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Mutual Information and Eigenvalue Distribution of MIMO Ricean Channels

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Abstract

This paper presents an explicit expression for the marginal probability density distribution of the unordered eigenvalues of a noncentral Wishart matrix $\mathbf{H}\mathbf{H}^\dagger$ where $\mathbf{H}$ can represent a multiple-input multiple-output channel obeying the Ricean law. By integrating over this marginal density distribution, the corresponding ergodic mutual information is characterized also in explicit form.

1. INTRODUCTION

The analytical characterization of the information rates that can be conveyed reliably through MIMO (multiple-input multiple-output) channels spanned by multiple transmit and receive antennas has been the object of intense research in recent years. The bulk of such work has focused on Rayleigh-faded channels with possibly correlated entries. Such channels are representative of the typical propagation conditions in mobile systems, where terminals tend to be located within the clutter, as well as indoor systems, where both the terminal and the access point are typically surrounded by clutter. The Rayleigh-faded channel, however, does not properly represent the most common propagation conditions in fixed wireless access, an application that appears most suited to the use of MIMO techniques.

In fixed wireless access, both transmitter and receiver tend to be located over the clutter. Hence, the resulting fading profile is usually not Rayleigh but Ricean in nature. Although it is widely perceived that Ricean components can only be detrimental, it has been shown that in some cases (namely if the faded component of the channel remains unchanged) the presence of Ricean components actually enhances the mutual information [1, 2]. Since Ricean channels are often advantageous in terms of SNR (signal-to-noise ratio), these findings reinforce the interest thereon.

Few analytical results, however, are currently available on the capacity and mutual information of MIMO Ricean channels. An integral expression for the ergodic mutual information achieved by isotropic inputs is provided in [3] and some simpler upper bounds are given in [4]. The behaviors at low and high SNR are studied in [5] and in [6, 7], respectively.

In this paper, we present a new analytical expression, valid for arbitrary SNR and numbers of antennas, for the ergodic mutual information achieved by isotropic inputs. This class of inputs is of interest because it embodies many established space-time coding schemes. As in all of the aforementioned contributions, we focus on the coherent regime where the receiver has instantaneous access to the channel state while the transmitter has only access to its distribution. In contrast with [3], where the mutual information is derived through the moment-generating function, in our approach it is obtained by integrating over the marginal distribution of the eigenvalues of $\mathbf{H}\mathbf{H}^\dagger$ (or, equivalently, the squared singular values of $\mathbf{H}$) where $\mathbf{H}$ is the channel matrix. As a result, in our route to the mutual information we uncover an explicit expression for the marginal density distribution of the unordered squared singular values of $\mathbf{H}$. This expression is of independent interest for it may be of use in characterizing other information-theoretical quantities that are functionals of this distribution, e.g., the signal-to-interference at the output of a minimum mean-square error receiver.

In expressing both the marginal distribution of the squared singular values and the mutual information, we consider Ricean channels where the unfaded (deterministic) component is of arbitrary rank, with emphasis on the cases where it is either full rank or unit rank. The latter is particularly relevant because the unfaded component is often associated with a dominant line-of-sight or diffracted wave.

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2. PROBLEM FORMULATION

Denoting by n_{T} and n_{R} the number of transmit and receive antennas, we consider the complex frequency-flat[1] linear model

$$\mathbf{y} = \mathbf{H}\,\mathbf{x} + \mathbf{n}$$

where $\mathbf{x}$ and $\mathbf{y}$ are the input and output vectors while $\mathbf{n}$ is white Gaussian noise. The input is isotropic and thus

$$E[\mathbf{x}\mathbf{x}^\dagger] = \frac{E[\|\mathbf{x}\|^2]}{n_{\mathrm{T}}}\mathbf{I}.$$

The channel, in turn, is represented by the $(n_{\mathrm{R}} \times n_{\mathrm{T}})$ random matrix $\mathbf{H}$ normalized such that

$$E[\mathrm{Tr}\{\mathbf{H}\mathbf{H}^\dagger\}] = n_{\mathrm{R}} n_{\mathrm{T}}. \tag{1}$$

The ergodic mutual information is

$$\mathcal{I}(\mathsf{SNR}) = E\left[\log_2 \det\left(\mathbf{I} + \tfrac{\mathsf{SNR}}{n_{\mathrm{T}}}\,\mathbf{H}\mathbf{H}^\dagger\right)\right] \tag{2}$$

where the expectation is with respect to the distribution of $\mathbf{H}$ while

$$\mathsf{SNR} = \frac{E[\|\mathbf{x}\|^2]}{\frac{1}{n_{\mathrm{R}}}E[\|\mathbf{n}\|^2]}$$

which, given (1), corresponds with the average signal-to-noise ratio per receive antenna.

The entries of $\mathbf{H}$ are modelled as independent nonzero-mean random variables. More precisely,

$$\mathbf{H} = \sqrt{\tfrac{K}{K+1}}\,\bar{\mathbf{H}} + \sqrt{\tfrac{1}{K+1}}\,\mathbf{H}_w$$

where K is the Ricean factor between the unfaded and faded channel components, $\bar{\mathbf{H}}$ is deterministic and the entries of $\mathbf{H}_w$ are independent zero-mean unit-variance complex Gaussian. Although the operational significance of SNR being the average signal-to-noise ratio per receive antenna is retained only if $\mathrm{Tr}\{\bar{\mathbf{H}}\bar{\mathbf{H}}^\dagger\}{=}n_{\mathrm{T}}n_{\mathrm{R}}$, which upholds (1), the formulation that follows remains valid even the unfaded component does not satisfy this condition.

For notational convenience, we define

$$m = \min\{n_{\mathrm{T}}, n_{\mathrm{R}}\}$$
$$n = \max\{n_{\mathrm{T}}, n_{\mathrm{R}}\}$$

3. MARGINAL DENSITY DISTRIBUTION OF THE SQUARED SINGULAR VALUES OF H

Theorem 1 *Let $\phi_1, \ldots, \phi_m$ be the m squared singular values of $\sqrt{K}\bar{\mathbf{H}}$. The marginal density distribution of an arbitrary (unordered) squared singular value of $\mathbf{H}$ is*

$$\begin{aligned} f(\lambda) \;=\; & \frac{e^{-\sum_i \phi_i}}{m\,((n-m)!)^m}\,\frac{e^{-\lambda(K+1)}}{\lambda}\sum_{j=1}^{m}((K+1)\lambda)^{n-m+j} \\ & \cdot \sum_{i=1}^{m} \frac{{}_0F_1(n-m+1, (K+1)\phi_i\lambda)\,\mathcal{D}_{i,j}}{\prod_{k<\ell}^{m}(\phi_\ell - \phi_k)} \end{aligned} \tag{3}$$

where $\mathcal{D}_{i,j}$ is the (i,j)-cofactor[2] of the $(m \times m)$ matrix $\mathbf{A}$ whose (ℓ,k)-th entry is

$$(\mathbf{A})_{\ell,k} = (n{-}m{+}k{-}1)!\;{}_1F_1(n{-}m{+}k, n{-}m{+}1, \phi_\ell)$$

and with ${}_0F_1(\cdot,\cdot)$ and ${}_1F_1(\cdot,\cdot,\cdot)$ hypergeometric functions.

Proof: *See Appendix A.1.*

The function ${}_0F_1(\cdot,\cdot)$ relates directly to the Bessel function of the first kind via [15]

$${}_0F_1(k+1, x^2) = \frac{k!}{(\mathrm{j}x)^k}\,J_k(2\mathrm{j}x)$$

where $\mathrm{j} = \sqrt{-1}$. The function ${}_1F_1(\cdot,\cdot,\cdot)$, in turn, is also known as the Kummer function and it can be represented in both series and integral forms [15]

$$\begin{aligned} {}_1F_1(\ell, k, x) &= \sum_{q=0}^{\infty} \frac{[\ell]_q}{[k]_q}\frac{x^q}{q!} \\ &= \int_0^1 e^{xt} t^{\ell-1}(1-t)^{k-\ell-1}\,dt \end{aligned}$$

where $[a]_b = \frac{(a+b-1)!}{(a-1)!}$.

Example 1 Let $n_{\mathrm{T}}{=}3$, $n_{\mathrm{R}}{=}2$ and $K{=}1$ with

$$\bar{\mathbf{H}} = \begin{bmatrix} 1 & \sqrt{2} & 1 \\ 1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

The eigenvalue distribution provided by Theorem 1 is depicted in Fig. 1 alongside the histogram obtained through Montecarlo simulation.

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[1] If the fading process is frequency selective, the channel can be decomposed into a number of parallel non-interacting subchannels, each experiencing frequency-flat fading and having the same ergodic mutual information as the aggregate channel.

[2] The (i,j)-th cofactor of $\mathbf{A}$ equals $(-1)^{i+j}$ times the determinant of the matrix obtained by excluding from $\mathbf{A}$ the i-th row and j-th column.

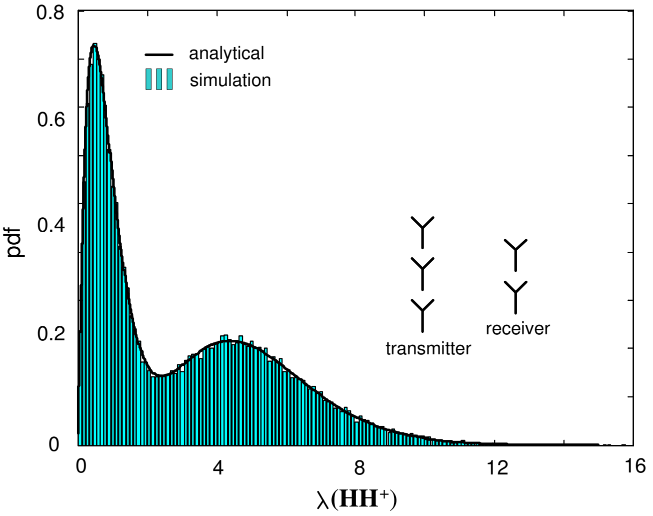

Figure 1: Marginal density distribution of an arbitrary eigenvalue of $\mathbf{H}\mathbf{H}^\dagger$ with n_{T}=3, n_{R}=2 and K=1.

If some of the singular values of $\bar{\mathbf{H}}$ are zero, the numerator and denominator in (3) vanish but their ratio remains well defined. In fact, $f(\lambda)$ can then be more conveniently expressed as function of only those singular values of $\bar{\mathbf{H}}$ that are nonzero using Lemma 2 in Appendix A.1. Singular values with plural multiplicity can be handled in a similar fashion.

In order to illustrate this procedure, we next particularize Theorem 1 to the relevant case of a Ricean channel whose deterministic component has unit rank.

Corollary 1 *If $\phi_i = 0$ for $i \in \{1, \ldots, m-1\}$ and $\phi_m > 0$, then*

$$f(\lambda) = \frac{e^{-\phi_m}}{m\,((n-m)!)^m} \frac{e^{-\lambda(K+1)}}{\lambda} \sum_{j=1}^{m} \frac{((K+1)\lambda)^{n-m+j}}{\phi_m^{m-1} \prod_{\ell=0}^{m-2} \ell!} \cdot \left(\frac{{}_0F_1(n-m+1, (K+1)\phi_i\lambda)}{(\tilde{\mathcal{D}}_{m,j})^{-1}} + \sum_{i=1}^{m-1} \frac{\lambda^{i-1}}{[n-m+1]_{i-1}} \tilde{\mathcal{D}}_{i,j} \right)$$

where $\tilde{\mathcal{D}}_{i,j}$ is the (i,j)-cofactor of the $(m \times m)$ matrix $\tilde{\mathbf{A}}$ whose (ℓ,k)-th entry is

$$(\tilde{\mathbf{A}})_{\ell,k} = \begin{cases} \dfrac{(n-m+k+\ell-2)!}{[n-m+1]_{\ell-1}} & 1 \leq \ell \leq m-1 \\[2ex] \dfrac{{}_1F_1(n-m+k, n-m+1, \phi_\ell)}{((n-m+k-1)!)^{-1}} & \ell = m \end{cases}$$

4. MUTUAL INFORMATION

Theorem 2 *Let $\phi_1, \ldots, \phi_m$ be the m squared singular values of $\sqrt{K}\bar{\mathbf{H}}$. The mutual information is*

$$\mathcal{I}(\mathsf{SNR}) = \kappa\, e^{\frac{K+1}{\mathsf{SNR}/n_{\mathrm{T}}}} \sum_{i=1}^{m} \sum_{j=1}^{m} \mathcal{D}_{i,j} \sum_{p=0}^{\infty} \frac{(u-1)!\, \phi_i^p}{p!\,(n-m+p)!} \cdot \sum_{k=1}^{u} E_{u-k+1}\left(\tfrac{K+1}{\mathsf{SNR}/n_{\mathrm{T}}}\right)$$

where u=$n-m+j+p$ and $\mathcal{D}_{i,j}$ is as in Theorem 1 while

$$E_\ell(x) = \int_1^\infty \frac{e^{-xt}}{t^\ell}\, dt$$

is the exponential integral and

$$\kappa = \frac{e^{-\sum_i \phi_i} \log_2 e}{((n-m)!)^{m-1} \prod_{k<\ell}^{m} (\phi_\ell - \phi_k)}$$

Proof: *See Appendix A.2.*

Example 2 *Consider the channel of Example 1. Displayed in Fig. 2 is $\mathcal{I}(\mathsf{SNR})$ as given by Theorem 2. Also shown is the result of a Montecarlo simulation where 20000 independent realizations are averaged.*

Although, in contrast with the integral solution in [3], Theorem 2 provides an explicit expression, it still involves an infinite series. Fortunately, for Ricean factors of practical interest its convergence is rapid. In Fig. 2, for instance, $\mathcal{I}(\mathsf{SNR})$ is evaluated with the series truncated to only 14 terms.

As with $f(\lambda)$ in Theorem 1, some of the terms in the expression for $\mathcal{I}(\mathsf{SNR})$ in Theorem 2 vanish if $\bar{\mathbf{H}}$ has less than m nonzero distinct singular values but the mutual information remains well defined and it can be posed as function of only the nonzero distinct singular values. It suffices, as the next corollary exemplifies, to perform the integration described in Appendix A.3 after having applied Lemma 2 to $f(\lambda)$.

Corollary 2 *If $\phi_i = 0$ for $i \in \{1, \ldots, m-1\}$ and $\phi_m > 0$, then*

$$\mathcal{I}(\mathsf{SNR}) = \tilde{\kappa} \sum_{j=1}^{m} \left(\sum_{i=1}^{m-1} \frac{[n-m+i]_{j-1}}{(\tilde{\mathcal{D}}_{i,j})^{-1}} \sum_{k=1}^{\theta} E_{\theta-k+1}\left(\tfrac{K+1}{\mathsf{SNR}/n_{\mathrm{T}}}\right) + \tilde{\mathcal{D}}_{m,j} \sum_{p=0}^{\infty} \frac{\phi_m^p\,(u-1)!}{p!\,(n-m+p)!} \sum_{k=1}^{u} E_{u-k+1}\left(\tfrac{K+1}{\mathsf{SNR}/n_{\mathrm{T}}}\right) \right)$$

where u=$n-m+j+p$, θ=$n-m+j+i-1$, $\tilde{\mathcal{D}}_{i,j}$ is as in Corollary 1 and

$$\tilde{\kappa} = \frac{e^{-\phi_m} \log_2 e}{((n-m)!)^{m-1}\, \phi_m^{m-1} \prod_{\ell=0}^{m-2} \ell!}\, e^{\frac{K+1}{\mathsf{SNR}/n_{\mathrm{T}}}}$$

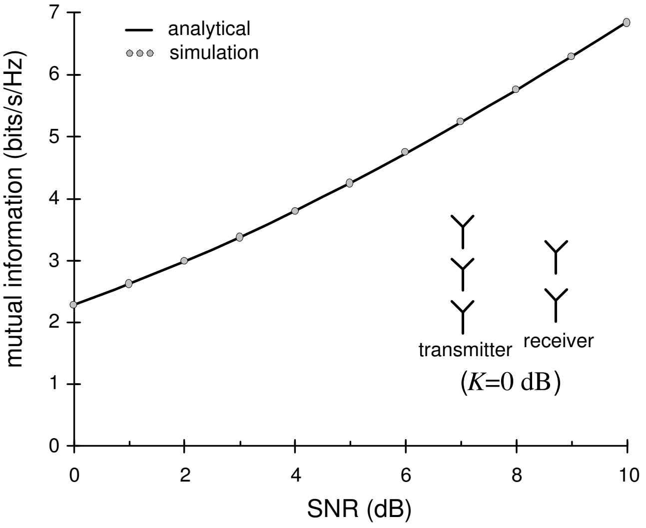

Figure 2: $\mathcal{I}(\text{SNR})$ with n_{T}=3, n_{R}=2 and K=1.

APPENDIX

A.1 Auxiliary Results

Lemma 1 *[12] (see also [13]) Let* $\mathbf{F}$ *and* $\mathbf{G}$ *be two* $(n{\times}n)$ *matrices whose (i,j)-th entries are, respectively,* $(\mathbf{F})_{i,j}{=}f_j(w_i)$ *and* $(\mathbf{G})_{i,j}{=}g_j(w_i)$ *where* f_j *and* g_j, $j{=}1,\ldots,n$, *are functions defined on* $\Re^+$. *Then, for* $b{>}a{>}0$,

$$\int_a^b \cdots \int_a^b \det(\mathbf{F})\,\det(\mathbf{G})\,dw_1,\ldots,dw_n = n!\det(\mathbf{A})$$

where $\mathbf{A}$ *is another* $(n{\times}n)$ *matrix whose (i,j)-th entry is*

$$\mathbf{A} = \int_a^b f_i(w) g_j(w)\,dw$$

Note that, in [12], the factor $n!$ *does not appear because the variables* $w_1,\ldots,w_n$ *are ordered.*

Lemma 2 *Consider the expression for* $f(\lambda)$ *in Theorem 1. Let* $\phi_i = 0$ *for* $i \in \{1,\ldots,m-\mathcal{Q}\}$ *while* $\phi_i > 0$ *for* $i \in \{m-\mathcal{Q}+1,\ldots,m\}$. *Given an* m*-dimensional vector* $\epsilon = \{\epsilon_i\}$ *whose components are distinct,*

$$\lim_{\epsilon\to\mathbf{0}} \frac{{}_0F_1(n-m+1,(K+1)(\phi_i+\epsilon_i)\lambda),\mathcal{D}_{i,j}(\phi_i+\epsilon_i)}{\prod_{k<\ell}^m((\phi_\ell+\epsilon_\ell)-(\phi_k+\epsilon_k))}$$

$$= \frac{f_i(\lambda)\,\mathcal{D}'_{i,j}}{\displaystyle\prod_{\ell=1}^{m-\mathcal{Q}}(\ell-1)! \prod_{\ell=m-\mathcal{Q}+1}^{m}\phi_\ell^{m-\mathcal{Q}} \prod_{m-\mathcal{Q}+1\le q<\ell\le m}(\phi_\ell-\phi_q)}$$

with

$$f_i(\lambda) = \begin{cases} \dfrac{\lambda^{i-1}}{[n-m+1]_{i-1}} & 1 \le i \le m-\mathcal{Q} \\ {}_0F_1(n-m+1,(K+1)\phi_i\lambda) & \text{otherwise} \end{cases}$$

and with $\mathcal{D}'_{i,j}$ *the* (i,j)*-cofactor of the* $(m{\times}m)$ *matrix* $\mathbf{A}$ *whose* (ℓ,k)*-th entry is*

$$(\mathbf{A})_{\ell,k} = \begin{cases} \dfrac{(n-m+k+\ell-2)!}{[n-m+1]_{\ell-1}} & 1 \le \ell \le m-\mathcal{Q} \\ \dfrac{{}_1F_1(n{-}m{+}k,n{-}m{+}1,\phi_\ell)}{((n-m+k-1)!)^{-1}} & \text{otherwise} \end{cases}$$

where $[a]_b = \frac{(a+b-1)!}{(a-1)!}$.

A.2 Proof of Theorem 1

Denoting by $\tilde{\mathbf{H}} = \sqrt{K}\,\bar{\mathbf{H}} + \mathbf{H}_w$ the $(n_{\mathrm{R}}{\times}n_{\mathrm{T}})$ noncentral Wishart matrix with mean $\sqrt{K}\,\bar{\mathbf{H}}$, the joint distribution of the ordered strictly positive eigenvalues of $\tilde{\mathbf{H}}\tilde{\mathbf{H}}^\dagger$ is given by [9]

$$f(\mathbf{\Lambda}) = \frac{e^{-\mathrm{Tr}\{K\,\bar{\mathbf{H}}^\dagger\bar{\mathbf{H}}\}}}{\prod_{\ell=1}^m(n-\ell)!\,(m-\ell)!}\prod_{k=1}^m\lambda_k^{n-m}$$

$$\cdot\,{}_0F_1(n,K\,\bar{\mathbf{H}}^\dagger\bar{\mathbf{H}},\mathbf{\Lambda})\,e^{-\mathrm{Tr}\{\mathbf{\Lambda}\}}\prod_{k<\ell}^m(\lambda_k-\lambda_\ell)^2$$

where $\mathbf{\Lambda}{=}\mathrm{diag}\{\lambda_k\}$. If $\bar{\mathbf{H}}^\dagger\bar{\mathbf{H}}$ has full rank, then we can express the hypergeometric function of matrix arguments ${}_0F_1(n,K\,\bar{\mathbf{H}}^\dagger\bar{\mathbf{H}},\mathbf{\Lambda})$ as [10]

$${}_0F_1(n,K\,\bar{\mathbf{H}}^\dagger\bar{\mathbf{H}},\mathbf{\Lambda}) = \frac{\prod_{\ell=1}^{m-1}(m-\ell)^\ell\,(n-\ell)^\ell}{\prod_{k<\ell}^m(\phi_k-\phi_\ell)} \cdot\frac{\det(\{{}_0F_1(n-m+1,\phi_i\lambda_j)\})}{\prod_{k<\ell}^m(\lambda_k-\lambda_\ell)}$$

where $\det(\{b_{i,j}\})$ indicates the determinant of a matrix whose (i,j)-th entry is $b_{i,j}$ while ${}_0F_1(\cdot,\cdot)$ is the scalar hypergeometric function. Plugging the above expression in (4) and dividing by $m!$, the joint probability density function of the strictly positive unordered eigenvalues is found to be

$$f(\mathbf{\Lambda}) = \frac{e^{-\mathrm{Tr}\{K\bar{\mathbf{H}}\bar{\mathbf{H}}^\dagger\}}}{m!\,((n-m)!)^m}\prod_{\ell=1}^{m-1}\prod_{k<\ell}^{m}\frac{\lambda_k-\lambda_\ell}{\phi_k-\phi_\ell}$$

$$\cdot\,\frac{e^{-\mathrm{Tr}\{\mathbf{\Lambda}\}}}{\prod_{k=1}^m\lambda_k^{m-n}}\det(\{{}_0F_1(n-m+1,\phi_i\lambda_j)\})$$

Let us now expand both determinants[3] in (4) via Laplace expansion and then integrate the result with

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[3] Note that $\prod_{k<\ell}^m(\lambda_k-\lambda_\ell) = \det(\{\lambda_i^{m-j}\})$.

respect to $m-1$ eigenvalues by using Lemma 1, which yields

$$f'(\lambda) = \frac{\kappa'}{e^{\lambda}} \sum_{j=1}^{m} \lambda^{n-m+j-1} \sum_{i=1}^{m} {}_0F_1(n-m+1, \phi_i \lambda)\, \mathcal{D}'_{i,j}$$

with

$$\kappa' = \frac{e^{-\mathrm{Tr}\{K \bar{\mathbf{H}} \bar{\mathbf{H}}^{\dagger}\}}}{m\,((n-m)!)^m \prod_{k<\ell}^{m} (\phi_\ell - \phi_k)}$$

and with $\mathcal{D}'_{i,j}$ the (i,j)-cofactor of the $(m \times m)$ matrix $\mathbf{A}'$ whose (ℓ, k)-th entry is

$$(\mathbf{A}')_{\ell,k} = (n-m+k-1)!\; {}_1F_1(n-m+k, n-m+1, \phi_\ell)$$

Noticing that

$$\mathbf{H} = \frac{\tilde{\mathbf{H}}}{\sqrt{K+1}},$$

the claimed marginal probability density distribution is easily obtained from $f'(\cdot)$ through a change of variables.

A.3 Proof of Theorem 2

The mutual information can be expressed as

$$\begin{aligned} \mathcal{I}(\mathsf{SNR}) &= m\, E\left[\log_2\left(1 + \tfrac{\mathsf{SNR}}{n_{\mathrm{T}}}\lambda\right)\right] \\ &= m \int_0^{\infty} \log_2\left(1 + \tfrac{\mathsf{SNR}}{n_{\mathrm{T}}}\lambda\right) f(\lambda)\, d\lambda \qquad (4) \end{aligned}$$

where the expectation is over λ, an unordered squared singular value of $\mathbf{H}$, whose marginal density distribution $f(\cdot)$ is given in Theorem 1. Recalling

$$[a]_b = \frac{(a+b-1)!}{(a-1)!},$$

the series expansion of the ${}_0F_1(\cdot,\cdot)$ scalar hypergeometric function yields [15]

$${}_0F_1(n-m+1, (K+1)\phi_i\lambda) = \sum_{p=0}^{\infty} \frac{((K+1)\phi_i\lambda)^p}{p!\,(n-m+1)_p}$$

which, plugged into (4), leads to

$$\begin{aligned} \mathcal{I}(\mathsf{SNR}) = \kappa \sum_{p=0}^{\infty} \sum_{i=1}^{m} \sum_{j=1}^{m} & \frac{\mathcal{D}_{i,j}\,(K+1)^{j+p}\ \phi_i^p}{p!\,(n-m+1)_p} \\ & \cdot \int_0^{\infty} \log\left(1 + \tfrac{\mathsf{SNR}}{n_{\mathrm{T}}}\lambda\right) \frac{\lambda^{p+n-m+j-1}}{e^{\lambda(K+1)}}\, d\lambda \end{aligned}$$

Finally, using [14]

$$\int_0^{\infty} \log(1+\alpha\lambda) \frac{\lambda^{q-1}}{e^{\gamma\lambda}}\, d\lambda = \frac{(q-1)!}{e^{-\frac{\gamma}{\alpha}}\, \gamma^q} \sum_{k=1}^{q} E_{q-k+1}\left(\tfrac{\gamma}{\alpha}\right)$$

the claimed expression is obtained.

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