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Optimum Multiuser Asymptotic Efficiency of CDMA with Random Spreading

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Abstract — **This paper analyzes the hign SNR performance of optimum multiuser detectors for synchronous direct-sequence spread spectrum with random spreading in an additive white Gaussian noise channel. We show that the asymptotic efficiency of a K-user system with spreading gain N converges to 1 almost surely as K and N go to infinity, and K/N is kept equal to an arbitrary constant.**

I. Introduction

In direct-sequence spread spectrum systems with spreading gain N, each user is assigned a signature waveform which is a linear combination (with $\pm$ coefficients) of N orthogonal waveforms. In the random spreading model studied in this paper the binary sequences are chosen randomly and independently with all choices being equally likely [1]. This is a simple model which lends itself to analysis and accurately models CDMA with long signature sequences. A number of recent works [1, 2, 3, 4, 5] have analyzed the asymptotic performance of direct-sequence spread spectrum with random spreading when the number of users K grows without bound and the ratio of number of users to the spreading gain N is kept fixed to $K/N = \beta$. More specifically, under that asymptotic regime, reference [4] has found the Shannon capacity achievable by the maximum-likelihood decoder, the single-user matched filter, the decorrelating detector, and the MMSE multiuser detector, for systems with perfect power control; and [5] has investigated the signal-to-interference ratio of the single-user matched filter, the decorrelating detector, and the MMSE multiuser detector without assuming equal received powers.

Uncoded bit-error-rate has received much attention as a performance measure of multiuser detectors. Of particular interest is the asymptotic multiuser efficiency [1] which characterizes the performance loss (in effective SNR) as the background noise vanishes. If a particular receiver achieves bit error rate $\mathsf{P}_k(\sigma)$ in the presence of multiple access interference and additive white Gaussian noise with power spectral level equal to σ^2, the asymptotic multiuser efficency is given by

$$\eta_k = \frac{2}{A_k^2} \lim_{\sigma^2} \sigma^2 \log 1/\mathsf{P}_k(\sigma).$$

where A_k^2 is the received power of the kth user.

Under the foregoing random spreading model, the following behavior is known for the asymptotic efficiency of the single-user matched filter η^c, decorrelator η^d, and linear MMSE detector η^m, as $K \to \infty$ and $K/N = \beta$:

$$\begin{aligned} \eta^c &\to 0 \quad \text{(Problem 3.39 in [1])} \\ \eta^d &\to [1-\beta]^+ \quad \text{(Problem 5.27 in [1])} \\ \eta^m &\to [1-\beta]^+ \quad \text{((6.62) in [1])} \end{aligned}$$

II. New results

In this paper we show that the optimum asymptotic efficiency converges to 1 almost surely under the following sufficient conditions:

- $0 < \beta < 1$ and $\inf A_i > 0$
- $\beta \geq 1$ and $A_1 = \ldots = A_K > 0$

This shows the surprising result that even if the number of users is a large multiple of the spreading gain, the odds are that the nearest neighbor to each of the transmitted signal is at the same distance as if no interferers were present in the channel. Naturally, as the number of users per degree of freedom β increases we expect that the number of nearest neighbors increases, and accordingly, the asymptotic efficiency is representative of the low-noise behavior only for increasingly higher SNR. In this regard, we caution that, in the setting considered in this paper, the bit-error-rate is investigated as the background noise σ vanishes for fixed K and N, and then the limits with respect to K and N are taken. An interesting open problem is the behavior of the bit-error-rate for fixed SNR as K and N grow.

References

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