

# Multiuser Detection with Random Spreading and Error-Correction Codes: Fundamental Limits

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## Abstract

The CDMA channel with randomly and independently chosen spreading sequences accurately models the situation where pseudo-noise sequences span many symbol periods. Its analysis provides a comparison baseline for CDMA channels with carefully designed signature waveforms that span one bit period. We analyze the spectral efficiency (total capacity per chip) as a function of the number of users, spreading gain and signal-to-noise ratio, and we quantify the loss in efficiency relative to an optimally chosen set of signature sequences and to a multiaccess system without spreading. White Gaussian background noise and equal-power synchronous users are assumed. The following receivers are analyzed: a) optimal joint processing, b) single-user matched filtering; c) decorrelation and d) MMSE linear processing.

## I Spectral Efficiency

Each signature in a direct-sequence spread-spectrum code division multiple access system can be viewed as a unit-norm vector in an  $N$ -dimensional signal space, where  $N$  is the spreading gain or number of chips per symbol. In the model considered in this paper,  $K$  users linearly modulate their signatures with the outputs of respective autonomous encoders which map information bits to channel symbols. The central question we address is the capacity loss incurred by the imposition of such a structure on the transmitted signals and by the adoption of several popular multiuser detectors. The fundamental figure of merit in a coded spread spectrum system is the *spectral efficiency*,  $C$ , defined as the total number of bits per chip that can be transmitted arbitrarily reliably. Since the bandwidth of the CDMA system is (roughly) equal to the reciprocal of the chip duration, the spectral efficiency can be viewed as the bits/s/Hz supported by the system. Note that if the individual code rates (bits/symbol) are identical and denoted by  $R$ , then the spectral efficiency is equal to the product:

$$C = \frac{K}{N} R. \quad (1)$$

In a system where no spreading is imposed, the encoders are able to control the symbols modulating each chip independently. Therefore, assuming chip-synchronism, the Cover-Wyner capacity region of the conventional Gaussian multiaccess channel [5] applies to this case and the spectral efficiency in the absence of spreading is given by

$$C^* = \frac{1}{2} \log\left(1 + \frac{K}{N} \text{SNR}\right). \quad (2)$$

where, for consistency with the results below, SNR denotes the energy per transmitted  $N$  chips divided by the Gaussian noise spectral level,  $\sigma^2$ . This means that the energy per

bit divided by  $N_0 = 2\sigma^2$  is

$$\frac{E_b}{N_0} = \frac{\text{SNR}}{2R}. \quad (3)$$

Once the spectral efficiency is determined, it is possible to obtain the minimum bandwidth necessary to transmit a predetermined information rate or the maximum information rate that can be supported by a given bandwidth. In order to compare different systems (with possibly different spreading gains and data rates), the spectral efficiency must be given as a function of  $\frac{E_b}{N_0}$ . According to (1) and (3), if the system operates at full capacity, then SNR in (2) can be substituted by

$$\text{SNR} = \frac{2N}{K} \frac{E_b}{N_0} C^*$$

so the maximum spectral efficiency  $C^* \left( \frac{E_b}{N_0} \right)$  in the absence of spreading is the solution to

$$C^* \left( \frac{E_b}{N_0} \right) = \frac{1}{2} \log \left( 1 + 2C^* \left( \frac{E_b}{N_0} \right) \frac{E_b}{N_0} \right) \quad (4)$$

or equivalently

$$\frac{4^{C^* \left( \frac{E_b}{N_0} \right)} - 1}{2C^* \left( \frac{E_b}{N_0} \right)} = \frac{E_b}{N_0}.$$

The solution is well-known to satisfy [3]

$$C^* \left( \frac{E_b}{N_0} \right) > 0 \quad \text{if and only if} \quad \frac{E_b}{N_0} > \log_e 2 = -1.6 \text{ dB},$$

and

$$\lim_{\frac{E_b}{N_0} \rightarrow \infty} \frac{C^* \left( \frac{E_b}{N_0} \right)}{10 \log_{10} \frac{E_b}{N_0}} = \frac{\log 10}{20} = 0.166 \text{ bits/dB}. \quad (5)$$

Since (4) does not depend on  $K$ , when the transmitted signals are not constrained to the spread-spectrum format, the spectral efficiency is the same as in a single-user system.

The total capacity (sum-rate) of the power-constrained synchronous CDMA channel

$$y(t) = \sum_{k=1}^K \sum_{i=1}^n A_k b_k[i] s_k(t - iT) + n(t),$$

is equal to [25]

$$\frac{1}{2} \log \left( \det \left[ \mathbf{I} + \sigma^{-2} \mathbf{A} \mathbf{R} \mathbf{A} \right] \right).$$

where  $\mathbf{A} = \text{diag}\{A_1, \dots, A_K\}$ , and  $\mathbf{R}$  is the matrix of normalized crosscorrelations.

If the users have equal power, then  $A_k = A$ ,  $\text{SNR} = A^2/\sigma^2$ , and the optimum spectral efficiency is equal to

$$C^{\text{opt}}(\text{SNR}, \mathbf{R}, K, N) = \frac{1}{2N} \log \left( \det \left[ \mathbf{I} + \text{SNR} \mathbf{R} \right] \right) \quad (6)$$

For example, in the case of orthogonal sequences the spectral efficiency is equal to

$$C^{\text{orth}} = \frac{K}{2N} \log(1 + \text{SNR}), \quad \text{if } K \leq N. \quad (7)$$

Substituting

$$\text{SNR} = C^{\text{orth}} \frac{2N E_b}{K N_0},$$

we obtain that if  $K \leq N$ , then

$$C^{\text{orth}} \left( \frac{K}{N}, \frac{E_b}{N_0} \right) = \frac{K}{N} C^* \left( \frac{E_b}{N_0} \right). \quad (8)$$

The equality of  $C^{\text{orth}}$  and  $C^*$  for  $K = N$  is a consequence of the well-known fact [5] that for equal-rate equal-power users orthogonal multiple access incurs no loss in capacity relative to unconstrained multiple access. It is also known [16] that even if  $K > N$ , there exist spreading codes (*Welch-Bound-Equality*) which incur no loss in capacity.

Optimal spectral efficiency in non-orthogonal CDMA requires joint processing and decoding of users. As advocated in a number of recent works [17, 6, 18, 1, 23, 10, 26, 4], it is sensible in terms of complexity-performance tradeoff to adopt as a front-end a (soft-output) *multiuser detector* [24] followed by autonomous single-user error-control decoders. In our analysis of spectral efficiency we consider, in addition to optimal decoding, some popular linear multiuser detector front-ends:

- Single-user matched filter.
- Decorrelator.
- Linear Minimum Mean-Square-Error (MMSE).

Our purpose is to evaluate the spectral efficiency of CDMA systems where *signature waveforms are assigned at random*. Denote the unit-norm signature of the  $k$ th user by  $[c_{k1}, \dots, c_{kN}]$ , and assume that  $c_{kj} \in \{-1/\sqrt{N}, +1/\sqrt{N}\}$  are chosen equally likely and independently for all  $(k, j)$ . The rationale for averaging capacity with respect to random signature waveforms is twofold:

- It accurately models CDMA systems (such as IS-95 [20]) where pseudo-noise sequences span many symbol periods.
- The spectral efficiency averaged with respect to the choice of signatures provides a lower bound to the optimum spectral efficiency achievable with a deterministic choice of signature waveforms.

In our asymptotic (in  $K$ ) analysis we do not just average spectral efficiency with respect to the spreading sequences, but we show convergence of the spectral efficiencies to deterministic quantities.

Most analyses of multiuser detectors have focused on the bit-error-rate of uncoded communication ([24]). The results found on this paper give the best achievable performance with error-control coding assuming random signature waveforms (Figure 1). This serves as a lower bound to the performance achievable through careful design of signatures with low-crosscorrelations. Furthermore this analysis is directly applicable to multiuser detectors operating with spreading codes whose periodicity is much larger than the spreading gain (e.g. [27],[13]).

## II Optimal Decoding

Optimal decoding can be performed by a bank of matched filters (which converts the received process to a discrete-time vector process) followed by joint maximum likelihood decoding of the error control code. The formula in [25] for capacity as a function of the signature waveforms was used in [16] to show that if so-called Welch-bound-equality signature waveforms are assigned, then the spectral efficiency of the CDMA system is equal to the case of no-spreading (2); a necessary condition for the existence of such signature waveforms is  $K \geq N$ . It had been conjectured in [14] that as  $K/N \rightarrow \infty$  and  $\text{SNR} \rightarrow \infty$  the loss incurred by a random choice of signatures vanishes. This was verified independently by MonteCarlo simulation in [26] and with an asymptotic  $K \geq N \rightarrow \infty$  lower bound on the average capacity for random signature waveforms in [7, 8]. We have shown that the gain in optimum spectral efficiency achievable by allocating instantaneous power as a function of the actual spreading waveform employed by other users is negligible. Henceforth, all the results assume that power is not allocated dynamically depending on the realization of  $\mathbf{R}$ . When  $K = N = 2$ , we have shown that binary

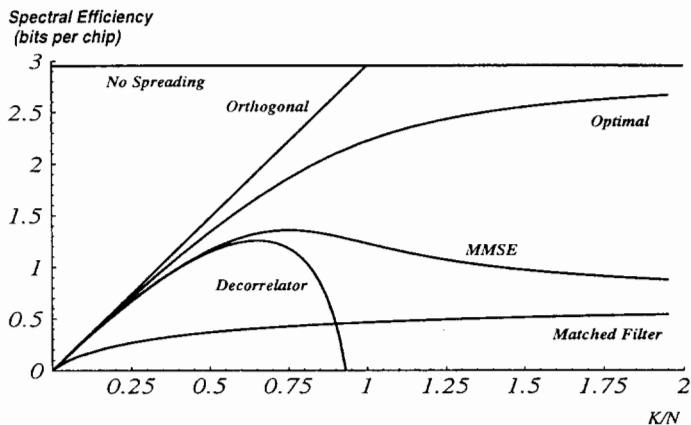


Figure 1: Large- $K$  Spectral Efficiencies for  $\frac{E_b}{N_0} = 10$  dB.

random sequences achieve 75% of the spectral efficiency of orthogonal sequences. Using recent results on the asymptotic spectral distribution of random covariance matrices [2] we have shown the following result (proofs are omitted in this conference version because of space limitations) in the asymptotic case of  $K \rightarrow \infty$  and  $K = \beta N$ .

**Proposition II.1.** *Let  $\beta > 0$ , then the following limit holds almost surely*

$$\lim_{K \rightarrow \infty} C^{\text{opt}}(\text{SNR}, \mathbf{R}, K, K/\beta) = \frac{1}{4\pi} \int_{a(\beta)}^{b(\beta)} \log(1 + \text{SNR}x) \sqrt{(b(\beta) - x)(x - a(\beta))} \frac{dx}{x}, \quad (9)$$

where  $a(\beta) = (\sqrt{\beta} - 1)^2$ , and  $b(\beta) = (\sqrt{\beta} + 1)^2$ .

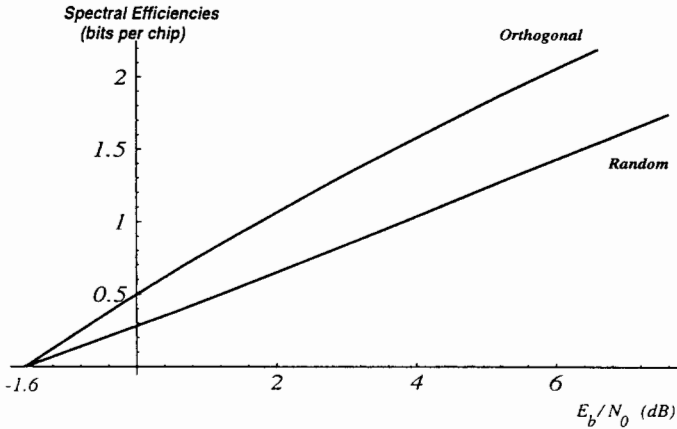


Figure 2: Optimum Spectral Efficiencies with Orthogonal and Random Sequences  $K = N \rightarrow \infty$ .

In terms of  $\frac{E_b}{N_0}$ , the spectral efficiency is the solution to

$$C^{\text{opt}}\left(\beta, \frac{E_b}{N_0}\right) = \frac{1}{4\pi} \int_{a(\beta)}^{b(\beta)} \log\left(1 + \frac{2}{\beta} \frac{E_b}{N_0} C^{\text{opt}}\left(\beta, \frac{E_b}{N_0}\right) x\right) \sqrt{(b(\beta) - x)(x - a(\beta))} \frac{dx}{x} \quad (10)$$

It can be checked from (10) that  $C^{\text{opt}}\left(\beta, \frac{E_b}{N_0}\right)$  satisfies

$$C^{\text{opt}}\left(\beta, \frac{E_b}{N_0}\right) = \beta C^{\text{opt}}\left(\frac{1}{\beta}, \frac{E_b}{N_0}\right). \quad (11)$$

By manipulation of (10) it can be checked that the behavior for small  $\beta$  is

$$\lim_{\beta \rightarrow 0} \frac{1}{\beta} C^{\text{opt}}\left(\beta, \frac{E_b}{N_0}\right) = C^* \left(\frac{E_b}{N_0}\right). \quad (12)$$

The limit for large  $\beta$  follows immediately from (11) and (12):

$$\lim_{\beta \rightarrow \infty} C^{\text{opt}}\left(\beta, \frac{E_b}{N_0}\right) = C^* \left(\frac{E_b}{N_0}\right). \quad (13)$$

It is straightforward to show that as  $\frac{E_b}{N_0} \rightarrow \infty$  the slope of the spectral efficiency achievable with random sequences as a function of  $\frac{E_b}{N_0}$  coincides with (5) for  $\beta \geq 1$ . In Figure 2 we have shown  $C^{\text{opt}}\left(1, \frac{E_b}{N_0}\right)$  and  $C^{\text{orth}}\left(1, \frac{E_b}{N_0}\right)$ . The slopes of both curves with the logarithm of  $\frac{E_b}{N_0}$  are asymptotically equal. However, there is a gap between both curves which can be shown to be half a nat (0.72 bits) asymptotically.

If  $\frac{E_b}{N_0}$  is close to its lower limit of  $\log_e(2) = -1.6$  dB, then random sequences only achieve 50% of the spectral efficiency of orthogonal sequences. Figure 3 displays the percentage of the spectral efficiency of optimum sequences which is achieved by random sequences. When  $K$  is large, the loss in spectral efficiency as a function of  $E_b/N_0$  due to a random choice of sequences (as opposed to optimal) vanishes as  $E_b/N_0 \rightarrow \infty$  or as  $\beta \rightarrow \infty$ . The maximum loss is 50% and occurs at  $K = N$ ,  $E_b/N_0 \downarrow \log_e 2$ .

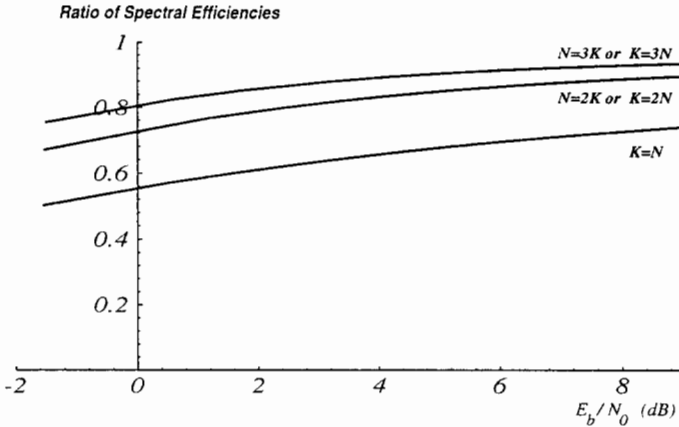


Figure 3: Optimal Processing: Spectral Efficiency with random signatures divided by spectral efficiency with optimally chosen signatures.

### III Single-User Matched Filter

The output of the matched filter of user 1 is the following discrete-time process:

$$y_1[i] = Ab_1[i] + A \sum_{j=2}^K \rho_{1j} b_j[i] + \sigma n_1[i] \quad (14)$$

where  $\{n_1[i]\}$  is an independent Gaussian sequence with unit variance and  $\{b_k[1], \dots, b_k[n]\}$  is the input codeword of user  $k$ . A rigorous analysis of this important channel has not been undertaken previously. Approximating the interference as an independent Gaussian sequence, [11] argued that the spectral efficiency as  $K/N \rightarrow \infty$  goes to 0.5 nat/sec/Hz = 0.72 bit/sec/Hz when the signatures are antipodally modulated. Achieving the capacity of (14) requires the receiver of user 1 to know the crosscorrelations and input distributions of all the interferers. However, the following result (proved using a recent central limit theorem [12]) shows that that information becomes useless as the number of users grows without bound. Furthermore, no averaging of capacity with respect to the random signature sequences is required.

**Proposition III.1.** Let  $C(\rho_{12}, \dots, \rho_{1K}, p_{b_2}, \dots, p_{b_K})$  denote the capacity of the single-user channel (14) where the input codeword  $\{b_1[1], \dots, b_1[n]\}$  is constrained to satisfy

$$\frac{1}{n} \sum_{i=1}^n b_i^2[i] \leq 1,$$

the random variables  $\{b_j[i]\}$  are independent with distribution  $p_{b_k}$ ,  $E\{b_j^2[i]\} = 1$ , and  $n_1[i]$  is Gaussian with unit variance. Then, as  $\beta N = K \rightarrow \infty$

$$C(\rho_{12}, \dots, \rho_{1K}, p_{b_2}, \dots, p_{b_K}) \xrightarrow{a.s.} \frac{1}{2} \log \left( 1 + \frac{A^2}{\sigma^2 + \beta A^2} \right). \quad (15)$$

Equation (15) gives the capacity per user and per symbol ( $N$  chips). To obtain the spectral efficiency, all we need to do is multiply by  $K$  and divide by  $N$ . Recalling that the energy per symbol divided by the noise spectral level is  $\text{SNR} = A^2/\sigma^2$ , we obtain that the asymptotic spectral efficiency for the single-user matched filter as a function of  $\beta = K/N$  and SNR is given by

$$C^{\text{sumf}} = \frac{\beta}{2} \log \left( 1 + \frac{\text{SNR}}{1 + \text{SNR}\beta} \right).$$

Upon substitution of

$$\text{SNR} = C^{\text{sumf}} \frac{2 E_b}{\beta N_0},$$

we obtain that the asymptotic spectral efficiency of the single-user matched filter  $C^{\text{sumf}}(\beta, \frac{E_b}{N_0})$ , is the solution to (cf. [15])

$$\frac{E_b}{N_0} = \frac{1}{2 C^{\text{sumf}}(\beta, \frac{E_b}{N_0})} \frac{4 C^{\text{sumf}}(\beta, \frac{E_b}{N_0})/\beta - 1}{\frac{1}{\beta} + 1 - 4 C^{\text{sumf}}(\beta, \frac{E_b}{N_0})/\beta}. \tag{16}$$

It follows from (16) that  $C^{\text{sumf}}(\beta, \frac{E_b}{N_0})$  is monotonically increasing with  $\beta$ , and

$$\lim_{\beta \rightarrow \infty} C^{\text{sumf}}(\beta, \frac{E_b}{N_0}) = \frac{\log_2 e}{2} - \frac{1}{2} \frac{N_0}{E_b} \quad \frac{E_b}{N_0} > \log_e 2. \tag{17}$$

which is seen to converge to one half nat as  $\frac{E_b}{N_0} \rightarrow \infty$ . The use of random signatures as opposed to optimally chosen sequences brings about substantial losses in spectral efficiency for the single-user matched filter, unless  $\frac{E_b}{N_0}$  is relatively low and  $K/N$  is high.

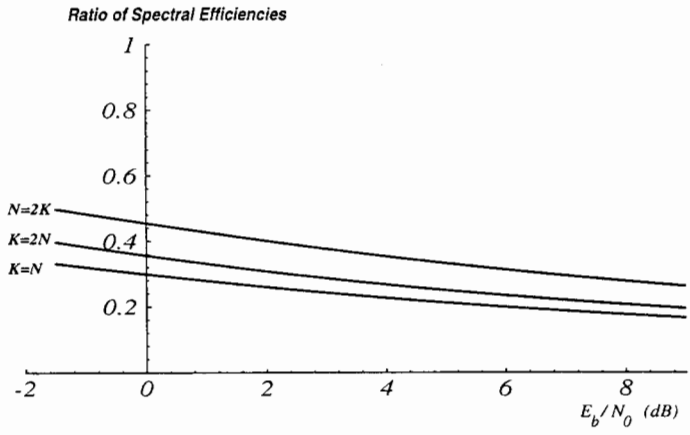


Figure 4: Single-user Matched Filtering: Spectral Efficiency with random signatures divided by spectral efficiency with optimally chosen signatures.

Figure 4 shows the loss due to a random choice of signatures relative to optimally designed sequences. Recall that if  $K \leq N$ , orthogonal sequences are optimal, and if

$K = mN$ , they remain optimal provided each is assigned to  $m$  users. In either case, the single-user matched filter is an optimal front-end. The ratio of spectral efficiencies is monotonically decreasing with  $\frac{E_b}{N_0}$ . At  $\frac{E_b}{N_0} \downarrow -1.6\text{dB}$ , the ratio is  $1/3$  at  $K = N$ , and higher for any other  $K/N$ .

We see in Figure 1 that the single-user matched filter is distinctly suboptimal in the scenario of perfect power control, large number of users, error control coding and random signature waveforms. The potential of multiuser detection in that scenario had been questioned in [22] (cf. [23]).

## IV Decorrelator

A front-end consisting of a bank of decorrelators [24] incurs no loss of information since it is a one-to-one transformation of the sufficient statistics and eliminates multiaccess interference from each of its outputs. Optimal decoding still requires joint processing of all  $K$  outputs due to the correlation among the noise components. The spectral efficiency of this method is maximum. The point of studying capacity with a decorrelating front-end is that it lends itself naturally to a suboptimal approach in which single-user decoding is based on each individual decorrelator (unquantized) output. Since the output of each single-user decorrelator is uncontaminated by multiaccess interference, the analysis of the single-user decorrelator capacity requires the single-user capacity formula evaluated at the decorrelator output signal-to-noise ratio, which is equal to the maximum near-far resistance,  $\bar{\eta}$ , [24]. Bounds and MonteCarlo simulation of capacity using the decorrelator were given in [26] and [4]. Random sequences with complex-valued chips where sequences are uniformly distributed on the surface of the unit-radius  $N$ -dimensional sphere are considered in [15, 19] where an expression is found for the density function of the maximum near-far resistance as a function of  $K$  and  $N$ . We have shown the following result:

**Proposition IV.1.** *For  $\beta \leq 1$  and binary random spreading, the spectral efficiency of the decorrelator converges in mean-square as  $K \rightarrow \infty$  to*

$$\lim_{K \rightarrow \infty} C^{\text{deco}} = \frac{\beta}{2} \log(1 + \text{SNR}(1 - \beta)), \tag{18}$$

Upon comparison to (4), we get

$$C^{\text{deco}}\left(\beta, \frac{E_b}{N_0}\right) = \beta C^* \left( (1 - \beta) \frac{E_b}{N_0} \right). \tag{19}$$

Notice that (cf. Figure 1)  $C^{\text{deco}} = 0$  if

$$\frac{E_b}{N_0} \leq \frac{\log_e 2}{1 - \beta}.$$

Since the decorrelator is an optimum front-end in the case of orthogonal sequences, the loss due to the use of random sequences is given by (via (8) and (19))

$$\gamma^{\text{deco}}\left(\beta, \frac{E_b}{N_0}\right) \stackrel{\text{def}}{=} \frac{C^{\text{deco}}\left(\beta, \frac{E_b}{N_0}\right)}{C^{\text{orth}}\left(\beta, \frac{E_b}{N_0}\right)} = \frac{C^* \left( (1 - \beta) \frac{E_b}{N_0} \right)}{C^* \left( \frac{E_b}{N_0} \right)} \quad \text{if } \beta \leq 1$$

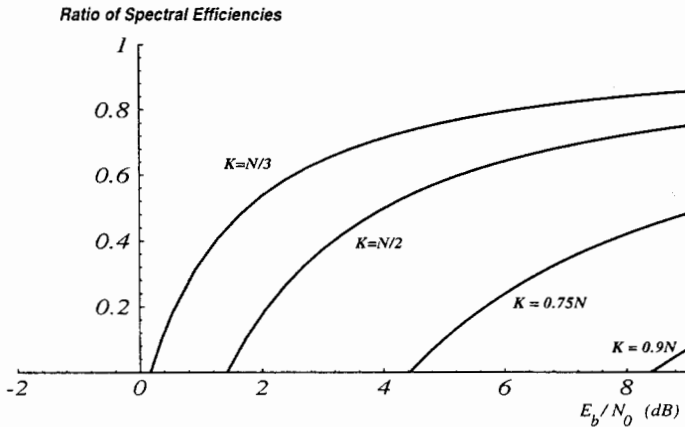


Figure 5: Decorrelator: Spectral Efficiency with random signatures divided by spectral efficiency with optimally chosen signatures.

which is shown in Figure 5. Comparing this figure to Figure 3 we can see that for high  $\frac{E_b}{N_0}$  and low  $\beta$  the decorrelator achieves near optimal spectral efficiency (see also Figure 1). As should be expected and in contrast to the single-user matched filter, the suboptimality of the random choice decreases with  $\frac{E_b}{N_0}$ .

## V MMSE Receiver

The linear MMSE receiver [24] offers a compromise between the multiaccess suppression capabilities of the decorrelator and the optimal background-noise combating capabilities of the single-user matched filter. Unlike the decorrelator, the MMSE filter is well-defined regardless of whether  $K$  is smaller or larger than  $N$ . As in the case of the decorrelator, we are interested in the spectral efficiency of the MMSE linear transformation followed by single-user decoding. When the channel symbols are binary and binary decisions are made at the output of the linear transformation, [21] shows that the spectral efficiency (of both the matched filter and of the MMSE transformation) tends to 0.46 bits/sec/Hz as  $K/N \rightarrow \infty$  in the synchronous case and to 0.69 bits/sec/Hz in the asynchronous case. MonteCarlo simulation of the expected capacity with binary sequences and (nonbinary) power-constrained codewords and the MMSE transformation was given in [26]. We have obtained the following result:

**Proposition V.1.** *For  $\beta > 0$  and binary random spreading, the spectral efficiency of the MMSE receiver converges in mean-square as  $K \rightarrow \infty$  to*

$$\lim_{K \rightarrow \infty} C^{\text{mmse}} = \frac{\beta}{2} \log \left( 1 + \text{SNR} - \frac{1}{4} \mathcal{F}(\text{SNR}, \beta) \right)$$

where

$$\mathcal{F}(x, z) \stackrel{\text{def}}{=} \left( \sqrt{x(1 + \sqrt{z})^2 + 1} - \sqrt{x(1 - \sqrt{z})^2 + 1} \right)^2.$$

Let us study the behavior of the spectral efficiency of the MMSE receiver for asymptotically large  $\frac{E_b}{N_0}$ . For  $0 < \beta < 1$ , the MMSE and decorrelator spectral efficiencies coincide (cf. Figure 7) :

$$\lim_{\frac{E_b}{N_0} \rightarrow \infty} C^{\text{mmse}} \left( \beta, \frac{E_b}{N_0} \right) - C^{\text{deco}} \left( \beta, \frac{E_b}{N_0} \right) = 0.$$

If  $\beta = 1$ , then it can be shown that

$$\lim_{\frac{E_b}{N_0} \rightarrow \infty} \frac{C^{\text{mmse}} \left( 1, \frac{E_b}{N_0} \right)}{C^* \left( \frac{1}{2} \frac{E_b}{N_0} \right)} = \frac{1}{2}.$$

If  $\beta > 1$ , then it can be shown that

$$\lim_{\frac{E_b}{N_0} \rightarrow \infty} C^{\text{mmse}} \left( \beta, \frac{E_b}{N_0} \right) = \frac{\beta}{2} \log \frac{\beta}{\beta - 1}.$$

The asymptotic behavior of the spectral efficiency of the MMSE receiver with  $\beta \rightarrow \infty$  is identical to that of the single-user matched filter:

$$\lim_{\beta \rightarrow \infty} C^{\text{mmse}} \left( \beta, \frac{E_b}{N_0} \right) = \frac{\log_2 e}{2} - \frac{1}{2} \frac{N_0}{E_b} \quad \frac{E_b}{N_0} > \log_e 2. \quad (20)$$

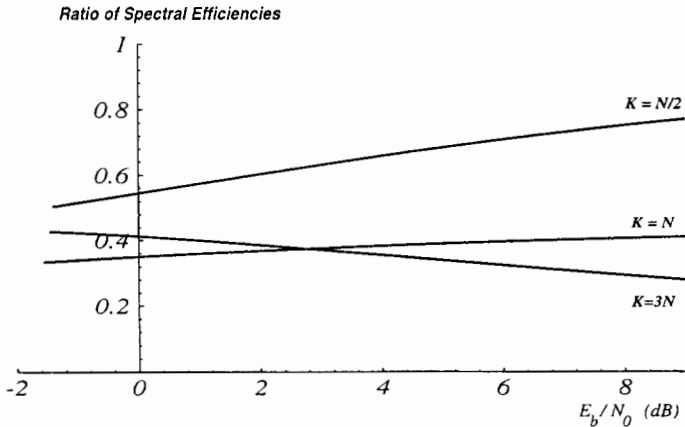


Figure 6: MMSE Receiver: Spectral Efficiency with random signatures divided by spectral efficiency with optimally chosen signatures.

Comparing Figure 6 to Figure 3 we see that a random choice of spreading codes (as opposed to optimal) brings about substantial losses for  $K = 3N$ . For  $K = N$ , and

in the range of  $\frac{E_b}{N_0}$  considered in Figure 6 a random choice achieves around 40% of the spectral efficiency achieved by orthogonal sequences. As the spreading gain  $N$  increases, the MMSE detector loss is more important at low  $\frac{E_b}{N_0}$  and approaches that in Figures 3 and 5 for large  $\frac{E_b}{N_0}$ . The deleterious effect of low  $\frac{E_b}{N_0}$  on the decorrelator (Figure 5) is not suffered by the MMSE receiver. Relative to Figure 4, we see that at low  $\frac{E_b}{N_0}$  the MMSE and single-user matched filter behave similarly; at high  $\frac{E_b}{N_0}$  the comparison depends heavily on  $K/N$ .

When  $K/N$  is large, linear multiuser detection is distinctly suboptimal (Figure 1); however, for lightly loaded systems, the performance improvement afforded by nonlinear techniques is quite small.

## VI Optimum Coding-Spreading Tradeoff

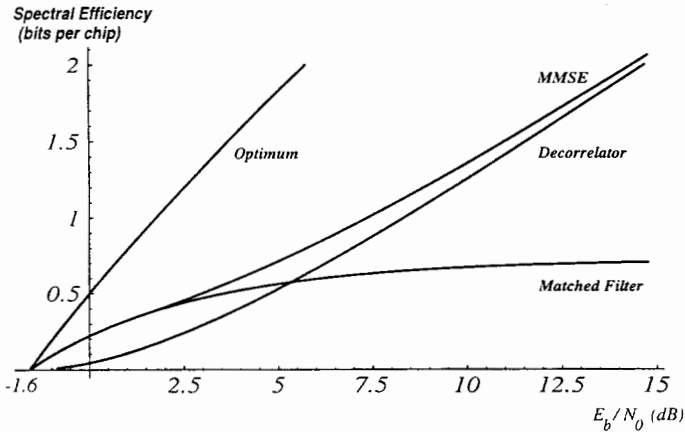


Figure 7: Large- $K$  Spectral Efficiencies with Optimum  $K/N$ .

When the spreading gain  $N$  is a free design parameter, it is of course interesting to analyze the  $N$  that optimizes the spectral efficiency with random spreading. The answer, as we can see in Figure 1, depends heavily on the type of receiver. For either optimum processing or matched-filtering followed by single-user decoding, spectral efficiency is maximized by letting  $K/N \rightarrow \infty$ . Thus for those receivers, the coding-spreading tradeoff favors coding: it is best to use error-correcting codes with very low rates (cf. (1)) and a negligible spreading gain with respect to the number of users. This conclusion was known to hold for the single-user matched filter [11]. Note, however, that the behavior of optimum processing and the conventional single-user matched filter at  $K/N \rightarrow \infty$  are quite different: the optimal spectral efficiency grows without bound with  $\frac{E_b}{N_0}$ , whereas the matched filter efficiency approaches 0.72 bit/sec/Hz monotonically as  $\frac{E_b}{N_0} \rightarrow \infty$ .

For large  $K$ , the optimum choice of  $K/N$  for the decorrelator ranges from 0 for  $\frac{E_b}{N_0} \downarrow$

-1.6 dB to 1 for  $\frac{E_b}{N_0} \rightarrow \infty$ . The optimum coding-spreading tradeoff of the decorrelator dictates using codes whose rates (bits/symbol) lie between 0 ( $\frac{E_b}{N_0} \downarrow -1.6$  dB) to  $C^{\text{deco}}$  ( $\frac{E_b}{N_0} \rightarrow \infty$ ). With an optimum choice of spreading gain, the decorrelator spectral efficiency with random signature waveforms is better than that of the single-user matched filter for  $\frac{E_b}{N_0} > 5.2$  dB (Figure 7). Unlike the single-user matched filter, the spectral efficiency of the decorrelator grows without bound as  $\frac{E_b}{N_0} \rightarrow \infty$ .

As far as the optimum coding-spreading tradeoff for the MMSE receiver, for low  $\frac{E_b}{N_0}$  it favors making  $K/N$  very large in which case the MMSE receiver achieves essentially the same spectral efficiency as the single-user matched filter (Figure 7). The optimum  $K/N$  reaches 1 at  $\frac{E_b}{N_0} = 4$  dB, and a minimum of 0.75 at  $\frac{E_b}{N_0} = 10$  dB.

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