the unique positive solution to the implicit equation

$$\begin{aligned}1 &= \eta_{\text{ms}} + \beta E_\nu \left\{ \frac{\bar{P}\nu\eta_{\text{ms}}}{1 + \bar{P}\nu\eta_{\text{ms}}} + \frac{2\alpha^2\bar{P}\nu\eta_{\text{ms}}}{1 + \alpha^2\bar{P}\nu\eta_{\text{ms}}} \right\} \\ &= \eta_{\text{ms}} + \beta\Bigg[3 - \frac{1}{\bar{P}\eta_{\text{ms}}}\, e^{\frac{1}{\bar{P}\eta_{\text{ms}}}} E_1\left(\frac{1}{\bar{P}\eta_{\text{ms}}}\right) - \\ &\qquad \frac{2}{\alpha^2\bar{P}\eta_{\text{ms}}}\, e^{\frac{1}{\alpha^2\bar{P}\eta_{\text{ms}}}} E_1\left(\frac{1}{\alpha^2\bar{P}\eta_{\text{ms}}}\right)\Bigg] .\end{aligned} \tag{3-6}$$

The spectral efficiency of the detector is then evaluated by substituting the result in (3-2).

3.3 The SCO Detector

The following Lemma is a result of [2] referring to single-cell systems and flat-fading channels.

Lemma 3.3 (Shamai-Verdú, [2] Theorem IV.1) *Let the empirical distribution of the received powers of all users converge as in Lemma 3.2 to some non-random limit $\mathcal{H}(P)$. Then, the spectral efficiency of the optimum multiuser detector is given by*

$$\tilde{C}_{sco} = \tilde{C}^{s\text{-}c}_{ms} + \log \frac{1}{\eta^{s\text{-}c}_{ms}} + (\eta^{s\text{-}c}_{ms} - 1) \log e \ , \qquad (3\text{-}7)$$

where $\eta^{s\text{-}c}_{ms}$ is the limiting multiuser efficiency of the corresponding linear MMSE detector, as given by Lemma 3.2 (Eq. (3-5)), and

$$\tilde{C}^{s\text{-}c}_{ms} = \beta E_{\mathcal{H}} \left\{ \log(1 + \eta^{s\text{-}c}_{ms} P) \right\} \qquad (3\text{-}8)$$

is the spectral efficiency of this detector.

In the multi-cell setting considered in this paper, adding a mild restriction that the additive adjacent-cell interference is ergodic in second moment, the SCO detector is equivalent to an optimum detector in a *single-cell* system, where the additive white Gaussian background noise process has spectral level given by $1 + 2\beta\alpha^2\bar{P}$ (see [7] and references therein for justification). The spectral efficiency of the SCO detector is hence given by

$$\bar{C}_{sco} = \beta \log e \, e^{\frac{1}{\bar{P}_{eq}\eta^{s\text{-}c}_{ms}}} E_1 \left(\frac{1}{\bar{P}_{eq}\eta^{s\text{-}c}_{ms}} \right) + \log \frac{1}{\eta^{s\text{-}c}_{ms}} + (\eta^{s\text{-}c}_{ms} - 1) \log e \ , \quad (3\text{-}9)$$

with $\eta^{s\text{-}c}_{ms}$ being the unique positive solution of

$$1 = \eta^{s\text{-}c}_{ms} + \beta \left[1 - \frac{1}{\bar{P}_{eq}\eta^{s\text{-}c}_{ms}} e^{\frac{1}{\bar{P}_{eq}\eta^{s\text{-}c}_{ms}}} E_1 \left(\frac{1}{\bar{P}_{eq}\eta^{s\text{-}c}_{ms}} \right) \right] , \qquad (3\text{-}10)$$

and $\bar{P}_{eq} \triangleq \frac{\bar{P}}{1+2\beta\alpha^2\bar{P}}$.

3.4 The MMSE-SC Detector

It has been recently shown [7], that the spectral efficiency of the MMSE-SC detector can be expressed as a difference of the spectral efficiencies of *full* MMSE-SC detectors in two single-cell settings. The first single-cell setting is identical to the $3K$-users single-cell interpretation, as described in Subsection 3.1. The second setting is of a single cell with $2K$ users, received with powers $\{\alpha^2 \nu \bar{P}\}$. The key tool in the derivation, at this point, is the equivalence, in terms of spectral efficiency, between the (full) MMSE-SC detector and the optimum detector in a *single-cell environment*, i.e., when *all* received transmissions are to be decoded at the receiver (see [12], [1] and references therein).

Hence, Lemma 3.3 can be applied, and the spectral efficiency of the MMSE-SC detector is given by

$$\tilde{C}_{m\text{-}sc} = \beta \log e \Bigg[e^{\frac{1}{\bar{P}\eta_{\mathfrak{C}}}} E_1 \left(\frac{1}{\bar{P}\eta_{\mathfrak{C}}} \right) + 2 e^{\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{C}}}} E_1 \left(\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{C}}} \right) - 2 e^{\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}}} E_1 \left(\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}} \right) \Bigg] + \log \frac{\eta_{\mathfrak{S}}}{\eta_{\mathfrak{C}}} + (\eta_{\mathfrak{C}} - \eta_{\mathfrak{S}}) \log e \ , \quad (3\text{-}11)$$

where $\eta_{\mathfrak{C}}$ and $\eta_{\mathfrak{S}}$ are uniquely determined by solving the following two implicit equations, respectively,

$$1 = \eta_{\mathfrak{C}} + \beta \Bigg[3 - \frac{1}{\bar{P}\eta_{\mathfrak{C}}} e^{\frac{1}{\bar{P}\eta_{\mathfrak{C}}}} E_1 \left(\frac{1}{\bar{P}\eta_{\mathfrak{C}}} \right) - \frac{2}{\alpha^2\bar{P}\eta_{\mathfrak{C}}} e^{\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{C}}}} E_1 \left(\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{C}}} \right) \Bigg] , \quad (3\text{-}12)$$

and

$$1 = \eta_{\mathfrak{S}} + 2\beta \left[1 - \frac{e^{\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}}}}{\alpha^2\bar{P}\eta_{\mathfrak{S}}} E_1 \left(\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}} \right) \right] . \quad (3\text{-}13)$$

It is important to note at this point, that in terms of spectral efficiency the MMSE-SC detector is in fact the *optimum multiuser detector*, under the assumption of single cell-site processing, and the assumption that the receiver has *no knowledge of the codebooks used in the adjacent cells*, and that those codebooks are randomly selected per message (as is indeed assumed in the system model considered, see Section 2). This is evident by noticing the information preserving property of the MMSE estimator in the Gaussian regime. By exactly the same arguments used in [12] for the full MMSE-SC detector in a single-cell scenario, the sum of rates attained in the present model by the successive cancellation process, can be shown to correspond to the chain decomposition rule for mutual information. Hence, the MMSE-SC detector achieves the maximum mutual information between the channel output and the channel input due to *intra*-cell users *only* (see (2-1)), with the above restrictions on information available with respect to adjacent-cell interference. It is noted, however, that in the case in which the codebooks of adjacent-cell users *are* known or chosen once for good, the MMSE-SC is *no longer* the optimum receiver, as the problem falls within the difficult framework of joint multiple-access/interference channels.

4. Summary of Results

Examining the spectral efficiency results of Section 3, it can be seen that both the matched-filter detector and the SCO detector are interference limited, i.e., their spectral efficiencies reach a limit as $\frac{E_b}{N_0}$ grows without bound. It is also observed that in terms of spectral efficiency, it is optimum, using the above two detectors, to increase the system load β to infinity (see also [2] and [7]). In such case, the effect of fading is *eliminated* and the spectral efficiency of the detectors coincides with that attained in *non-fading* channels. This result holds, in fact, regardless of the fading distribution. Increasing the system load without bound also eliminates the penalty due to the use of random spreading. This is observed by comparison to the spectral efficiency of the SCO detector, and that of a detector equivalent to the matched-filter detector, when no spreading is employed and *all* bandwidth is available for coding, as given in [9] (it is noted that the effect of fading in the no-spreading setting is also eliminated in the infinite number of users regime).

In contrast to the above two detectors, the linear MMSE detector and the MMSE-SC detector are not interference limited, provided that the system load β is appropriately chosen. For low $\frac{E_b}{N_0}$, it is optimum, with both detectors, to increase the system load without bound. In such case, the spectral efficiencies of the linear MMSE and MMSE-SC detectors coincide, respectively, with those of the matched-filter and SCO detectors (this equivalence holds for $\beta \to \infty$ regardless of $\frac{E_b}{N_0}$). However, beyond some critical $\frac{E_b}{N_0}$ the optimum system load starts to decrease from infinity, eventually becoming lower than $\frac{1}{3}$, and the spectral efficiencies of both detectors grow without bound with $\frac{E_b}{N_0}$.

Comparative spectral efficiency results of all four multiuser detectors are plotted in Fig. 1 for the *optimum choice of system load* β. The interference factor α was set to $\frac{1}{2}$ to mimic the case in which the average inter-cell interference power equals one half of the average power of intra-cell transmissions ($2\alpha^2 = \frac{1}{2}$), which is in agreement with the early reports on IS-95 systems. The above described behavior of the spectral efficiency of the detectors is clearly observed in the figure. The sort of "knee effect" in the linear MMSE and MMSE-SC curves designates the region in which the optimum choice for system load starts to decrease from infinity. It is also observed that beyond some critical $\frac{E_b}{N_0}$, the relatively simpler *linear* MMSE detector, which is more informed regarding adjacent-cell interference, is preferable over the interference limited SCO detector which employs *non-linear* detection of intra-cell users, while treating inter-cell interference as noise. For the sake of comparison, the spectral efficiencies obtained in non-fading channels are also provided in Fig. 1, demonstrating the performance degradation with both linear MMSE and MMSE-SC detectors due to the presence of fading. It is noted however that for the MMSE type detectors and a *fixed* system load $\beta > \frac{1}{3}$, Rayleigh fading becomes, in fact, *beneficial* in terms of spectral efficiency beyond some critical (β depended) $\frac{E_b}{N_0}$, and the spectral efficiency *with* fading surpasses that of a non-fading channel. This result is explained by the "interference population control" effect of fading, effectively reducing the system load as seen by the receiver (see an elaboration on this phenomena in [2]). This behavior is demonstrated in Fig. 2, where the spectral efficiency of the linear MMSE detector is plotted taking $\beta = 0.2 < \frac{1}{3}$, for which the spectral efficiency without fading always surpasses the spectral efficiency in Rayleigh fading channels, and taking $\beta = 0.5 > \frac{1}{3}$, for which Rayleigh fading becomes beneficial beyond some critical $\frac{E_b}{N_0}$. Corresponding results for the MMSE-SC detector are of similar nature.

To complete the comparison (in view of [7]), the spectral efficiency of what is referred to in [9] as the *adjacent-cell decoder (ACD)* was also considered. This detector, that *employs no-spreading*, also knows the *codebooks* of users in adjacent cells, and either decodes their transmissions, or treats them as additive Gaussian noise, whichever is preferable in terms of spectral efficiency. The spectral efficiency of this detector is given by (note that here $K = \beta$)

$$\tilde{C}^{\text{n.s.}}_{\text{ACD}} = \max\Bigg[\log\left(1 + \frac{K\bar{P}}{1 + 2K\alpha^2\bar{P}}\right), \min\left(\frac{1}{3}\log\left(1 + (1 + 2\alpha^2)K\bar{P}\right), \frac{1}{2}\log\left(1 + 2\alpha^2 K\bar{P}\right)\right)\Bigg], \quad (4\text{-}1)$$

and the corresponding numerical results are provided in Fig. 1. As can be seen, for low $\frac{E_b}{N_0}$ it is preferable not to decode adjacent-cell transmissions, and the spectral efficiency of this detector coincides with that of the SCO, and MMSE-SC detectors, for the optimum choice of β (which is $\beta \to \infty$). However beyond some critical $\frac{E_b}{N_0}$, where decoding is preferable, the curves depart and the spectral efficiency of the ACD grows quite rapidly with $\frac{E_b}{N_0}$ (as compared to the other detectors).

5. Concluding Remarks

This paper demonstrates the dramatic effect of information about interfering signals on system performance. The effect is most clearly seen by comparing the linear MMSE detector and the SCO detector, which represent a tradeoff between intra-cell transmissions processing complexity, and additional information on adjacent-cell interference. It was shown that one can gain even without trying to decode the transmissions of the interfering users in adjacent cells (which enables interference *cancellation*), or treating them optimally in the setting of an interference channel (see [9]). The gain emerges by the very fact that

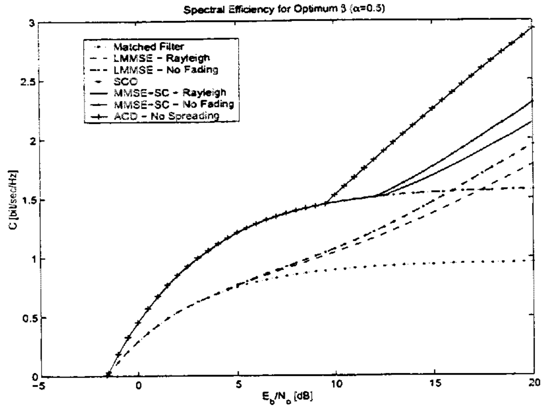

Figure 1. Spectral efficiency comparison for $\alpha = \frac{1}{2}$, and optimized system load β.

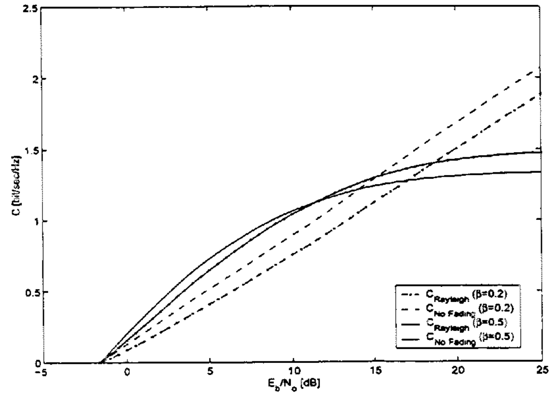

Figure 2. Spectral efficiency of the linear MMSE detector, with β fixed and $\alpha = \frac{1}{2}$.

the linear MMSE filter accounts for the reduction of interference, provided that the signatures of interfering users are known not only at the intended cell-site, but at those cell-sites where they cause interference. It may be concluded that for high data rates, inherently demanding high $\frac{E_b}{N_0}$, it is advantageous to mitigate out-of-cell interference through linear MMSE processing.

Assuming *equal* transmit powers, it was shown that the matched-filter and SCO detectors are asymptotically unaffected by the presence of fading, in the large (optimum) system load region. In contrast, the linear MMSE and the MMSE-SC detectors experience performance degradation, when fading is present, at the high $\frac{E_b}{N_0}$ region where the system load β is lower than $\frac{1}{3}$. However when *fixing* $\beta > \frac{1}{3}$, both detectors *benefit* from the presence of fading due to its "population control" effect, and attain higher spectral efficiency, as compared to the non-fading case, beyond a critical $\frac{E_b}{N_0}$ value.

Finally, it is noted that the analysis of the *optimum* power control policy in the presence of fading, as well as the optimum and suboptimum multi-cell-site processing detectors (see [6], [8]), are currently investigated.

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