

On Strongest-Users-Only Detectors for DS-CDMA Cellular Systems

Benjamin M. Zaidel
The Department of EE,
Technion - IIT,
Haifa 32000, Israel.
e-mail: benny@inter.net.il

Shlomo Shamai (Shitz)
The Department of EE,
Technion - IIT,
Haifa 32000, Israel.
e-mail: sshlomo@ee.technion.ac.il

Sergio Verdú
The Department of EE,
Princeton University,
Princeton, NJ 08544 USA.
e-mail: verdu@ee.princeton.edu

1 Introduction

The fundamental figure of merit that specifies the *ultimate* performance of cellular systems is the per-cell spectral efficiency, defined as the maximum number of bits/sec/Hz that can be transmitted arbitrarily reliably in each of the cells. The spectral efficiency of single-cell, randomly spread, direct sequence code division multiple access (DS-CDMA) systems has been thoroughly analyzed in [1] and [2]. Focusing on the asymptotic setup, where both the number of users per cell K and the processing gain N go to infinity, while $\frac{K}{N} \rightarrow \beta < \infty$ (β is referred to as the "cell load"), analytical and deterministic limiting expressions for the spectral efficiency of various multiuser detectors were obtained, using relevant random matrix theory results.

The underlying tacit assumption in the above spectral efficiency analyses is that all active users are (reliably) decoded *regardless of their received powers*. In slow-fading channels, where the channel fading gains are assumed constant within the codeword duration, the above assumption actually implies that the users adjust their rates (and possibly also their powers) as a function of the channel fading level they experience, via delayless feedback from the receiver. Unfortunately, in practice, such feedback and ideal tuning of the users' transmissions cannot always be accomplished, and motivated by *practical* considerations we consider in this extended abstract an alternative approach.

Independent (over users) identically distributed slow flat-fading channels are assumed, and a simple multi-cell model is considered, adhering to Wyner's infinite linear cell-array setting [3], [4], where only adjacent-cell interference is present, and characterized by a *single* parameter $0 \leq \alpha \leq 1$. As in [1], [2], the discussion is confined to asymptotic analysis. Excluding any form of feedback from the receiving cell-site, but an ACK/NACK type, it is assumed that all users transmit at *equal rates* and *equal powers* regardless of the individual fade levels. The receiver ranks the intra-cell users according to their received powers, and decodes only a subset of the users with strongest received powers, equal in number to the largest integer for which decoding is successful [5], [6]. The scheme is hence referred to as "*strongest-users decoding*". In this setup, the receiver at the cell-site can no longer guarantee reliable decoding of all active users due to the presence of fading (a scheduling protocol can be combined in this setup so that undecoded users are eventually decoded reliably). The total achievable sum-rate over all *decodable* users is referred to as the "*outage constrained capacity*", and considered as the figure of merit of interest for system performance. The *fraction of undecodable users (FUU) per cell* is assumed to be a system design parameter, and can be controlled by the equal rate employed by each of the users. System performance can be further optimized if the cell load is also a degree of freedom.

Assuming single cell-site processing, four types of multiuser detection strategies are considered. The first is the "*conventional*" *matched-filter detector* that treats *all* (either intra-cell or inter-cell) interference as additive white Gaussian noise (AWGN). The second is the *linear MMSE detector* that knows the signatures (spreading sequences) of all interfering users (both intra-cell and in adjacent cells), and mitigates their interference by means of a linear MMSE filter. The third is a detector that "*optimally*" decodes the transmissions of the maximum decodable subset of intra-cell users, while taking into account the structure of the multiuser interference generated by all remaining undecodable *intra-cell* users (assuming their signatures are known at the receiver). *Out-of-cell interference* is treated as AWGN. This detector is referred to henceforth as the "*single-cell optimum (SCO) detector*". Finally, an "optimum" detector is considered (analogous to the SCO detector) that takes into account the structure of *all* undecodable multiuser interference (*both intra-cell and out-of-cell*), assuming all signatures are known at the receiver. This detector is referred to henceforth as the "*single cell-site processing optimum (SCPO) detector*". The above four detection strategies are analyzed and compared in terms of their outage constrained capacities, analytically derived as functions of the FUU (equivalent to the outage probability). The reader is referred to [7] and [8] for more details on the derivations.

2 Summary of Results

Examining the outage constrained capacity expressions, and focusing on the low signal to noise ratio (SNR) regime, an important property of the strongest-users decoding scheme can be observed. Denoting by \bar{R}_T the outage constrained

capacity, the transmit SNR by \bar{P} , and defining the system average $\frac{E_b}{N_0}$ through the relation $\bar{P} = \frac{1}{\beta} \bar{R}_T \frac{E_b}{N_0}$, the minimum received $\frac{E_b}{N_0}$ that enables reliable communications is found to be

$$\frac{E_b}{N_{0, \min}} = \frac{\ln 2}{(1-Q)\mathcal{F}_v^{-1}(Q)} \stackrel{\text{Rayleigh}}{=} -\frac{\ln 2}{(1-Q)\ln(1-Q)}, \quad (2-1)$$

where Q denotes the limiting per-cell FUU, and $\mathcal{F}_v^{-1}(\cdot)$ denotes the inverse function of cumulative distribution function of the fade power levels. Comparing this result to $\frac{E_b}{N_{0, \min}} = \ln 2 \approx -1.59\text{dB}$, which corresponds to the case where the rates are such that *all* users are decoded [2], the penalty in $\frac{E_b}{N_{0, \min}}$ induced by the strongest-users decoding scheme is found to be a factor of $[(1-Q)\mathcal{F}_v^{-1}(Q)]^{-1}$. This penalty is attributed to the power "wasted" when users fail to be decoded. For the particular case of Rayleigh fading, while minimizing (2-1) with respect to Q , it follows that $\frac{E_b}{N_{0, \min}} = e \ln 2 \approx 2.75\text{dB}$, which is achieved for $Q^* = 1 - 1/e$ (being hence the *optimum FUU* in the low SNR regime, for Rayleigh fading channels). As can be observed, a severe penalty of $10 \log_{10} e \approx 4.34\text{dB}$ is induced to $\frac{E_b}{N_{0, \min}}$ by the strongest-users decoding scheme, in Rayleigh fading channels.

Comparative outage-constrained capacity results in Rayleigh fading channels, are presented in Fig. 1. An interference factor of $\alpha = \frac{1}{2}$ was assumed, to mimic the case in which the total out-of-cell interference power equals one half of the intra-cell interference power. The outage constrained capacity for all detectors was evaluated for the (numerically obtained unless otherwise stated) optimum values of both the cell load β and the FUU. Both optimum parameters are in general functions of $\frac{E_b}{N_0}$. For the sake of comparison the *spectral efficiencies* of the four detectors (reproduced from [9]) are also included in Fig. 1.

The outage constrained capacity of the matched-filter detector monotonically increases with the cell load β . It is therefore optimum to take $\beta \rightarrow \infty$, and it can be shown that for *all* $\frac{E_b}{N_0}$ values the optimum FUU for this detector, in Rayleigh fading channels, is $Q^* = 1 - 1/e$. The detector is interference limited, as can be seen from Fig. 1, and the outage constrained capacity reaches the limit of $\frac{\log e}{e(1+2\alpha^2)} = 0.3538 \text{ bits/sec/Hz}$ (for $\alpha = \frac{1}{2}$) as $\frac{E_b}{N_0} \rightarrow \infty$ (as opposed to $\frac{\log e}{1+2\alpha^2} = 0.9618 \text{ bits/sec/Hz}$ when all users are decoded).

The SCO detector is also interference limited. The results show that while optimizing with respect to *both* β and Q , it is optimum in terms of the outage constrained capacity to take $\beta \rightarrow \infty$ for all $\frac{E_b}{N_0}$ values. The optimum Q monotonically decreases with $\frac{E_b}{N_0}$, eventually reaching a limit of $Q^* = 0.4795$, for $\alpha = \frac{1}{2}$, as $\frac{E_b}{N_0} \rightarrow \infty$. However, fixing the FUU, the optimum choice of cell load β (as a function of $\frac{E_b}{N_0}$) depends on the value of the adjacent-cell interference factor α , and may take on finite values in the high $\frac{E_b}{N_0}$ region (decreasing from infinity beyond some critical $\frac{E_b}{N_0}$), provided that the factor α is low enough. This behavior is in agreement with the behavior of the detector in a *single-cell* setup [7] corresponding to $\alpha = 0$ (in which case it is the "optimum" strongest-users-only detector), where it was shown that the detector is interference limited when $\beta \rightarrow \infty$ for any *fixed* FUU, and therefore finite system loads are optimum beyond some critical $\frac{E_b}{N_0}$. For the particular case of $\alpha = \frac{1}{2}$, however, the results show that taking $\beta \rightarrow \infty$ is optimum for *all* $\frac{E_b}{N_0}$ values.

In contrast to the above two detectors, the linear MMSE detector is not interference limited, and its *optimized* outage constrained capacity grows without bound with $\frac{E_b}{N_0}$. For low $\frac{E_b}{N_0}$ values, it is optimum to take $\beta \rightarrow \infty$, in which case the outage constrained capacity of the linear MMSE detector coincides with that of the matched-filter detector. However, with the increase of $\frac{E_b}{N_0}$ the optimum cell load β starts to decrease beyond a critical $\frac{E_b}{N_0} \triangleq \frac{E_b}{N_{0, \text{cr}}}$, eventually becoming lower than $\frac{1}{3}$ (note that the detector processes the signals of users of *three cells*). Assuming $E_{\mathcal{F}_v} \{\nu^2\} < \infty$ (denoting by $E_{\mathcal{F}_v} \{\cdot\}$ the expectation with respect to \mathcal{F}_v), it can be shown that

$$\frac{E_b}{N_{0, \text{cr}}} = \frac{2(1+2\alpha^4)E_{\mathcal{F}_v} \{\nu^2\} \frac{E_b}{N_{0, \min}}}{2(1+2\alpha^4)E_{\mathcal{F}_v} \{\nu^2\} - (1+2\alpha^2)\mathcal{F}_v^{-1}(Q)} \stackrel{\text{Rayleigh}}{=} \frac{4(1+2\alpha^4) \frac{E_b}{N_{0, \min}}}{4(1+2\alpha^4) + (1+2\alpha^2)\ln(1-Q)}, \quad (2-2)$$

where $\frac{E_b}{N_{0, \min}}$ is given by (2-1). The linear MMSE detector outperforms the matched-filter detector for $\frac{E_b}{N_0} > \frac{E_b}{N_{0, \text{cr}}}$. The optimum FUU also decreases with $\frac{E_b}{N_0}$ beyond $\frac{E_b}{N_{0, \text{cr}}}$, below which it equals $Q^* = 1 - 1/e$ for Rayleigh fading channels. For Rayleigh fading and $\alpha = \frac{1}{2}$, (2-2) yields $\frac{E_b}{N_{0, \text{cr}}} = 1.5e \ln 2 \approx 4.51\text{dB}$.

The SCPO detector is also not interference limited, provided that *both* the cell load β and the FUU are appropriately chosen. For low $\frac{E_b}{N_0}$ values it is optimum to take $\beta \rightarrow \infty$. However, beyond some critical $\frac{E_b}{N_0}$ the optimum cell load starts to decrease monotonically with $\frac{E_b}{N_0}$, while the outage constrained capacity grows without bound. With $\beta \rightarrow \infty$ the outage constrained capacity of the SCPO detector coincides with that of the SCO detector, and the detector becomes interference limited, regardless of the FUU. This behavior is in sheer contrast to single-cell setup [7], in which $\beta \rightarrow \infty$ is optimum for *all* $\frac{E_b}{N_0}$ values if the optimization is performed over *both* Q and β . In fact, in the single-cell setup, letting the FUU vanish with $\frac{E_b}{N_0}$ at an appropriate rate, the outage constrained capacity of the detector approaches the *optimum spectral efficiency* at the high $\frac{E_b}{N_0}$ region, as shown in Fig. 2 (please note that a significant performance degradation is still observed in the single-cell setup for the two *linear* detectors). As can be observed from Fig. 1, this is no longer the case

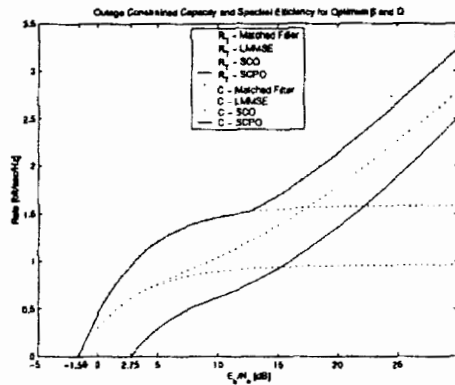


Figure 1: Optimized spectral efficiencies and outage constrained capacities for $\alpha = \frac{1}{2}$ (Rayleigh fading).

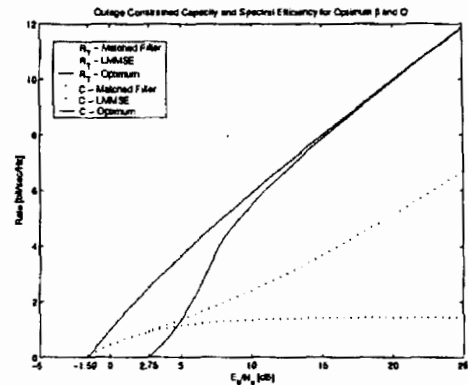


Figure 2: Optimized spectral efficiencies and outage constrained capacities for $\alpha = 0$ (Rayleigh fading).

when undecodable out-of-cell interference is introduced, and a clear performance degradation is observed as compared to the ultimate spectral efficiency of the detector. Examining the optimum FUU of the detector, it is observed that in the low $\frac{E_b}{N_0}$ region where it is optimum to take $\beta \rightarrow \infty$, the optimum FUU of the SCPO detector coincides with that of the SCO detector. However, beyond a critical $\frac{E_b}{N_0}$, which corresponds to the point beyond which the optimum cell load decreases to finite values, the optimum FUU of the SCPO detector starts to decrease more rapidly with $\frac{E_b}{N_0}$ as compared to that of the SCO detector, which goes to a limit.

To conclude, comparing the outage constrained capacities of all four multiuser detectors to their corresponding optimum spectral efficiencies [9], the strongest-users decoding scheme is observed to induce a severe penalty in system performance, emphasizing the crucial role of *rate-adjustment feedback* in the presence of undecodable out-of-cell interference.

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