Cognitive Interference Channels with State Information

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***Abstract*— Cognitive state-dependent interference channels are analyzed. We focus on the two-user case with two message sources. One of the transmitters, referred to as the cognitive informed user, knows both messages and also the states of the channel in a non-causal manner. The other transmitter knows only one of the messages and does not know the channel states. Each of the two decoders is supposed to decode only its intended message. Inner and outer bounds on the capacity region of this channel are provided for the general finite input alphabet case. The asymmetric state-dependent Gaussian weak interference channel with non-causal state information is then considered, and a closed form formula for the capacity region is established in the regime of weak interference.**

I. Introduction

Wireless channels are dynamic and their propagation characteristics change in time. A useful model for a time-varying wireless channel is a channel whose instantaneous parameters depend on a random state sequence. Channel states can also model interfering signals. The concept of channel states available at the transmitter, was introduced and studied by Shannon [1], in the setup of a state-dependent memoryless channel whose states are i.i.d. and available causally to the transmitter. Gel'fand and Pinsker [2] established a single-letter formula for the capacity of the same channel where the transmitter observes the channel states non-causally. For a review on the subject of channels with state information and related work see [3] and references therein.

Similarly to the single-user Gaussian channel with non-causal channel state information (CSI) studied by Costa [4], it has been shown that interference cancelation is possible also for certain Gaussian state-dependent multi-user channels with non-causal CSI including: MAC with CSI known to the two transmitters, and the broadcast channel [5], physically degraded Gaussian relay channel with CSI both at the relay and the transmitter, [6]. An exception to this principle is noted in [6] where it is argued that interference cancelation is impossible for a state-dependent non-cognitive strong interference Gaussian channel with CSI available to the two transmitters.

Wireless communication systems in which signals intended to one receiver are also received at other receivers can be modeled by an interference channel. Understanding interference channels, whose capacity region remains unknown to date, has been the subject of much research activity (see, e.g., [7], [8], [9], [10], [11], [12] and the references therein).

Interference channels with a common message model situations in which transmitters can cooperate. Interference channels with cooperating encoders were studied in [13]. An interference channel with one user having full knowledge of the other user's message, a.k.a. cognitive interference channel or interference channel with degraded message sets, was studied in [14], [15] for the strong interference case and in [16] for the weak interference case. A different setup where the common message is also required to be decoded by the two receivers was analyzed in [17].

In [18], [19], [20] a generalization of the Gel'fand-Pinsker setup to a model which encompasses the setup of a cognitive memoryless multiple-access channel (MAC) is presented. According to this setup, only one of the encoders knows the states of the channel (non-causally), which is also unknown to the receiver. Two independent messages are transmitted: a common message and a message transmitted by the informed encoder. An explicit characterization of the capacity region of this channel is established. Further, the general formula is specialized to the Gaussian case with non-causal channel state information, under a power constraint. In this case, the capacity region is achievable by a generalized writing-on-dirty-paper scheme.

The motivation for this paper is the exploration of the interplay between interferences and transmitter channel state information and their effect on achievable communication rates.

We study the state-dependent cognitive interference channel, with two transmitters and two receivers (see Fig. 1). Transmitter 1 knows only message 1, and transmitter 2 (the cognitive transmitter) knows both messages 1 and 2 and the channel's states sequence non-causally. As in the ordinary interference channel, receiver 1 needs to decode only message 1 and receiver 2 needs to decode message 2.

This channel can model a cognitive radio (see [21] and references therein, [16], [22], [23], [24]), which is a device capable of sensing its environment and making use of the gained knowledge to increase the spectral efficiency of the wireless system by exploiting unused spectral resources. In the model we study, transmitter 2 models the cognitive radio, which can help transmitting message 1 of the primary transmitter, and also transmit an additional message source.

The channel model studied in this paper generalizes (a) the

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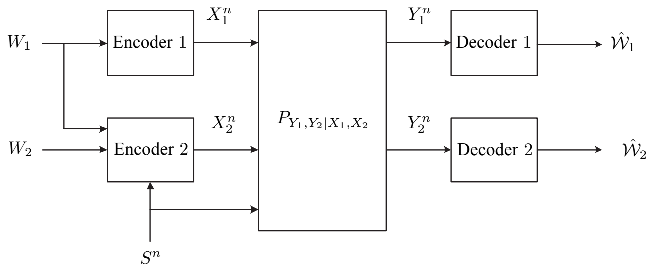

Fig. 1. Cooperative State-Dependent Interference Channel.

cognitive interference channel of [16] to the case where the channel is state-dependent and the cognitive transmitter has non-causal CSI, and (b) the cognitive MAC with asymmetric CSI [20] which is a special case of interference channel with identical receivers inputs.

In this paper, we establish inner and outer bounds to the capacity region of the cognitive interference channel with asymmetric non-causal CSI. We further derive the capacity region for the Gaussian weak interference case.

The paper is organized as follows. In Section II we introduce the cognitive state-dependent interference channel with asymmetric CSI. Section III presents the inner and outer bounds on the capacity region, and in Section IV we establish the capacity region for the Gaussian weak interference case.

II. Problem Setup

A memoryless stationary state-dependent interference channel is defined by the input alphabets $\mathcal{X}_1, \mathcal{X}_2$, the states alphabet $\mathcal{S}$ and distribution $\mathcal{Q}_S$, the outputs alphabets $\mathcal{Y}_2, \mathcal{Y}_2$, and a conditional probability measure $P_{Y_1,Y_2|X_1,X_2,S}$. The state sequence is assumed to be emitted by a memoryless stationary source $\mathcal{Q}_S$. The conditional distribution of the channel output n-sequences Y_1^n, Y_2^n given the input and states n-sequences X_1^n, X_2^n, S^n takes the product form

$$\begin{aligned} & P_{Y_1^n,Y_2^n|X_1^n,X_2^n,S^n}(y^n,\tilde{y}^n|x^n,\tilde{x}^n,s^n) \\ = & \prod_{i=1}^{n} P_{Y_1,Y_2|X_1,X_2,S}(y_i,\tilde{y}_i|x_i,\tilde{x}_i,s_i). \end{aligned} \quad (1)$$

Let W_1, W_2 designate the messages which are independent random variables, uniformly distributed on the sets $\mathcal{W}_1 = \{1, ..., M_1\}, \mathcal{W}_2 = \{1, ..., M_2\}$, respectively. The interference channel is said to be *asymmetric cognitive* if only encoder 2 knows the channel states sequence, and the encoders for the channel are defined by the mappings

$$\begin{aligned} \varphi_{1,n} &: \mathcal{W}_1 \to \mathcal{X}_1^n \\ \varphi_{2,n} &: \mathcal{W}_1 \times \mathcal{W}_2 \times \mathcal{S}^n \to \mathcal{X}_2^n. \end{aligned} \quad (2)$$

A decoder for the asymmetric cognitive interference channel is defined by the mappings

$$\begin{aligned} \psi_{1,n} &: \mathcal{Y}_1^n \to \mathcal{W}_1 \\ \psi_{1,n} &: \mathcal{Y}_2^n \to \mathcal{W}_2. \end{aligned} \quad (3)$$

We denote $R_i = \lfloor \frac{1}{n} \log M_i \rfloor$, and say that (R_1, R_2) is the rate-pair of the code $(\varphi_{1,n}, \varphi_{2,n}, \psi_{1,n}, \psi_{2,n})$. A code is said to be an (ϵ, R_1, R_2)-code if

$$P_e^{(n)} = \Pr\{\psi_{1,n}(Y_1^n) \neq W_1 \text{ or } \psi_{2,n}(Y_2^n) \neq W_2\} \leq \epsilon. \quad (4)$$

A rate-pair (R_1, R_2) is achievable if there exists a sequence of (ϵ_n, R_1, R_2) codes, $\{(\varphi_{1,n}, \varphi_{2,n}, \psi_{1,n}, \psi_{2,n})\}_{i=1}^n$, with $\epsilon_n \to 0$. The capacity region is the closure of the set of achievable rate-pairs.

III. Bound on the Capacity Region – Finite Alphabet Case

The following result gives an achievable region for the finite alphabet cognitive asymmetric non-causal CSI interference channel.

Theorem 1 *(inner bound) The closure of the convex hull of the set of rate-pairs that satisfy*

$$\begin{aligned} R_1 &\leq I(U, X_1; Y_1) - I(U, X_1; S) \\ R_2 &\leq I(V; Y_2) - I(V; U, X_1, S), \end{aligned} \quad (5)$$

for some joint law

$$P_{S,X_1,U,V,X_2,Y_1,Y_2} = P_S P_{X_1} P_{U,V,X_2|S,X_1} P_{Y_1,Y_2|S,X_1,X_2}, \quad (6)$$

is achievable for the finite alphabet cognitive asymmetric non-causal CSI interference channel.

The random-coding scheme relies on the generalized binning principle of [18], [20]. We next describe the random coding scheme that achieve this region.

Generation of the Codebooks: Let $M_1 = M_{1,a} \times M_{1,b}$. Encoder 1's codebook is $\{\mathbf{x}_1, ..., \mathbf{x}_{M_{1,a}}\}$ whose codewords are selected randomly and independently with i.i.d. components drawn from distribution P_{X_1}. For each codeword, $\mathbf{x}_\ell$, $\ell = 1, ..., M_{1,a}$, a codebook (referred to as a U-codebook) consisting of $M_{1,b} \times J_1$ auxiliary vectors, denoted $\{\mathbf{u}_{\ell,k,j}\}$ $j \in \{1, ..., J_1\}, k \in \{1, ..., M_{1,b}\}$ is drawn, independently and with i.i.d. components given $\mathbf{x}_\ell$, with the i-th symbol distributed according to $P_{U|X_1}(\cdot|x_i)$ given x_i where x_i is the i-th symbol of $\mathbf{x}_\ell$. Hence, each codeword in Encoder 1's codebook is associated with a U-codebook of auxiliary codewords.

Further, a codebook (referred to as the V-codebook) consisting of $M_2 \times J_2$ i.i.d. independent codewords $\{\mathbf{v}_{m,\tilde{j}}\}, m \in \{1, ..., M_2\}, \tilde{j} \in \{1, ..., J_2\}$ is drawn where each symbol is distributed according to P_V.

Encoding: Let $m_1 = (\ell, k)$ and m_2 be the message indices to be transmitted. Encoder 1 transmits $\mathbf{x}_\ell$. Encoder 2 searches the k-th bin of the ℓ-th U-codebook for a vector $\mathbf{u} = \mathbf{u}_{\ell,k,j_0}$ that is jointly typical with $(\mathbf{x}_\ell, \mathbf{s})$. Having done that, it searches the V-codebook for a vector $\mathbf{v} = \mathbf{v}_{m,\tilde{j}_0}$ that is jointly typical with $(\mathbf{x}_\ell, \mathbf{u}, \mathbf{s})$. Finally, encoder 2 transmits a vector that is i.i.d. conditioned on $(\mathbf{x}_\ell, \mathbf{u}, \mathbf{s}, \mathbf{v})$ with conditional marginal distribution $P_{X_2|X_1,S,U,V}$.

If such j_0 or $\tilde{j}_0$ are not found, or if the observed state sequence $\mathbf{s}$ is not typical, an error is declared.

Decoding: Upon observing $\mathbf{y}_1$, decoder 1 searches for a triplet of indices, ℓ', k', j', such that $\mathbf{x}_{\ell'}, \mathbf{u}_{\ell',k',j'}$ are jointly typical with $\mathbf{y}_{(i)}$ and outputs $\hat{m}_1 = (\ell', k')$. Decoder 2 which observes $\mathbf{y}_2$ searches for a pair of indices $(m', \tilde{j}')$ such that $\mathbf{v}_{m',\tilde{j}'}$ is typical with $\mathbf{y}_2$ and outputs $\hat{m}_2 = m'$. If either of the decoders does no find appropriate codewords jointly typical with the observed date, or if the typical codewords are not unique, an error is declared.

We note that the achievable region described in Theorem 1 with no states reduces to the achievable region presented in [16], that is, the closure of the convex hull of the union of rate pairs that satisfy

$$\begin{aligned} R_1 &\leq I(U, X_1; Y_1) \\ R_2 &\leq I(V; Y_2) - I(V; U, X_1) \end{aligned} \tag{7}$$

for some $P_{X_1,U,V,X_2,Y_1,Y_2} = P_{X_1,U,V,X_2} P_{Y_1,Y_2|X_1,X_2}$.

The following result presents a rather straightforward outer bound on the capacity region.

Theorem 2 *(outer bound) The set of achievable rate-pairs of the cognitive interference channel with asymmetric CSI is contained in the closure of the set of rate-pairs (R_1, R_2) that satisfy*

$$\begin{aligned} R_1 &\leq I(U, X_1; Y_1) - I(U, X_1; S) \\ R_2 &\leq I(X_2; Y_2|X_1, S) \end{aligned} \tag{8}$$

for some joint law $P_{S,X_1,U,V,X_2,Y_1,Y_2}$ satisfying (6).

The bound on R_1 is derived similarly to [20] and the bound on R_2 follows from a genie-aided decoder 2 which observes the other user's signal as well as the states sequence.

Another outer bound is the capacity region of the same channel where the state sequence is known to the receivers, and is given in the following theorem.

Theorem 3 *(outer bound) The set of achievable rate-pairs of the cognitive interference channel with asymmetric CSI is contained in the closure of the set of rate-pairs (R_1, R_2) that satisfy*

$$\begin{aligned} R_1 &\leq I(U, X_1; Y_1|S) \\ R_2 &\leq I(X_2; Y_2|X_1, S) \\ R_1 + R_2 &\leq I(X_2; Y_2|U, X_1, S) + I(U, X_1; Y_1|S) \end{aligned} \tag{9}$$

for some joint law $P_{S,X_1,U,V,X_2,Y_1,Y_2}$ satisfying (6).

The proof of Theorem 3 is a straightforward extension of the proof of Theorem 3.2 of [16].

IV. The Gaussian Cognitive Interference Channel with Asymmetric CSI

In this section, we consider the Gaussian cognitive interference channel. The channel outputs at receivers 1 and 2 at time instant i are given, respectively, by

$$\begin{aligned} Y_{1,i} &= X_{1,i} + aX_{2,i} + S_{1,i} + N_{1,i} \\ Y_{2,i} &= bX_{1,i} + X_{2,i} + S_{2,i} + N_{2,i} \end{aligned} \tag{10}$$

where $\{N_{1,i}\}_{i=1}^{\infty}$ and $\{N_{2,i}\}_{i=1}^{\infty}$ are independent i.i.d. unit variance Gaussian processes, $\{(S_{1,i}, S_{2,i})\}_{i=1}^{\infty}$ is an i.i.d. Gaussian process that is independent of $\{(N_{1,i}, N_{2,i})\}_{i=1}^{\infty}$ and a and b are real constants. We note that $S_{1,i}$ and $S_{2,i}$ may be correlated.

Assuming that the state sequence $S_i \triangleq (S_{1,i}, S_{2,i}), i = 1, ..., n$ is known to the second transmitter non-causally, one realizes that this channel is a state dependent interference channel of the form (1).

We assume that the transmitters are subject to the following power constraints

$$\frac{1}{n}\sum_{i=1}^{n} X_{1,i}^2 \leq P_1 \quad \text{and} \quad \frac{1}{n}\sum_{i=1}^{n} X_{2,i}^2 \leq P_2. \tag{11}$$

It is easy to realize that, at least when $|a| \leq 1$, if both transmitters know $S^n = (S_1^n, S_2^n)$ non-causally, they can cancel the interference S^n, that is, the capacity region reduces to that of the channel without the interference S^n as in [16]. To realize this, assume for convenience $S = S_1 = S_2$, and consider the encoding scheme used in [16] which also uses dirty paper coding w.r.t. S^n. More specifically, encoder 2 devotes a fraction β of its power to transmit the signal $X_{2,1}$ to help encoder 1, and a fraction $(1-\beta)$ to transmit its own message by the signal $X_{2,2}$. The resulting signal of encoder 2 is $X_2 = X_{2,1} + X_{2,2}$. The signal $X_1 + aX_{2,1}$ is dirty paper coded against S. Then, $X_{2,2}$ is dirty paper coded against the effective interference $bX_1 + X_{2,1} + S$.

While interference cancelation is achievable in this cognitive regime with users observing S, it is impossible in the non-cognitive Gaussian interference channel [6]. This demonstrates the importance of cooperation for successful interference cancelation. In the sequel we show that when only the cognitive transmitter knows S^n, complete cancelation of the interference S^n is impossible (unless S_1^n is 0). We next derive a single letter outer bound on the capacity of an the interference channel of the form (10) with full asymmetric CSI. We consider the weak interference case, i.e., $|a| \leq 1$.

Theorem 4 *(outer bound) The set of achievable rate-pairs of the cognitive interference channel (10) with $|a| \leq 1$ and where $S^n = (S_1^n, S_2^n)$ is known non-causally to transmitter 2 is contained in the closure of the set of rate-pairs (R_1, R_2) that satisfy*

$$\begin{aligned} R_1 &\leq I(U, X_1; Y_1) - I(U, X_1; S) \\ R_2 &\leq I(X_2; Y_2|U, X_1, S) \end{aligned} \tag{12}$$

for some joint law $P_{S,X_1,U,V,X_2,Y_1,Y_2}$ satisfying (6).

The proof of Theorem 4 is omitted and can be found in [25].

It is easy to verify that the inner bound of Theorem 1 is contained in the outer bound of Theorem 4 by noting that,

$$\begin{aligned} & I(V; Y_2) - I(V; U, X_1, S) \\ = \ & I(V; Y_2|U, X_1, S) - I(V; U, X_1, S|Y_2) \\ \leq \ & I(X_2; Y_2|U, X_1, S) - I(V; U, X_1, S|Y_2) \\ \leq \ & I(X_2; Y_2|U, X_1, S), \end{aligned} \tag{13}$$

where the first inequality holds since $(U,V) \leftrightarrow (X_1,X_2,S) \leftrightarrow Y_2$ is a Markov chain (due to (6)).

Next, using Theorem 4 we can establish the capacity region for Gaussian case. Denote without loss of generality

$$S_{2,i} = dS_{1,i} + \tilde{S} \tag{14}$$

where d is some real constant and $S_{1,i}$ and $\tilde{S}_i$ are uncorrelated, and let

$$Q = E\left[S_1^2\right],\ \tilde{Q} = E\left[\tilde{S}^2\right], \tag{15}$$

$$\tilde{\sigma}_{2s} = E[X_2\tilde{S}],\ \tilde{\rho}_{2s} = \frac{\tilde{\sigma}_{2s}}{\sqrt{P_2\tilde{Q}}}. \tag{16}$$

Having defined the alternative noises, $S_1, \tilde{S}$, we can refer to the alternative channel representation of (10)

$$Y_{1,i} = X_{1,i} + aX_{2,i} + S_{1,i} + N_{1,i} \tag{17}$$

$$Y_{2,i} = bX_{1,i} + X_{2,i} + dS_{1,i} + \tilde{S}_i + N_{2,i}. \tag{18}$$

Denote

$$C(\Delta,\rho,P_1,P_2,Q,a) = \frac{1}{2}\log\left(1 + \frac{\left(\sqrt{P_1} + \sqrt{1-\Delta-\rho^2}a\sqrt{P_2}\right)^2}{a^2P_2\Delta + (\sqrt{P_2}a\rho + \sqrt{Q})^2 + 1}\right) \tag{19}$$

Theorem 5 *The capacity region of the cognitive asymmetric Gaussian interference channel with non-causal CSI and* $|a| \le 1$ *is given by the set of rate-pairs satisfying*

$$R_1 \le \max_{\Delta \ge [\gamma,1], \rho \in [-\sqrt{1-\Delta},0]} \left[C(\Delta,\rho,P_1,P_2,Q,a) + \frac{1}{2}\log\left(1+\Delta a^2P_2\right)\right] - \frac{1}{2}\log\left(1+\gamma a^2P_2\right)$$
$$R_2 \le \frac{1}{2}\log\left(1+\gamma P_2\right), \tag{20}$$

for some $\gamma \in [0,1]$.

The proof of Theorem 5 is omitted and can be found in [25]. The following Corollary provides an equivalent expression for the capacity region.

Corollary 1 *Denote* $\gamma_0 = 1 - \frac{P_1(a^2P_2+1)^2}{a^2P_2Q(P_1+Q)}$. *The capacity region of the cognitive asymmetric Gaussian interference channel with non-causal CSI and* $|a| \le 1$ *is given by the set of rate-pairs satisfying*

$$R_1 \le \begin{cases} \max_{\rho\in[-\sqrt{1-\gamma},0]} C(\gamma,\rho,P_1,P_2,Q,a) & \gamma \le \gamma_0 \\ \frac{1}{2}\log\left(1+\frac{P_1}{Q}\right) + \frac{1}{2}\log\left(1+a^2P_2\right) - \frac{1}{2}\log\left(1+\gamma a^2P_2\right) & \text{otherwise} \end{cases}$$
$$R_2 \le \frac{1}{2}\log\left(1+\gamma P_2\right), \tag{21}$$

for some $\gamma \in [0,1]$. *The maximization over* ρ *can be limited to either* $\rho = -\sqrt{1-\gamma}$, $\rho = 0$ *or any real root of* $h_\gamma(\rho)$ *that satisfies* $\rho \in [-\sqrt{1-\gamma},0]$ *with*

$$\begin{aligned} h_\gamma(\rho) = &- a^2P_2(P_1+Q)\rho^4 - 2\sqrt{Qa^2P_2}(a^2P_2+Q+1+P_1)\rho^3 \\ &+ \left[(P_1-2Q)a^2P_2(1-\gamma) - (a^2P_2+Q+1)^2 - P_1Q\right]\rho^2 \\ &+ 2\sqrt{a^2P_2Q}(1-\gamma)\left[P_1 - (a^2P_2+Q+1)\right]\rho \\ &+ (1-\gamma)Q\left[P_1 - a^2P_2(1-\gamma)\right]. \end{aligned} \tag{22}$$

Discussion: We note that the proof of the direct part of Theorem 5 does not rely on the fact that $|a| \le 1$, so the capacity region of Theorem 5 is achievable also when $|a| > 1$.

The proof of the converse part of Theorem 5 includes an upper bound that relies on the entropy power inequality, used also in [16]. To prove the direct part of Theorem 5 we choose $P_{X_1,X_2,U,V|S_1,\tilde{S}} = P_{X_1}P_{X_2,U,V|S_1,\tilde{S},X_1}$ that is Gaussian, with

$$E[X_1X_2] = \sigma_{12} = \frac{\rho_{12}}{\sqrt{P_1P_2}}, \quad E[X_2S_1] = \sigma_{2s} = \frac{\rho_{2s}}{\sqrt{P_2Q}}$$
$$E[X_2\tilde{S}] = \tilde{\sigma}_{2,s} = 0, \quad U = aX_2 + \alpha S_1 + Z, \tag{23}$$

where Z is a zero-mean Gaussian RV with variance $P_z > 0$, independent of $(X_1,X_2,S_1,\tilde{S},U)$, and α is given by

$$\alpha_{opt} = \frac{a^2P_2P_1Q - a^2P_1\sigma_{2s}^2 - P_1a\sigma_{2s} - a^2\sigma_{12}^2Q}{a^2P_2P_1Q + P_1Q - P_1a^2\sigma_{2s}^2 - a^2\sigma_{12}^2Q} \tag{24}$$

It is worth noting that α_{opt} is identical to the optimal value of α for the MAC channel [20] because α affects only the term $I(U,X_1;Y_1) - I(U,X_1;S)$ (for $U = X_2 + \alpha S$) in (5) while it does not affect $I(V;Y_2) - I(V;U,X_1,S)$.

It is also interesting to note that the capacity depends on the interference S_1 but it does not depend on the part of the interference S_2 which is uncorrelated with S_1, i.e., $\tilde{S}$ (see (14)). Hence, interference cancelation is achieved in the sense that the noise $\tilde{S}$ is eliminated.

The proof of the direct part also involves showing that in the Gaussian case the two inequalities in (13) can be met with equality simultaneously without loss of optimality. It is easy to observe that this happens when there exists V which is a deterministic function of (U,X_1,S,X_2) and at the same time satisfies $I(V;U,X_1,S|Y_2) = 0$. The existence of such V which takes on the form $V = X_2 + c_1X_1 + c_2S_1 + c_3\tilde{S} + c_4U$ for some constants $c_i, i = 1,2,3,4$, is guaranteed by the following more general result, whose proof appears in [25].

Lemma 1 *Let* X *and* N *be independent Gaussian random variables with variances* $\sigma_x^2 > 0$ *and* $\sigma_n^2 > 0$, *respectively. Let* $\mathbf{f}$ *be a* $k \times 1$ *random vector, with covariance matrix* $\Lambda_{\mathbf{f}}$, *independent of* N *and such that* $(X,\mathbf{f})$ *is a Gaussian vector. Let* $\mathbf{g}$ *be a deterministic* $k \times 1$ *vector, and let* $\mathbf{r}$ *be the covariance vector between* X *and* $\mathbf{f}$. *If the covariance matrix of* $(X,\mathbf{f})$ *is positive definite,* $\mathbf{g}^T\mathbf{r} + \sigma_x^2 + \sigma_n^2 \neq 0$, *and* $\mathbf{r}^T\Lambda_{\mathbf{f}}^{-1}\mathbf{r} \neq \sigma_x^2 + \sigma_n^2$, *then,*

$$I(X + \mathbf{c}^T\mathbf{f};\mathbf{f}|X + \mathbf{g}^T\mathbf{f} + N) = 0 \tag{25}$$

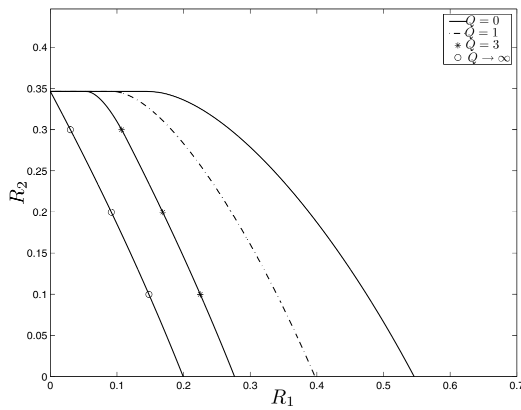

Fig. 2. Capacity Region of the Asymmetric Cooperative State-Dependent Gaussian Interference Channel for $P_1 = \frac{1}{2}, P_2 = 1$.

for $\mathbf{c}^T = \left[\sigma_x^2 \mathbf{g}^T \Lambda_{\mathbf{f}} - (\mathbf{g}^T\mathbf{r} + \sigma_n^2)\mathbf{r}^T\right] \mathbf{B}$, *with*

$$\begin{aligned} \mathbf{B} &= \mathbf{H} + \frac{\mathbf{H}\mathbf{r}\mathbf{r}^T\mathbf{H}}{1 - \mathbf{r}^T\mathbf{H}\mathbf{r}} \\ \mathbf{H} &= \Lambda_{\mathbf{f}}^{-1} \frac{1}{(\mathbf{g}^T\mathbf{r} + \sigma_x^2 + \sigma_n^2)} \left[I + \frac{\mathbf{r}\mathbf{g}^T}{\sigma_x^2 + \sigma_n^2}\right]. \end{aligned} \quad (26)$$

If the covariance matrix of $(X, \mathbf{f})$ *is positive definite,* $\mathbf{g}^T\mathbf{r} + \sigma_x^2 + \sigma_n^2 = 0$, *and* $\mathbf{g}^T\mathbf{r} + \mathbf{r}^T\Lambda_{\mathbf{f}}^{-1}\mathbf{r} \neq 0$ *then (25) holds for* $c^T = q(\mathbf{g}^T + \mathbf{r}^T\Lambda_{\mathbf{f}}^{-1}) - \mathbf{r}^T\Lambda_{\mathbf{f}}^{-1}$ *with* $q = \frac{\mathbf{r}^T\Lambda_{\mathbf{f}}^{-1}\mathbf{r} - \sigma_x^2}{\mathbf{g}^T\mathbf{r} + \mathbf{r}^T\Lambda_{\mathbf{f}}^{-1}\mathbf{r}}$.

We apply Lemma 1 with $\mathbf{f}^T = [\begin{array}{cccc} X_1 & S_1 & \tilde{S} & U \end{array}]$, $X = X_2$, $N = N_2$, $g^T = [b\ d\ 1\ 0]$. Hence

$$\begin{aligned} Y_2 &= X + g^T\mathbf{f} + N, \\ \mathbf{r}^T &= E[X\mathbf{f}^T] = \left(\begin{array}{cccc} \sigma_{12} & \sigma_{2s} & 0 & aP_2 + \alpha\sigma_{2s} \end{array}\right). \end{aligned} \quad (27)$$

We verify that provided that $P_z > 0, \rho_{12}^2 + \rho_{2s}^2 < 1$, the existence of the constants $c_i, i = 1, 2, 3, 4$ satisfying $I(V; U, X_1, S, \tilde{S}|Y_2) = 0$ is guaranteed and we calculate them. For the sake of brevity they are omitted.

In Fig. 2, the capacity region is plotted for $P_1 = \frac{1}{2}, P_2 = 1$ and several values of Q. The achievable rates are clearly decreasing as a function of Q. In the high interference regime, $Q \to \infty$, only the cognitive encoder functions, and hence we have an interference inflicted broadcast setting, for which the interference can be absolutely removed, [26]. This result persists also for strong interferences $1 < |a| < \infty$.

ACKNOWLEDGMENT

This research was supported by a Marie Curie International Fellowship within the 6th European Community Framework Programme. The work of S. Shamai and S. Verdú has also been supported by the Binational US-Israel Scientific Foundation, Grant 2004140.

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