On Information Theoretic Aspects of Multi-Cell Wireless Systems

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Abstract— **Some information theoretic aspects of cellular communication systems are addressed, focusing on a simple multi-cell model suggested by Wyner (1994). Accordingly, the system cells are ordered in an infinite linear array, and only adjacent cell interference is present, characterized by a single parameter $0 \leq \alpha \leq 1$. Starting with the downlink channel, it is assumed that only a single user is to be served in each cell. A linear *joint* preprocessing plus encoding scheme is proposed, which significantly enhances performance, while putting the complexity burden on the transmitting end. The approach is based on *LQ* factorization of the channel transfer matrix, combined with the "writing on dirty paper" approach (Costa (1983)) for eliminating the effect of uncorrelated interference, while fully known at the *transmitter* but unknown at the receiver. The attainable average rates with the proposed scheme approach those associated with receiver based optimum joint processing at the high SNR region. Extensions and applications are also discussed. Turning to the uplink channel, we investigate an optimally coded randomly spread DS-CDMA system with multiuser detection. The discussion is confined to asymptotic analysis where both the number of users per cell and the processing gain go to infinity, while their ratio goes to some finite constant. The spectral efficiency of various multiuser detection strategies is evaluated assuming single cell-site processing, and equal transmit powers for all users in all cells. Comparative results demonstrate how performance is affected by the introduction of inter-cell interference, and what is the penalty associated with the randomly spread coded DS-CDMA strategy.**

I. Introduction

The increasing penetration and widespread deployment of cellular systems throughout the world, during the last decade, made them the subject of numerous scientific researches considering both practical and theoretical system aspects, and in particular means for enhancing system performance. The available literature can be divided into works dealing with the downlink channel, e.g., [1], [2], [3] (see also references therein) and [4], and works dealing with the uplink channel, e.g., [5] – [12]. Exploring the possible methods for enhancing system performance, *joint processing* of signals related to different users is evidently the most appealing approach. However, a fundamental difference emerges in this respect between the downlink and the uplink channels. The *downlink* channel is a *broadcast* channel. In the common downlink channel model, each mobile user receives transmissions of neighboring cell-sites in addition to the transmissions from its "own" cell-site, plus an additive white Gaussian noise (AWGN). Since practical considerations usually limit the mobile receiver's complexity, it is reasonable to assume that it is restricted to *single-user detection*, and hence a suitable channel model is that of a Gaussian noise plus interference channel. The complexity considerations at the cell-site are however much less restrictive, and the use of *transmitter based joint preprocessing* for system performance enhancement is called for. The *uplink* channel is a *multiple access* channel. In the common uplink model each cell-site receives the transmissions of users operating in its "own" cell, with the addition of signals originated from users operating in neighboring cells (accompanied by AWGN). Here, *joint processing of the received signals* at the cell-site (by means of various multiuser detection techniques [13]) is the appropriate approach, reasonably assuming *non-cooperative transmissions* of the mobile users.

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In this paper we demonstrate the impact of employing joint processing at the cell-site on system performance, by reviewing some recently obtained results, as presented in [4] and [12]. The focus is on a simple multi-cell system model, based on the model suggested by Wyner in [5] (see also [6]). Accordingly, a *fully synchronous* cellular system is assumed, where the cells compose an infinite *linear* array. In the uplink channel model, the received signal at each cell-site is the sum of the signals received from intra-cell users, plus a factor α ($0 \leq \alpha \leq 1$) times the sum of the signals generated by users in the two adjacent cells. Non-adjacent cell users are assumed to produce no interference. The received signal is embedded in ambient Gaussian noise. The multi-cell effect on performance is thus specified by a *single* parameter (α). The downlink channel model is completely analogous, and the signal received by each of the users is composed of the signal received from the "local" cell-site, plus a factor α times the signals originated from the two *adjacent* (only) cell-sites, and an ambient Gaussian noise.

Section II is devoted to the downlink channel, where we review a novel transmitter based preprocessing plus encoding scheme [4]. The setup is restricted to either non-fading or slowly changing ergodic flat-fading channels, considering the case in which a *single* user is to be served in each of the cells. This model represents intra-cell TDMA, or alternatively a direct sequence code division multiple access (DS-CDMA) scheme employing orthogonal spreading sequences (assuming no multipath fading is present), as is the case for example in the downlink of IS-95 systems. It is assumed that all cell-sites can *cooperate* via a central Cell-Site Controller (CSC), that is fully informed by the cell-sites regarding the channel gains between *each* of the transmitting cell-sites and *each* of the receiving users. This CSC can arbitrarily process and encode the messages to be transmitted to the users, and determine in advance the signals to be transmitted by each of the cell-sites at each discrete time. However, restricting the discussion to single-user decoding, the users' receivers are assumed to be uninformed regarding the codebooks of users in adjacent cells, and thus joint decoding and possibly cancellation of interfering signals addressed to other users is not applicable. The setup is hence of a "broadcast channel" in the classical information theoretic sense.

In the proposed scheme the signals transmitted by the cell-sites are a linear transformation of the outputs of the encoders corresponding to each of the users, based on the *LQ* factorization of the channel transfer matrix, and equivalent to the noise whitening matched filter in single-user inter-symbol-interference (ISI) channels. A coding scheme is then applied, based on the "writing on dirty paper" concept suggested in [14] (see also [15], additional references in [16], and [17] where the multi-antenna broadcast channel is considered). The "dirty paper" approach can be used to eliminate the effect of uncorrelated additive interference on the capacity of a Gaussian noise plus interference channel, in which the interfering sequence is *unknown* at the *receiver*, provided that it is *known before hand* at the *transmitter*, thus achieving the AWGN channel capacity [15]. The combined preprocessing plus encoding scheme can be shown to approach the average rates (where the average is taken over all users) attained with optimum joint processing of the *received* signals, in the high signal-to-noise ratio (SNR) region [17]. The proposed scheme shall henceforth be referred to as "LP-DP" (stands for "Linear Preprocessing Dirty Paper"). This approach is clearly superior to the common practice in current systems, where each cell-site transmits information to the users operating in its own cell only, the mobile receivers treat interfering signals from neighboring cell-sites as an additive noise (henceforth referred to as the "conventional" approach), and the system is thus interference limited. However, performance enhancement is achieved also with respect to the zero-forcing approach of [2] and references therein (see [17] for a proof), and with respect to a spatial Tomlinson-Harashima based precoding [18], [3]. Various extensions of the basic technique are also discussed.

Section III is devoted to the uplink channel, and considers an *optimally* coded (in the information-theoretic sense) *randomly spread* DS-CDMA system, with multiuser detection at the cell-site receiver. Some recent results [12] on the spectral efficiency of multiuser detection strategies are reviewed, considering the linear cell-array multi-cell model, and assuming flat-fading channels (Rayleigh fading is assumed whenever explicit results are presented). DS-CDMA systems with random spreading are the focus of many information-theoretic analyses of recent years, in view of their wide-spread practical use (e.g., IS-95 and 3G systems). Particularly, the limiting scenario is examined, where both the number of users and the processing gain go to infinity, while their ratio goes to some *finite* constant. Thus, deterministic system performance measures of interest can be obtained, using relevant random matrix theory results (e.g., [19], [20]). Analyses of a *single-cell* DS-CDMA system, were recently presented in [7], [8] and [10] (see also references therein). Assuming equal received powers and no fading, expressions are presented in [7] for the spectral efficiencies of the optimum detector, the matched-filter detector, the decorrelator, and the linear minimum mean squared error (MMSE) detector. The impact of frequency-flat fading on these detectors is analyzed in [10]. Signal-to-interference-plus-noise ratios (SINRs) at the output of the above mentioned linear detectors are presented in [8]. The results of [7] and [10] are extended in [11] and [12] to the linear cell-array model.

The asymptotic scenario is considered in the following (as in [7], [8], [10]), in which denoting by K the number of intra-cell users (assumed constant and equal in all cells), and by N the spreading factor (processing gain), $K, N \to \infty$, while $\frac{K}{N} \to \beta < \infty$. The factor β is commonly referred to as the "system load". Assuming *single cell-site processing*, four types of multiuser detection strategies are considered:

1). The *"conventional" matched-filter detector* that treats *all* interference (either intra-cell or inter-cell) as AWGN;

2). A *single-cell optimum (SCO)* detector that "optimally" detects the transmissions of *intra-cell* users, while treating *inter-cell* interference as AWGN;

3). The *linear MMSE detector* that knows the signature sequences of all interfering users (both intra-cell and in adjacent cells) and mitigates their interference by means of a linear MMSE filter;

4). A detector that employs *MMSE based succes-*

sive interference cancellation (MMSE-SC) to decode transmissions of intra-cell users, while *inter-cell* interference is mitigated by means of a linear MMSE filter (the MMSE-SC detector is, in fact, optimum in terms of spectral efficiency in the setting considered in this paper, as briefly explained in Subsection III-B).

It is emphasized that neither the linear MMSE detector, nor the MMSE-SC detector, try to *decode* the transmissions of adjacent cell users (which might be prohibitive if α is small). In fact, the receiver at the cell-site may actually be ignorant regarding codebooks or code-mask sequences employed in other cells, but is aware, as usually is the case in practice, of the signature sequences of all users in adjacent cells. In addition to the above, it is also assumed that all detectors are provided with the required knowledge regarding the received powers of the interfering signals. The use of adaptive MMSE detection may be attractive in this respect, as it makes no distinction between intra-cell and adjacent-cell interference.

Identifying the spectral efficiency as the fundamental measure of system performance for coded systems [7], the spectral efficiency of all four detection strategies is obtained, and comparatively examined assuming a *constant* (fading independent) transmit power. The results are then compared to analogous results without fading (appropriately reproduced from [11]). Finally, the penalty in system performance due to random spreading is also examined, by comparison (following [6]) to the spectral efficiency of a few corresponding detectors, in the setting in which *all* bandwidth is available for coding (as opposed to bandwidth expansion by DS-spreading).

II. Downlink Channel Results

A. Downlink System Model

Let N denote the number of cells (which we eventually assume to go to infinity). Omitting time indices for simplicity of notations, an equivalent discrete time channel model for the setting in concern is given by the following equation:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \ . \tag{2-1}$$

In this model $\boldsymbol{y} \in \mathbb{C}^N$ denotes the vector of signals *received* by each of the *users* at some arbitrary discrete time, and $\boldsymbol{x} \in \mathbb{C}^N$ denotes the vector of *transmitted* signals by each of the *cell-sites*. The entries of the $N \times N$ channel transfer matrix $\boldsymbol{H}$, denoted $\{h_{ij}\}_{i,j=1}^N$, represent the (generally complex) channel gains, with h_{ij} being the gain of the channel between the jth cell-site and the ith user. Finally, $\boldsymbol{n}$ represents the N independent AWGNs received by each of the users, with underlying distribution $\mathcal{N}_c(0,1)$. We thus normalize without loss of generality all powers with respect to the noise spectral density. The cell-site transmissions are also assumed to be subject to the following average *system* power constraint (which in fact represents an SNR constraint following the above normalization):

$$\frac{1}{N}\sum_{k=1}^{N} E\left\{|x_k(i)|^2\right\} \leq P\,,\forall i\ , \tag{2-2}$$

where $x_k(i)$ denotes the output of the kth cell-site at the ith discrete time.

B. Linear Preprocessing

Applying the LQ factorization on the $N \times N$ channel transfer matrix $\boldsymbol{H}$ yields $\boldsymbol{H} = \boldsymbol{L}\boldsymbol{Q}$, where $\boldsymbol{L}$ is lower triangular and $\boldsymbol{Q}$ is unitary. Hence $\boldsymbol{H}\boldsymbol{H}^\dagger = \boldsymbol{L}\boldsymbol{L}^\dagger$ (with $(\cdot)^\dagger$ denoting conjugate transpose). Let $\boldsymbol{z} \in \mathbb{C}^N$ denote the vector comprising the encoders' outputs (each encoder being designated to a different user). We now propose the following linear one-to-one transformation between $\boldsymbol{z}$ and the cell-sites output vector $\boldsymbol{x}$:

$$\boldsymbol{x} = \boldsymbol{Q}^\dagger \boldsymbol{z} \ . \tag{2-3}$$

Assuming the encoders' outputs to be zero mean random variables with variance $E\left\{z_i z_i^\dagger\right\} = P_i$, it is easily seen that the power constraint of (2-2) is retained as long as we identically constrain the vector of *encoder* outputs (in fact the stronger relation $\frac{1}{N}\sum_{k=1}^{N}|x_k(i)|^2 = \frac{1}{N}\sum_{k=1}^{N}|z_k(i)|^2$ holds). The resulting *overall* discrete time equivalent channel is thus given by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x}(\boldsymbol{z}) + \boldsymbol{n} = \boldsymbol{L}\boldsymbol{z} + \boldsymbol{n} \ . \tag{2-4}$$

C. The Attainable Rates

Observing the expression for the received signal vector as given by (2-4) and recalling that $\boldsymbol{L}$ is a lower triangular matrix, one immediately sees that the signal received by the kth user is given by the following equation:

$$y_k = l_{kk}z_k + \sum_{i=1}^{k-1} l_{ki}z_i + n_k\ ,\ k \in \{1,2,\ldots,N\}\ , \tag{2-5}$$

where $\{l_{ij}\}_{i,j=1}^N$ represent the entries of $\boldsymbol{L}$. Hence, each of the users indexed $k = 2,3,\ldots,N$ experiences an additive Gaussian noise plus interference channel, with the additive noise term being white, and the additive *interference* term being a linear combination of the code symbols transmitted to users of preceding indices. The first user ($k = 1$) experiences a standard Gaussian noise channel with no interference at all. It is important to note here that with the above structure *any* coding scheme employed by *any* of the users has no effect on the performance of users of *preceding* indices.

The key observation at this point is that following the underlying assumptions regarding the CSC, as presented in Section I, the CSC has *full a priori knowledge of the interference terms received by each of the users*, as given in (2-5). The system hence falls exactly within the setup of "writing on dirty paper" as depicted and analyzed in [14] and [15], where with an appropriate encoding scheme the capacity of the channel experienced by each user is exactly that of a *standard* Gaussian channel, without *any* effect of the additive interference term. The capacity achieving statistics is therefore Gaussian, and the rate achievable by the nth user is given by

$$R_n = E_{\mathcal{H}}\left\{\log(1+|l_{nn}|^2 P_n)\right\} , \qquad (2\text{-}6)$$

where the expectation is with respect to the realizations of the channel transfer matrix $\boldsymbol{H}$. Focusing on the average rate (throughput) over all users we get

$$\bar{R} = E_{\mathcal{H}}\left\{\frac{1}{N}\sum_{n=1}^{N}\log(1+|l_{nn}|^2 P_n)\right\} . \qquad (2\text{-}7)$$

The above average rate can now be optimized with respect to the encoders' output power allocation, subject to the average power constraint of (2-2). The optimum average rate is given by

$$\bar{R}^{\rm opt} = E_{\mathcal{H}}\left\{\frac{1}{N}\sum_{n=1}^{N}\log\left[\max(\theta\,|l_{nn}|^2,1)\right]\right\} , \qquad (2\text{-}8)$$

where θ is a constant determined by the equation

$$E_{\mathcal{H}}\left\{\frac{1}{N}\sum_{n=1}^{N}\max\left[(\theta-\frac{1}{|l_{nn}|^2}),0\right]\right\} = P. \qquad (2\text{-}9)$$

For the sake of comparison, the average rate attained when all received signals are *jointly* processed is also considered, given assuming no linear preprocessing by

$$\bar{R}_{\rm jp} = E_{\mathcal{H}}\left\{\frac{1}{N}\log\det(\boldsymbol{I}+\boldsymbol{HPH}^{\dagger})\right\}, \qquad (2\text{-}10)$$

where $\boldsymbol{P} = E\{\boldsymbol{zz}^{\dagger}\}$. The above average rate can also be optimized with respect to the choice of input covariance matrix $\boldsymbol{P}$, while satisfying the input power constraint (2-2) (in such case it can be shown that the linear preprocessing of (2-3) induces no loss in the average attained rate, when joint processing is employed, which follows from the one-to-one correspondence between $\boldsymbol{x}$ and $\boldsymbol{z}$). The expressions for the maximum attainable average rate with joint processing are completely analogous to (2-8) and (2-9), with the absolute squares of the diagonal entries of $\boldsymbol{L}$ replaced by $\{\lambda_n\}_{n=1}^{N}$, the eigenvalues of $\boldsymbol{HH}^{\dagger} = \boldsymbol{LL}^{\dagger}$.

As mentioned in Section I, it was established that the LP-DP scheme achieves the maximal possible asymptotic throughput for $N = 2$ [17], and in general [21].

D. Particular examples

In order to demonstrate the effect of LP-DP on system performance, we first consider the linear cell-array model (as described in Section I) assuming no-fading is present. Omitting time indices, the received signal of the k-th cell user is given by (c.f. (2-1)):

$$y_k = \alpha x_{k-1} + x_k + \alpha x_{k+1} + n_k. \qquad (2\text{-}11)$$

It hence follows that the matrix $\boldsymbol{H}$ has a Toeplitz form satisfying:

$$[\boldsymbol{H}]_{ij} = h_{i-j} = \begin{cases}\alpha & |i-j| = 1,\\ 1 & i=j,\\ 0 & \text{otherwise.}\end{cases} \qquad (2\text{-}12)$$

Focusing on the asymptotic scenario in which $N \to \infty$, we can apply well known results on the asymptotic behavior of Toeplitz matrices (e.g., [22]) to obtain analytical expressions for the average attained rates, both with LP-DP and with joint processing of the received signals (for conciseness we bring here only final results and the interested reader is referred to [16] for more details on the derivations). Accordingly, the average attainable rates with LP-DP are given by (the equal power allocation is optimum in this case):

$$\bar{R}^{\rm opt} = \begin{cases}\log\left(1+\left(\frac{1+\sqrt{1-4\alpha^2}}{2}\right)^2 P\right) & 0\le\alpha\le\frac{1}{2},\\ \log\left(1+\alpha^2 P\right) & \alpha\ge\frac{1}{2}.\end{cases}$$
$$= \bar{R}^{\rm ep} . \qquad (2\text{-}13)$$

With joint processing the corresponding rates are given by

$$\bar{R}_{\rm jp}^{\rm ep} = \frac{1}{2\pi}\int_0^{2\pi}\log(1+(1+2\alpha\cos\omega)^2 P)\,d\omega , \qquad (2\text{-}14)$$

for the case of equal power allocation, and with optimum power allocation one gets

$$\bar{R}_{\rm jp}^{\rm opt} = \frac{1}{2\pi}\int_0^{2\pi}\log\left[\max(\theta(1+2\alpha\cos\omega)^2,1)\right]d\omega , \qquad (2\text{-}15)$$

where θ is determined by the equation

$$\frac{1}{2\pi}\int_0^{2\pi}\max\left[(\theta-\frac{1}{(1+2\alpha\cos\omega)^2}),0\right]d\omega = P . \qquad (2\text{-}16)$$

Finally, we also state here, for comparison, the expression for the average attained rate with the "conventional" approach given by (neglecting boundary

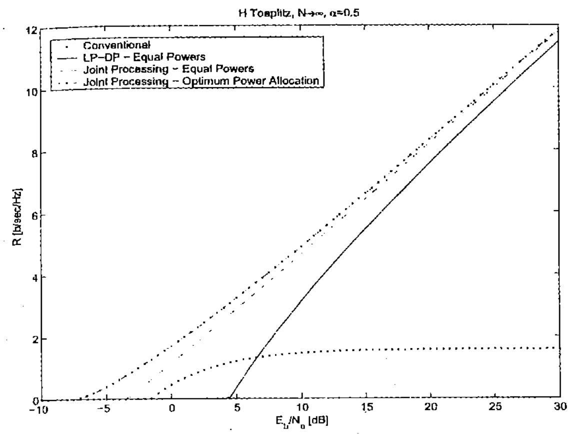

Fig. 1. Average attainable rates without fading, for $N \to \infty$.

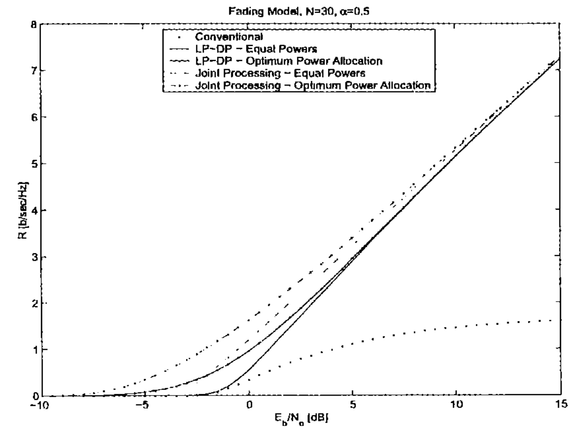

Fig. 2. Average attainable rates with Rayleigh fading, for $N = 30$.

cells for $N \to \infty$):

$$\bar{R}_{\text{con}} = \log\left(1 + \frac{P}{1 + 2\alpha^2 P}\right) . \qquad (2\text{-}17)$$

In Fig. 1 we show some numerical results for the above setup, assuming an interference factor of $\alpha = \frac{1}{2}$ (in such case, with equal power allocation, the power of adjacent cell interference equals one half of the received intra-cell power). The attainable rates are shown as a function of the *system average* $\frac{E_b}{N_0}$, defined through $P = \bar{R}\frac{E_b}{N_0}$. The results clearly demonstrate the dramatic performance enhancement of LP-DP, as compared to the interference limited conventional approach. LP-DP is not interference limited, and the attainable average rate grows without bound with $\frac{E_b}{N_0}$, approaching the rates attained with optimum joint processing, as we take $\frac{E_b}{N_0} \to \infty$. An interesting observation is that for *low* values of $\frac{E_b}{N_0}$, the "conventional" approach is preferable over LP-DP, which is readily explained by comparing (2-17) and (2-13), while taking $P \to 0$.

We now turn to consider system performance in Rayleigh flat-fading channels. In this setup, the received signal of the k-th cell user is given by (c.f. (2-1)):

$$y_k = \alpha\phi_k x_{k-1} + \xi_k x_k + \alpha\psi_k x_{k+1} + n_k, \qquad (2\text{-}18)$$

where $\{\phi_k\}_{k=2}^{N}$, $\{\xi_k\}_{k=1}^{N}$, and $\{\psi_k\}_{k=1}^{N-1}$ are i.i.d. zero mean complex Gaussian random variables with unit variance.

Unfortunately, analytical derivation of the average attainable rate in this model, either with LP-DP or with joint processing, is still an open problem. Upper and lower bounds for the attainable rate with *joint processing* can be found in [9]. Although the *uplink* channel is considered in [9], the performance of the super multi-cell receiver with intra-cell TDMA discussed therein is completely equivalent to the *downlink* channel considered here, when joint processing is employed. The average attained rate with the "conventional" approach is given by ($E_{\phi,\xi,\psi}\{\cdot\}$ denotes the expectation with respect to the fading gains)

$$\bar{R}_{\text{con}} = E_{\phi,\xi,\psi}\left\{\log\left(1 + \frac{|\xi|^2 P}{1 + \alpha^2(|\phi|^2 + |\psi|^2)P}\right)\right\} , \qquad (2\text{-}19)$$

which for Rayleigh fading can be analytically expressed by means of the exponential integral function (see [6]).

In Fig. 2 we show some numerical results for the above fading model, assuming a *finite* linear cell-array of $N = 30$ cells. The presented rates were evaluated using "Monte-Carlo" simulations, averaging over 500 random generations of the channel transfer matrix $\boldsymbol{H}$. As in the non-fading scenario, the significant improvement with LP-DP over the "conventional" approach, and the approach to the optimum joint processing results for high $\frac{E_b}{N_0}$ are clearly demonstrated. It is also observed that when fading *is* present, LP-DP performs no worse than the "conventional" approach even in the low $\frac{E_b}{N_0}$ region (as opposed to the non-fading scenario). We note here that this observation holds while averaging the attained rates over many random generation of the channel transfer matrix. However for *individual* realizations of $\boldsymbol{H}$, preference of the "conventional" scheme for low $\frac{E_b}{N_0}$ may be observed. Another observation is that for very low values of $\frac{E_b}{N_0}$ the attained rates with LP-DP and *optimum power allocation*, surpass the ones attained with *joint processing* but with *equal power allocation*.

For the sake of comparison we also bring here in Fig. 3 the corresponding results *without fading* for the finite dimensional system of $N = 30$ cells (*please note the scale difference between Fig. 3 and Fig. 2*). The slight improvement in system perfor-

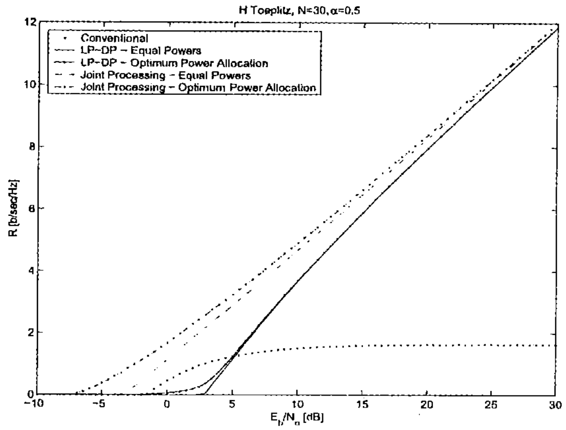

Fig. 3. Average attainable rates without fading, for $N = 30$.

mance with LP-DP, as compared to the limiting results of Fig. 1, is due to non-negligible boundary effects in the finite dimensional case. As observed, the attained rates *with fading* are *higher* than the rates attained *without* fading. This result should come in no surprise in view of [9], where the beneficial effect of independent fading was shown in this setup (beyond some critical value of α), considering joint processing. Since LP-DP approaches optimum performance as $\frac{E_b}{N_0} \to \infty$, the beneficial effect of fading holds as well. This phenomena is explained (see also [9]) by the fact that the diversity effect of multiple transmitters together with fluctuations of the received signal, *whose realizations are known at the transmitter*, helps rather than degrades performance when the interference factor is high enough. For the "conventional" approach the results can go either way, depending on $\frac{E_b}{N_0}$ and the interference factor α (for low α no fading is beneficial). An explicit expression for the high SNR limit is given in [6], by which indeed fading is seen to be beneficial for $\alpha = \frac{1}{2}$.

E. Discussion and Extensions

We have presented the basic precoding technique based on the "dirty paper" paradigm. This technique has been called "Ranked Known Interference" (RKI) in [17], where also an improved modified RKI technique has been introduced. In the modified RKI, relation (2-3) is replaced by: $\boldsymbol{x} = \boldsymbol{Q}^{\dagger}\boldsymbol{R}\boldsymbol{z}$, where $\boldsymbol{R}$ is an upper triangular matrix subjected to optimization. It has been established in [17] that for the two user case this approach achieves the optimal possible throughput (sum-rate). In [17], an improved Sato type outer bound on the sum-rate, which is tighter as the one in (2-10) has been introduced. It is based on optimizing the cooperative bound with respect to all possible correlations of the noise components of all users maintaining the marginal distribution of these noises, that is standard unit variance circularly symmetric Gaussian. A compact form of this optimization is given in [23]. This result has been extended to the K user case in [21], establishing the optimality of this technique as far as sum rates are concerned. This modified technique can be applied here in a straightforward fashion.

The basic RKI procedure outlined in this paper raises the issue of optimal user ordering, that is particularly important in the presence of fading. Cell-sites ordering manifests itself in permutation of rows in the matrix $\boldsymbol{H}$, and affects the relevant entries of the $\boldsymbol{L}$ matrix in the LQ factorization. Pivoting techniques can be harnessed as to optimize the users' order with respect to a desired optimization criterion, as sum-rate for example [21].

The actual implementation of the "dirty paper" coding can be based on algebraic rather than random binnig, utilizing the features of certain classes of nested lattice codes. See the general treatment in [24] and a specific application to the vector broadcast setting in [17].

The vector "dirty paper" technique [25] can be applied as to extend the results to channels with multi-receive antennas at the user's site. In this respect see also [21] which is based on the basic RKI procedure. The insightful duality between the vector broadcast channel and a corresponding multiple access channel [26] is most useful in the actual calculation of the associated rates of the general "dirty paper" broadcast capacity region, [21], [25], [26] and [23] where it is shown that these extended techniques are also optimal in terms of sum rates.

While the basic setting here addresses a single user per cell (TDMA) scheduling, it is easily extended to a modified TDMA technique where the user per cell is selected by some optimal scheduling algorithms that consider its instantaneous channel realization, as is done for example in the recent IS-856 standard. Further, the technique of co-cell-site processing is by no means limited to TDMA, and is extended in a straightforward fashion to other intra-cell multiple access procedures that allow a simultaneous transmission of more than a single user per cell. This fact increases the number of users to be serviced, but does not modify the principle structure to which the proposed approach applies. Further, also coded DS-CDMA signals can be treated within this framework of "dirty paper" precoding. See [3] for a Tomlinson-Harashima based approach in a single-cell scenario with suboptimal matched-filter based receivers. Our approach can also accommodate multiple receiving antennas of the users, see [21].

III. Uplink Channel Results

A. Uplink System Model

Using the standard discrete time equivalent channel representation, the signal vector received at an arbitrary cell site, at the discrete time related to the transmission of the ith symbol, is given by

$$\boldsymbol{y}_i = \boldsymbol{S}_i\boldsymbol{H}_i\boldsymbol{x}_i + \alpha\boldsymbol{S}_i^-\boldsymbol{H}_i^-\boldsymbol{x}_i^- + \alpha\boldsymbol{S}_i^+\boldsymbol{H}_i^+\boldsymbol{x}_i^+ + \boldsymbol{n}_i \ . \qquad (3\text{-}1)$$

The vector $\boldsymbol{x}_i = [x_{1,i}, \ldots, x_{K,i}]^T$ in (3-1) comprises the K code symbols transmitted by intra-cell users at the ith discrete time. The vectors $\boldsymbol{x}_i^\pm = \left[x_{1,i}^\pm, \ldots, x_{K,i}^\pm\right]^T$ denote the vectors of code symbols originated from users operating in adjacent cells. These symbols are assumed to be i.i.d., proper complex Gaussian (which conforms with the capacity achieving statistics), with $E\{x_{k,j}\} = 0$ and $E\{|x_{k,j}|^2\} = \bar{P}\ \forall\, k, j$, where $\bar{P}$ is the equal transmit power of all users. This model is justified by assuming that the codebooks of all users are chosen randomly, governed by an underlying i.i.d. Gaussian distribution per symbol, and *independently* for each message transmission [6].

The matrices $\boldsymbol{S}_i$ and $\boldsymbol{S}_i^\pm$ are $N \times K$ matrices, whose columns are the N-chip long spreading sequences (signatures) of the K users in the considered cell and in its adjacent cells, respectively. The entries of the above matrices are treated as i.i.d. zero mean random variables, with variance $1/N$. The vector $\boldsymbol{n}_i$ represents a zero mean white proper complex Gaussian noise vector, with $E\{\boldsymbol{n}_i\boldsymbol{n}_i^\dagger\} = \boldsymbol{I}$, $\forall i$. Without loss of generality all received powers are thus normalized with respect to the noise spectral level, and represent in fact the SNRs at the input to the multiuser detectors (as in Section II).

Finally, $\boldsymbol{H}_i \triangleq \text{diag}\ (h_{1,i}, \ldots, h_{K,i})$ and $\boldsymbol{H}_i^\pm \triangleq \text{diag}\ (h_{1,i}^\pm, \ldots, h_{K,i}^\pm)$, where $\{h_{k,i}\}_{k=1}^K$ and $\{h_{k,i}^\pm\}_{k=1}^K$ designate the assumed i.i.d. zero-mean channel fading gains associated with the signals of the different users, at the ith discrete time. It shall be assumed henceforth that as the system size becomes large ($N, K \to \infty$, $\frac{K}{N} \to \beta < \infty$), the empirical distribution of the channel fading (power) levels, $\nu_{k,i}^{(\pm)} \triangleq \left|h_{k,i}^{(\pm)}\right|^2$, converges a.s. to a distribution $\mathcal{F}_\nu$. The fading levels are assumed throughout to be normalized so that their expectation satisfies $E_{\mathcal{F}_\nu}\{\nu\} = 1$ (where ν denotes some arbitrary fading level).

B. Spectral Efficiency of the Multiuser Detectors

The per cell *spectral efficiency* [7], [10], or throughput, is defined as the total number of bits/sec/Hz that can be transmitted arbitrarily reliably in each cell. The spectral efficiency of a linear detector is conveniently expressed in terms of its *multiuser efficiency* [13], defined as the ratio between the detector's output SINR and the SNR (note that multiuser efficiency may depend, as is the case with the MMSE based detectors, on the presence of fading, see [10] and equation (3-4)). Denoting the multiuser efficiency by η, and following central limit results showing that the interference at the output of each of the two linear detectors, i.e., the matched-filter detector and the linear MMSE detector, is well approximated by a Gaussian noise (see [7], [11] and references therein for justification of this Gaussian approximation), the spectral efficiency of these detectors equals (bit/sec/Hz)

$$\begin{aligned}\tilde{C} &= \beta E_{\mathcal{F}_\nu}\{\log(1 + \nu\eta\bar{P})\} \\ &\underset{\text{Rayleigh}}{=} \beta\, e^{\frac{1}{\bar{P}\eta}}\mathcal{E}_1\!\left(\frac{1}{\bar{P}\eta}\right)\log e\ ,\end{aligned} \qquad (3\text{-}2)$$

where $\mathcal{E}_1(x) \triangleq \int_x^\infty \frac{e^{-t}}{t}\,dt$, $(t > 0)$ is the exponential integral function. The spectral efficiency of the two non-linear detectors, i.e., the SCO detector and the MMSE-SC detector, is most conveniently evaluated using inter-relations between the spectral efficiency of the optimum multiuser detector and that of the *linear* MMSE detector, as derived in [10] and [11].

Due to space limitations, only the final results for the spectral efficiency in flat-fading channels of all four multiuser detectors are presented in this review. The interested reader is referred to [12] for more details on the derivation of the above results, and to [11] for the corresponding results in non-fading channels. The notation $(\cdot)_{\text{mf}}$, $(\cdot)_{\text{sco}}$, $(\cdot)_{\text{ms}}$, and $(\cdot)_{\text{m-sc}}$ is used in the following to designate entries related to the matched-filter detector, the SCO detector, the linear MMSE detector, and the MMSE-SC detector, respectively. It is noted that when different systems are to be compared (with possibly different spreading gains and data rates), it is useful to express the spectral efficiency in terms of $\frac{E_b}{N_0}$, which is done through the relation $\bar{P} = \frac{1}{\beta}\tilde{C}\frac{E_b}{N_0}$ [10]. However, for simplicity of notation, equations are expressed in terms of the received power (which is in fact the SNR, following the normalization with respect to the noise spectral level).

In the limiting scenario considered here, the spectral efficiency of the matched-filter detector is given by:

$$\begin{aligned}\tilde{C}_{\text{mf}} &= \beta E_{\mathcal{F}_\nu}\left\{\log\left(1 + \frac{\bar{P}\nu}{1 + \beta(1 + 2\alpha^2)\bar{P}}\right)\right\} \\ &\underset{\text{Rayleigh}}{=} \beta e^{\left[\frac{1}{\bar{P}} + \beta(1+2\alpha^2)\right]}\mathcal{E}_1\!\left(\frac{1}{\bar{P}} + \beta(1 + 2\alpha^2)\right)\log e\ .\end{aligned} \qquad (3\text{-}3)$$

The multiuser efficiency of the linear MMSE detector converges a.s. to a non-random limit η_{ms}, equal for all users, given by the unique positive solution to the implicit equation

$$
\begin{aligned}
1 &= \eta_{\text{ms}} + \beta E_{\mathcal{F}_\nu}\left\{\frac{\bar{P}\nu\eta_{\text{ms}}}{1+\bar{P}\nu\eta_{\text{ms}}} + \frac{2\alpha^2\bar{P}\nu\eta_{\text{ms}}}{1+\alpha^2\bar{P}\nu\eta_{\text{ms}}}\right\} \\
&\underset{\text{Rayleigh}}{=} \eta_{\text{ms}} + \beta\left[3 - \frac{1}{\bar{P}\eta_{\text{ms}}}\, e^{\frac{1}{\bar{P}\eta_{\text{ms}}}}\,\mathcal{E}_1\!\left(\frac{1}{\bar{P}\eta_{\text{ms}}}\right) - \right. \\
&\left. \frac{2}{\alpha^2\bar{P}\eta_{\text{ms}}}\, e^{\frac{1}{\alpha^2\bar{P}\eta_{\text{ms}}}}\,\mathcal{E}_1\!\left(\frac{1}{\alpha^2\bar{P}\eta_{\text{ms}}}\right)\right] .
\end{aligned}
\tag{3-4}
$$

The spectral efficiency of the detector is then evaluated by substituting the result in (3-2).

In the multi-cell setting considered in this paper, adding a mild restriction that the additive adjacent-cell interference is ergodic in second moment, the SCO detector is equivalent to an optimum detector in a *single-cell* system, where the additive white Gaussian background noise process has spectral level given by $1+2\beta\alpha^2\bar{P}$ (see [11] and references therein for justification). Following [10], the spectral efficiency of the SCO detector is hence given by

$$
\begin{aligned}
\tilde{C}_{\text{sco}} &= \beta E_{\mathcal{F}_\nu}\left\{\log\left[1+\bar{P}_{\text{eq}}\nu\eta^{\text{s-c}}_{\text{ms}}\right]\right\} + \log\frac{1}{\eta^{\text{s-c}}_{\text{ms}}} \\
&\quad + (\eta^{\text{s-c}}_{\text{ms}} - 1)\log e \\
&\underset{\text{Rayleigh}}{=} \beta e^{\frac{1}{\bar{P}_{\text{eq}}\eta^{\text{s-c}}_{\text{ms}}}}\,\mathcal{E}_1\!\left(\frac{1}{\bar{P}_{\text{eq}}\eta^{\text{s-c}}_{\text{ms}}}\right)\log e + \log\frac{1}{\eta^{\text{s-c}}_{\text{ms}}} \\
&\quad + (\eta^{\text{s-c}}_{\text{ms}} - 1)\log e \ ,
\end{aligned}
\tag{3-5}
$$

with $\eta^{\text{s-c}}_{\text{ms}}$ being the unique positive solution of

$$
\begin{aligned}
1 &= \eta^{\text{s-c}}_{\text{ms}} + \beta E_{\mathcal{F}_\nu}\left\{\frac{\bar{P}_{\text{eq}}\nu\eta^{\text{s-c}}_{\text{ms}}}{1+\bar{P}_{\text{eq}}\nu\eta^{\text{s-c}}_{\text{ms}}}\right\} \\
&\underset{\text{Rayleigh}}{=} \eta^{\text{s-c}}_{\text{ms}} + \beta\left[1 - \frac{1}{\bar{P}_{\text{eq}}\eta^{\text{s-c}}_{\text{ms}}}\, e^{\frac{1}{\bar{P}_{\text{eq}}\eta^{\text{s-c}}_{\text{ms}}}}\,\mathcal{E}_1\!\left(\frac{1}{\bar{P}_{\text{eq}}\eta^{\text{s-c}}_{\text{ms}}}\right)\right] ,
\end{aligned}
\tag{3-6}
$$

and $\bar{P}_{\text{eq}} \triangleq \frac{\bar{P}}{1+2\beta\alpha^2\bar{P}}$.

Finally, the spectral efficiency of the MMSE-SC detector is given by

$$
\begin{aligned}
\tilde{C}_{\text{m-sc}} &= \beta E_{\mathcal{F}_\nu}\left\{\log\left(1+\bar{P}\nu\eta_{\mathfrak{C}}\right)\right\} \\
&\quad + 2\beta E_{\mathcal{F}_\nu}\left\{\log\left(1+\alpha^2\bar{P}\nu\eta_{\mathfrak{C}}\right)\right\} \\
&\quad + \log\frac{1}{\eta_{\mathfrak{C}}} + (\eta_{\mathfrak{C}} - 1)\log e \\
&\quad - \left[2\beta E_{\mathcal{F}_\nu}\left\{\log\left(1+\alpha^2\bar{P}\nu\eta_{\mathfrak{S}}\right)\right\} + \log\frac{1}{\eta_{\mathfrak{S}}}\right. \\
&\quad \left. + (\eta_{\mathfrak{S}} - 1)\log e\right] ,
\end{aligned}
\tag{3-7}
$$

which in the case of Rayleigh fading channels yields

$$
\begin{aligned}
\tilde{C}_{\text{m-sc}} &\underset{\text{Rayleigh}}{=} \beta\log e\left[e^{\frac{1}{\bar{P}\eta_{\mathfrak{C}}}}\,\mathcal{E}_1\!\left(\frac{1}{\bar{P}\eta_{\mathfrak{C}}}\right)\right. \\
&\quad + 2e^{\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{C}}}}\,\mathcal{E}_1\!\left(\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{C}}}\right) \\
&\quad \left. - 2e^{\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}}}\,\mathcal{E}_1\!\left(\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}}\right)\right] \\
&\quad + \log\frac{\eta_{\mathfrak{S}}}{\eta_{\mathfrak{C}}} + (\eta_{\mathfrak{C}} - \eta_{\mathfrak{S}})\log e \ .
\end{aligned}
\tag{3-8}
$$

$\eta_{\mathfrak{C}}$ and $\eta_{\mathfrak{S}}$ in (3-7) (3-8) above are uniquely determined by solving the following two implicit equations, respectively,

$$
\begin{aligned}
1 &= \eta_{\mathfrak{C}} + \beta E_{\mathcal{F}_\nu}\left\{\frac{\bar{P}\nu\eta_{\mathfrak{C}}}{1+\bar{P}\nu\eta_{\mathfrak{C}}}\right\} \\
&\quad + 2\beta E_{\mathcal{F}_\nu}\left\{\frac{\alpha^2\bar{P}\nu\eta_{\mathfrak{C}}}{1+\alpha^2\bar{P}\nu\eta_{\mathfrak{C}}}\right\} \\
&\underset{\text{Rayleigh}}{=} \eta_{\mathfrak{C}} + \beta\left[3 - \frac{1}{\bar{P}\eta_{\mathfrak{C}}}\, e^{\frac{1}{\bar{P}\eta_{\mathfrak{C}}}}\,\mathcal{E}_1\!\left(\frac{1}{\bar{P}\eta_{\mathfrak{C}}}\right)\right. \\
&\quad \left. - \frac{2}{\alpha^2\bar{P}\eta_{\mathfrak{C}}}\, e^{\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{C}}}}\,\mathcal{E}_1\!\left(\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{C}}}\right)\right] ,
\end{aligned}
\tag{3-9}
$$

and

$$
\begin{aligned}
1 &= \eta_{\mathfrak{S}} + 2\beta E_{\mathcal{F}_\nu}\left\{\frac{\alpha^2\bar{P}\nu\eta_{\mathfrak{S}}}{1+\alpha^2\bar{P}\nu\eta_{\mathfrak{S}}}\right\} \\
&\underset{\text{Rayleigh}}{=} \eta_{\mathfrak{S}} + 2\beta\left[1 - \frac{e^{\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}}}}{\alpha^2\bar{P}\eta_{\mathfrak{S}}}\,\mathcal{E}_1\!\left(\frac{1}{\alpha^2\bar{P}\eta_{\mathfrak{S}}}\right)\right] .
\end{aligned}
\tag{3-10}
$$

It is important to note at this point, that in terms of spectral efficiency the MMSE-SC detector is in fact the *optimum multiuser detector*, under the assumption of single cell-site processing, and the assumption that the receiver has *no knowledge of the codebooks used in the adjacent cells*, and those codebooks are randomly selected per message (as is indeed assumed in the system model considered, see Subsection III-A). This is evident by noticing the information preserving property of the MMSE estimator in the Gaussian regime, and that the sum of rates attained in the present model by the successive cancellation process, can be shown to correspond to the chain decomposition rule for mutual information (see [11] for more details).

C. Summary of Results

Examining the spectral efficiency results of Subsection III-B, it can be seen that both the matched-filter detector and the SCO detector are interference limited, i.e., their spectral efficiencies reach a limit as $\frac{E_b}{N_0}$ grows without bound. It is also observed that in terms of spectral efficiency, it is optimum, using the above two detectors, to increase the system load β to infinity (see also [10] and [11]). In such case, the effect of fading is *eliminated* and the spectral

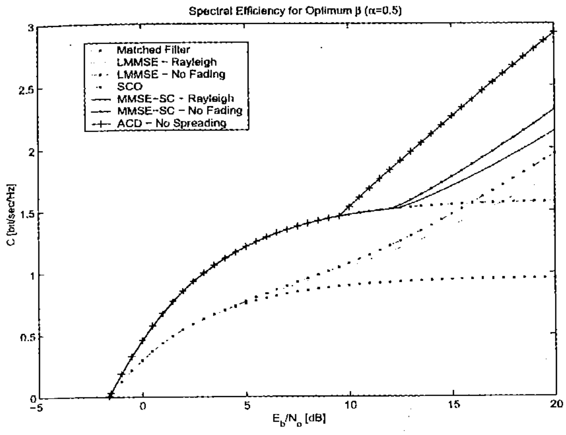

Fig. 4. Spectral efficiency comparison for $\alpha = \frac{1}{2}$, and optimized system load β (Rayleigh fading).

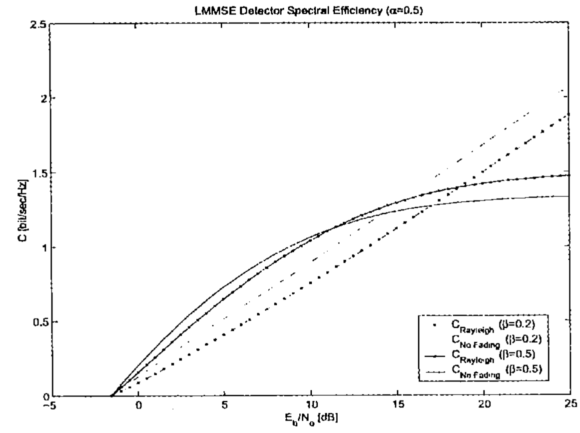

Fig. 5. Spectral efficiency of the linear MMSE detector, with β fixed and $\alpha = \frac{1}{2}$ (Rayleigh fading).

efficiency of the detectors coincides with that attained in *non-fading* channels. This result holds, in fact, regardless of the fading distribution. Increasing the system load without bound also eliminates the penalty due to the use of random spreading. This is observed by comparison to the spectral efficiency of the SCO detector, and that of a detector equivalent to the matched-filter detector, when no spreading is employed and *all* bandwidth is available for coding, as given in [6] (it is noted that the effect of fading in the no-spreading setting is also eliminated in the infinite number of users regime).

In contrast to the above two detectors, the linear MMSE detector and the MMSE-SC detector are not interference limited, provided that the system load β is appropriately chosen. For low $\frac{E_b}{N_0}$, it is optimum, with both detectors, to increase the system load without bound. In such case, the spectral efficiencies of the linear MMSE and MMSE-SC detectors coincide, respectively, with those of the matched-filter and SCO detectors (this equivalence holds for $\beta \to \infty$ regardless of $\frac{E_b}{N_0}$). However, beyond some critical $\frac{E_b}{N_0}$ the optimum system load starts to decrease from infinity, eventually becoming lower than $\frac{1}{3}$, and the spectral efficiencies of both detectors grow without bound with $\frac{E_b}{N_0}$.

Comparative spectral efficiency results of all four multiuser detectors are plotted in Fig. 4 for the case of Rayleigh fading channels an for the *optimum choice of system load* β. Analogously to Section II, the interference factor α was set to $\frac{1}{2}$ to mimic the case in which the average inter-cell interference power equals one half of the average power of intra-cell transmissions ($2\alpha^2 = \frac{1}{2}$), which is also in agreement with the early reports on IS-95 systems. The above described behavior of the spectral efficiency of the detectors is clearly observed in the figure. The sort of "knee effect" in the linear MMSE and MMSE-SC curves designates the region in which the optimum choice for system load starts to decrease from infinity. It is also observed that beyond some critical $\frac{E_b}{N_0}$, the relatively simpler *linear* MMSE detector, that is more informed regarding adjacent-cell interference, is preferable over the interference limited SCO detector that employs *nonlinear* detection of intra-cell users, while treating inter-cell interference as noise.

For the sake of comparison, the spectral efficiencies obtained in non-fading channels are also provided in Fig. 4, demonstrating the performance degradation with both linear MMSE and MMSE-SC detectors due to the presence of fading. It is noted however that for the MMSE type detectors and a *fixed* system load $\beta > \frac{1}{3}$, Rayleigh fading becomes, in fact, *beneficial* in terms of spectral efficiency beyond some critical (β depended) $\frac{E_b}{N_0}$, and the spectral efficiency *with* fading surpasses that of non-fading channels. This result is explained by the "interference population control" effect of fading, effectively reducing the system load as seen by the receiver (see an elaboration on this phenomena in [10]). This behavior is demonstrated in Fig. 5, where the spectral efficiency of the linear MMSE detector is plotted taking $\beta = 0.2 < \frac{1}{3}$, for which the spectral efficiency without fading always surpasses the spectral efficiency in Rayleigh fading channels, and taking $\beta = 0.5 > \frac{1}{3}$, for which Rayleigh fading becomes beneficial beyond some critical $\frac{E_b}{N_0}$. Corresponding results for the MMSE-SC detector are of similar nature.

To complete the comparison (in view of [11]), the spectral efficiency of what is referred to in [6] as the *adjacent-cell decoder (ACD)* was also considered. This detector, that *employs no-spreading*, also knows the *codebooks* of users in adjacent cells, and either decodes their transmissions, or treats them as additive Gaussian noise, whichever is preferable in terms of spectral efficiency. The spectral efficiency of this detector is given by (note the equivalence of K and β in the non-spreading setting)

$$\tilde{C}^{\text{n.s.}}_{\text{ACD}} = \max\Bigg[\log\left(1+\frac{K\bar{P}}{1+2K\alpha^2\bar{P}}\right), \min\left(\frac{1}{3}\log\left(1+(1+2\alpha^2)K\bar{P}\right), \frac{1}{2}\log\left(1+2\alpha^2K\bar{P}\right)\right)\Bigg], \quad (3\text{-}11)$$

and the corresponding numerical results are provided in Fig. 4. As can be seen, for low $\frac{E_b}{N_0}$ it is preferable not to decode adjacent-cell transmissions, and the spectral efficiency of this detector coincides with that of the SCO, and MMSE-SC detectors, for the optimum choice of β (which is $\beta \to \infty$). However beyond some critical $\frac{E_b}{N_0}$, where decoding is preferable, the curves depart and the spectral efficiency of the ACD grows quite rapidly with $\frac{E_b}{N_0}$ (as compared to the other detectors).

IV. Concluding Remarks

We demonstrated in this review the significant enhancement of system performance that can be achieved by employing *joint processing at the cell-site*, for both downlink and uplink channels. Starting with the cellular downlink channel, we discussed a linear preprocessing and encoding scheme, that dramatically improves system performance as compared to the "conventional" approach, eliminating its interference limited behavior and approaching optimum performance at the high SNR region. The main advantage of the scheme is that the whole complexity burden is put on the transmitting end, assuming the availability of full channel state information, while keeping the complexity of mobile receivers as low as possible. It is noted that the single user per cell assumption taken in this respect, is in fact optimal (as far as throughput is concerned) in the non-fading regime, and that is even if optimal decoding for intra-cell users is allowed. This is immediately seen by noticing that the problem reduces to the single user framework, if we allow joint treatment (decoding) of all intra-cell only users together. Clearly in practice this will not be the case, as different users in different locations experience different channel conditions. It is also noted that system performance attained with LP-DP can be further improved by choosing a more general linear transformation, termed as modified RKI [17]. Uniformity in user rates can be achieved by appropriately changing the cell-site ordering. These issues are under current investigation, as well as the application of LP-DP to more complex channel models. Further generalization as to account for general non-TDMA intra-cell scheduling, multi-antenna receivers, and DS-CDMA signalling were also shortly discussed.

Turning to the uplink channel, the benefit of utilizing information about interfering signals in terms of system performance has been demonstrated. The effect is most clearly seen by comparing the linear MMSE detector and the SCO detector, that represent a tradeoff between intra-cell transmissions processing complexity, and additional information on adjacent-cell interference. It was shown that one can gain even without trying to decode the transmissions of the interfering users in adjacent cells (which enables interference *cancellation*), or treating them optimally in the setting of an interference channel (see discussion in [6]). The gain emerges by the very fact that the linear MMSE filter accounts for the reduction of interference, provided that the signatures of interfering users are known not only at the intended cell-site, but at those cell-sites where they cause interference. It may be concluded that for high data rates, inherently demanding high $\frac{E_b}{N_0}$, it is advantageous to mitigate out-of-cell interference through linear MMSE processing. Assuming *equal* transmit powers, it was shown that the matched-filter and SCO detectors are asymptotically unaffected by the presence of fading, in the large (optimum) system load region. In contrast, the linear MMSE and the MMSE-SC detectors experience performance degradation, when fading is present, at the high $\frac{E_b}{N_0}$ region where the system load β is lower than $\frac{1}{3}$. However when *fixing* $\beta > \frac{1}{3}$, both detectors *benefit* from the presence of fading due to its "population control" effect, and attain higher spectral efficiency, as compared to the non-fading case, beyond a critical $\frac{E_b}{N_0}$ value. Finally, it is noted that the analysis of the *optimum* power control policy in the presence of fading, as well as the optimum and suboptimum multi-cell-site processing detectors (see [5], [9]), are currently investigated.

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