

Random CDMA in the Multiple Cell Uplink Environment: The Effect of Fading on Various Receivers

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Abstract — *A simple multi-cell Rayleigh fading uplink communication model is suggested and analyzed for optimally coded randomly spread DS-CDMA with multiuser detection. The model adheres to Wyner's (1994) infinite linear cell-array setting, according to which only adjacent-cell interference is present, and characterized by a single parameter $0 \leq \alpha \leq 1$. The discussion is confined to asymptotic analysis where both the number of users per cell and the processing gain go to infinity, while their ratio goes to some finite constant. The spectral efficiency of various multiuser detection strategies is evaluated assuming single cell-site processing, and equal transmit powers for all users in all cells. Comparative results demonstrate how performance is affected by the introduction of inter-cell interference (with and without fading), and what is the penalty associated with the randomly spread coded DS-CDMA strategy.*

I. INTRODUCTION

Direct sequence code division multiple access (DS-CDMA) systems employing random spreading signatures, are in the focus of many information-theoretic analyses of recent years, in view of their wide-spread practical use. Particularly, the limiting scenario is examined, where both the number of users and the processing gain go to infinity, while their ratio goes to some finite constant. Thus, deterministic system performance measures of interest can be obtained, using relevant random matrix theory results. Analyses of a *single-cell* DS-CDMA system, were recently presented in [1] – [3] (see also references therein). This abstract addresses *multi-cell* systems, using the attractive cellular model suggested by Wyner in [4]. This simple model allows for analytical tractability on one hand, while giving insight to practical systems on the other. Accordingly, a *fully synchronous* cellular system is considered, where the cells compose an infinite *linear* array, and where the received signal at each cell-site is the sum of the Rayleigh faded signals received from intra-cell users, plus a factor α ($0 \leq \alpha \leq 1$) times the sum of the faded signals generated by users in the two adjacent cells. Non-adjacent cell users are assumed to produce no interference. The received signal is embedded in ambient Gaussian noise. The multi-cell effect on

performance is thus specified by a *single* parameter (α). The same model is also used by the authors in [5], which is devoted to non-fading channels.

Flat-fading channels are assumed, as in [2], [6] and [7], and the spectral efficiency obtained by employing *optimally* coded randomly spread DS-CDMA with multiuser detection is analyzed. As in [1] – [3], the limiting case is considered in which the number of intra-cell users K (assumed constant and equal in all cells), and the spreading factor N (processing gain), both go to infinity, while $\frac{K}{N} \rightarrow \beta < \infty$. The factor β is commonly referred to as the “system load”. Assuming *single cell-site processing*, four types of multiuser detection strategies are considered (as in [5]): 1) The “conventional” *matched-filter detector* that treats *all* interference (either intra-cell or inter-cell) as additive white Gaussian noise; 2) A *single-cell optimum (SCO)* detector that “optimally” detects the transmissions of *intra-cell* users, while treating *inter-cell* interference as additive white Gaussian noise; 3) The *linear MMSE detector* that knows the signature sequences of all interfering users (both intra-cell and in adjacent cells) and mitigates their interference by means of a linear MMSE filter; 4) A detector that employs *MMSE based successive interference cancellation (MMSE-SC)* to decode transmissions of intra-cell users, while *inter-cell* interference is mitigated by means of a linear MMSE filter. It is emphasized that neither the linear MMSE detector, nor the MMSE-SC detector, try to *decode* the transmissions of adjacent cell users (which might be prohibitive if α is small), and are only aware of the signature sequences of all users in adjacent cells. In addition to the above, it is also assumed that all detectors are provided with the required knowledge regarding the received powers of the interfering signals.

Identifying the spectral efficiency as the fundamental measure of system performance for coded systems (see [1]), the spectral efficiency of all four detection strategies is obtained, and comparatively examined assuming a *constant* (fading independent) transmit power. The results are then compared to analogous results without fading (appropriately reproduced from [5]). Finally, the penalty in system performance due to random spreading is also examined, by comparison (following [7]) to the spectral efficiency of a few corresponding detectors, in the setting in which all bandwidth is available for coding (as opposed to bandwidth expansion by DS-spreading).

II. MULTI-CELL SYSTEM MODEL

Using the standard discrete time equivalent channel representation, the signal vector received at an arbitrary cell site, at the discrete time related to the transmission of the i th symbol, is given by

$$\mathbf{y}_i = \mathbf{S}_i \mathbf{H}_i \mathbf{x}_i + \alpha \mathbf{S}_i^- \mathbf{H}_i^- \mathbf{x}_i^- + \alpha \mathbf{S}_i^+ \mathbf{H}_i^+ \mathbf{x}_i^+ + \mathbf{n}_i. \quad (1)$$

The vectors $\mathbf{x}_i = [x_{1,i}, \dots, x_{K,i}]^T$, $\mathbf{x}_i^\pm = [x_{1,i}^\pm, \dots, x_{K,i}^\pm]^T$, comprise the K code symbols transmitted by intra-cell users and adjacent-cell users, respectively, at the i th discrete time. All code symbols are assumed to be i.i.d. zero-mean proper complex Gaussian random variables (which conforms with the capacity-achieving statistics), with variance \bar{P} designating the equal transmit power of all users. This model is justified by assuming that the codebooks of all users are chosen randomly, governed by an underlying i.i.d. Gaussian distribution per symbol, and *independently* for each message transmission (see [7]). The matrices \mathbf{S}_i and \mathbf{S}_i^\pm are $N \times K$ matrices, whose columns are the N -chip spreading (signature) sequences of the K users in the considered cell and in its adjacent cells, respectively. The entries of the above matrices are treated as i.i.d. zero-mean random variables, with variance $1/N$. The vector \mathbf{n}_i represents a zero-mean white proper complex Gaussian noise vector, with $E\{\mathbf{n}_i \mathbf{n}_i^\dagger\} = \mathbf{I}$, $\forall i$. Without loss of generality all received powers are thus normalized with respect to the noise spectral level, and represent in fact the signal to noise ratios (SNRs) at the input to the multiuser detectors. Finally, $\mathbf{H}_i \triangleq \text{diag}(h_{1,i}, \dots, h_{K,i})$ and $\mathbf{H}_i^\pm \triangleq \text{diag}(h_{1,i}^\pm, \dots, h_{K,i}^\pm)$, where $\{h_{k,i}\}_{k=1}^K$ and $\{h_{k,i}^\pm\}_{k=1}^K$ designate the i.i.d., zero-mean, complex Gaussian channel fading gains associated with the signals of the different users, at the i th discrete time. It shall be assumed henceforth that as the system size becomes large ($N, K \rightarrow \infty, \frac{K}{N} \rightarrow \beta < \infty$), the empirical distribution of the channel fading gains converges a.s. to a distribution \mathcal{H} . For regularity reasons it is assumed that $E_{\mathcal{H}}\{|h|^2\} = 1$ (where h denotes some arbitrary fading gain). The fading power is henceforth denoted by $\nu \triangleq |h|^2$.

III. SPECTRAL EFFICIENCY OF THE MULTIUSER DETECTORS

The per cell *spectral efficiency* (see [1], [2]), or throughput, is defined as the total number of bits/sec/Hz that can be transmitted arbitrarily reliably in each cell. The spectral efficiency of a linear detector is conveniently expressed in terms its *multiuser efficiency* (see [8]), defined as the ratio between the detector's output SIR and the SNR. Denoting the multiuser efficiency by η , and following central limit results showing that the interference at the output of each of the two linear detectors, i.e., the matched-filter detector and the linear MMSE detector, is well approximated by a Gaussian noise (see [1], [5] and references therein for justification of this Gaussian approximation), the spectral efficiency of these detectors is

given by

$$\tilde{C} = \beta E_\nu \{\log(1 + \nu \eta \bar{P})\} = \beta e^{\frac{1}{\beta \eta}} E_1 \left(\frac{1}{\bar{P} \eta} \right) \log e, \quad (2)$$

where $E_1(x) \triangleq \int_x^\infty \frac{e^{-t}}{t} dt$, ($t > 0$) is the exponential integral function, and $E_\nu \{\cdot\}$ denotes the expectation with respect to the fading power ν . It is noted, that when different systems are to be compared (with possibly different spreading gains and data rates), it is useful to express the spectral efficiency in terms of $\frac{E_b}{N_0}$, which is done through the relation $\bar{P} = \frac{1}{\beta} \tilde{C} \frac{E_b}{N_0}$.

The spectral efficiency of the two non-linear detectors, i.e., the SCO detector and the MMSE-SC detector, is most conveniently evaluated using inter-relations between the spectral efficiency of the optimum multiuser detector and that of the *linear* MMSE detector. It is important to note at this point, that in terms of spectral efficiency the MMSE-SC detector is in fact the *optimum multiuser detector*, under the assumption of single cell-site processing, and the assumption that the receiver has *no knowledge of the codebooks used in the adjacent cells*, and those codebooks are randomly selected per message (as is indeed assumed in the system model considered, see Section II). This is evident by noticing the information preserving property of the MMSE estimator in the Gaussian regime, and that the sum of rates attained in the present model by the successive cancellation process, can be shown to correspond to the chain decomposition rule for mutual information (see [5] for more details).

Due to space limitations, explicit spectral efficiency results were not included in this abstract. The interested reader is referred to [9] for detailed results, and to [5] for the corresponding results in non-fading channels.

IV. SUMMARY OF RESULTS

Examining the spectral efficiency results, it is observed that both the matched-filter detector and the SCO detector are interference limited, and that in terms of spectral efficiency it is optimum, using the above two detectors, to increase the system load β to infinity (see also [2] and [5]). In such case, the effect of fading is *eliminated* and the spectral efficiency of the detectors coincides with that attained in *non-fading* channels. Taking $\beta \rightarrow \infty$ also eliminates the penalty due to the use of random spreading, as observed by comparison to the results of [7].

In contrast to the above two detectors, the linear MMSE detector and the MMSE-SC detector are not interference limited, provided that the system load β is appropriately chosen. For low $\frac{E_b}{N_0}$, it is optimum, with both detectors, to take $\beta \rightarrow \infty$, and the spectral efficiencies of these detectors coincide, respectively, with those of the matched-filter and SCO detectors (an equivalence that holds in general when $\beta \rightarrow \infty$). However, beyond some critical $\frac{E_b}{N_0}$ the optimum system load starts to decrease from infinity, eventually becoming lower than $\frac{1}{3}$, and the spectral efficiencies of both detectors grow without bound with $\frac{E_b}{N_0}$.

Comparative spectral efficiency results of all four multiuser detectors are plotted in Fig. 1 for the *optimum choice of system load* β . The interference factor α was set to $\frac{1}{2}$ to mimic the case in which the average inter-cell interference power equals one half of the average power of intra-cell transmissions ($2\alpha^2 = \frac{1}{2}$), which is in agreement with the early reports on IS-95 systems. The sort of “knee effect” in the linear MMSE and MMSE-SC curves designates the region in which the optimum choice for system load starts to decrease from infinity. The results demonstrate the dramatic effect of information about interfering signals on system performance. The effect is most clearly seen by comparing the linear MMSE detector and the SCO detector, that represent a tradeoff between intra-cell transmissions processing complexity, and additional information on adjacent-cell interference. It is observed that one can gain even without trying to decode the transmissions of the interfering users in adjacent cells (which enables interference *cancellation*), or without treating them optimally in the setting of an interference channel (see [7]). The gain emerges by the very fact that the linear MMSE filter accounts for the reduction of interference, provided that the signatures of interfering users are known not only at the intended cell-site, but at those cell-sites where they cause interference. It may be concluded that for high data rates, inherently demanding high $\frac{E_b}{N_0}$, it is advantageous to mitigate out-of-cell interference through linear MMSE processing.

For the sake of comparison, the spectral efficiencies obtained in non-fading channels are also provided in Fig. 1, demonstrating the performance degradation with both linear MMSE and MMSE-SC detectors due to the presence of fading. It is noted however that with the MMSE type detectors and a *fixed* system load $\beta > \frac{1}{3}$, Rayleigh fading becomes, in fact, *beneficial* in terms of spectral efficiency beyond some critical (β depended) $\frac{E_b}{N_0}$, and the spectral efficiency *with* fading surpasses that of a non-fading channel. This result is explained by the “interference population control” effect of fading, effectively reducing the system load as seen by the receiver (see an elaboration on this phenomena in [2]). Fig. 1 also shows the spectral efficiency of the *adjacent-cell decoder* (ACD) (see [7]), that also knows the *codebooks* of users in adjacent cells, and either decodes their transmissions (beyond a critical $\frac{E_b}{N_0}$), or treats them as additive Gaussian noise (at lower $\frac{E_b}{N_0}$ values), whichever is preferable in terms of spectral efficiency.

V. CONCLUDING REMARKS

Assuming *equal* transmit powers, it was shown that the matched-filter and SCO detectors are asymptotically unaffected by the presence of fading, in the large (optimum) system load region. In contrast, the linear MMSE and the MMSE-SC detectors experience performance degradation, when fading is present, at the high $\frac{E_b}{N_0}$ region where the *optimum* system load β is lower than $\frac{1}{3}$. However when *fixing* $\beta > \frac{1}{3}$, both detectors *benefit* from the presence of fading, due to its “population control” effect, and attain higher spectral efficiency, as compared

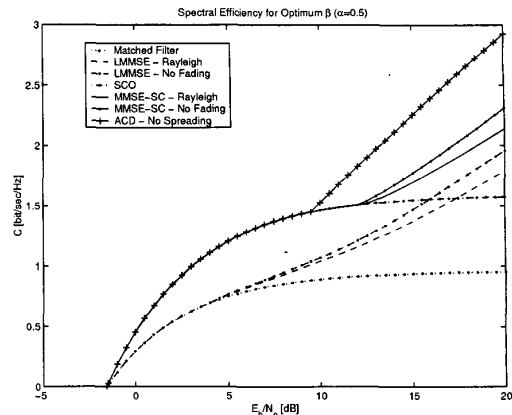


Figure 1: Spectral efficiency comparison for $\alpha = \frac{1}{2}$, and optimized system load β .

to the non-fading case, beyond a critical $\frac{E_b}{N_0}$ value. It is noted that whenever the actual fixed system load is higher than the optimum load, “population control” can also be achieved by simplified power control, i.e., by allocating non-zero equal transmit power to only a *fraction* of the intra-cell users (with appropriate power scaling), and thus reducing the effective system load till it reaches the optimum load.

Finally, it is noted that the analysis of the *optimum* power control policy in the presence of fading, as well as the optimum and suboptimum multi-cell-site processing detectors (see [4], [6]), are currently investigated.

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