Spectral Efficiency of Low–Complexity Multiuser Detectors

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Abstract — **A family of multiuser detectors is analyzed which require neither matrix inversions nor other operations with significant complexity. The time complexity per bit of most of them is independent of the number of users. Nevertheless, their spectral efficiency for random spreading sequences is shown to be not far behind that of linear MMSE detection.**

I. Introduction

Recently, the performances of well-known linear and nonlinear multiuser detectors in random environments were analyzed in [1, 2, 3] revealing important gains over the spectral efficiency of the single-user matched filter. The price for those improvements is receiver complexity.

An important class of multiuser receivers with lower complexity is based on the idea of approximate decorrelation (AD) [4] (a generalization to approx. MMSE detectors is straightforward): Matrix inversion in approximated via L^{th} order polynomial expansion $\boldsymbol{M}^{-1} \approx \sum_{\ell=0}^{L} w_\ell \boldsymbol{M}^\ell$, see e.g. [5], with properly chosen weights w_ℓ.

II. Main Results

Let $\boldsymbol{y} = \boldsymbol{S}^{\text{H}}(\boldsymbol{S}\boldsymbol{x} + \boldsymbol{n})$ denote the vector notation of a synchronous K user Gaussian CDMA channel with $\boldsymbol{x}, \boldsymbol{y}$ denoting the transmitted and received symbols, respectively, $\boldsymbol{n}$ the complex additive white Gaussian noise of variance σ^2 and $\boldsymbol{S}$ the $L \times K$ matrix of complex signature sequences. In this summary, we restrict attention to equal received powers and we assume that the diagonal elements of the matrix $\boldsymbol{R} = \boldsymbol{S}^{\text{H}}\boldsymbol{S}$ equal unity.

Theorem 1 *Let $K, N \to \infty$, but $0 < \beta = \frac{K}{N} < \infty$ and the random components of $\boldsymbol{S}$ be independent with finite variance. Then, the signal–to–interference–and–noise ratio at the output $\boldsymbol{d} = \boldsymbol{T}\boldsymbol{y}$ of any linear equalizer described by a matrix $\boldsymbol{T} = \sum_{\ell=0}^{L} w_\ell(\beta,\sigma)\boldsymbol{R}^\ell$ converges almost surely to a deterministic scalar for arbitrary weight functions $w_\ell(\beta,\sigma), 0 \le \ell \le L$, and arbitrary order L.*

Theorem 1 allows to give explicit expressions for the SIR of L^{th} order approximation to the MMSE multiuser detector. The results for $L = 1, 2, 3$ are the following:

$$\max_{w_i} SIR_1 \to \frac{1+\beta+\sigma^2}{\beta^2+\sigma^2(1+2\beta)+\sigma^4} > \frac{1-2\beta+\beta^2}{\beta^2+\beta^3+\sigma^2(1-\beta+\beta^2)} \leftarrow SIR_{\text{AD}}$$

$$\max_{w_i} SIR_2 \to \frac{1+\beta+\beta^2+\sigma^2(2+2\beta)+\sigma^4}{\beta^3+\sigma^2(1+2\beta+3\beta^2)+\sigma^4(2+3\beta)+\sigma^6}$$

$$\max_{w_i} SIR_3 \to \frac{1+\beta+\beta^2+\beta^3+\sigma^2(3+4\beta+3\beta^2)+\sigma^4(3+3\beta)+\sigma^6}{\beta^4+\sigma^2(1+2\beta+3\beta^2+4\beta^3)+\sigma^4(3+6\beta+6\beta^2)+\sigma^6(3+4\beta)+\sigma^8}$$

The 0^{th} order approx. is equivalent to the conventional matched filter. The first order approximation (cf. [4, Prob. 5.28.d] is better than the approximate decorrelator analyzed in [4, p. 281], where the weights were based on a Taylor expansion and not optimized with respect to the maximum achievable SIR. The optimum weights can be expressed as L^{th} order polynomials in β and σ^2 and calculated, recursively. Thus, their computation is very simple in real–time applications.

For a fair comparison to the performances of the decorrelator, the MMSE detector, and the matched filter, the spectral efficiency $\mathit{\Gamma} = \beta C = \beta \log_2(1 + SIR)$ and power efficiency $\frac{N_0}{E_b} = \sigma^2 C$ are calculated as in [1, 2]. Averaging capacity results over the load, our results are extended to re–encoded successive cancellation (SC) receivers [1, 2] via $\mathit{\Gamma}_{\text{SC}}(\beta) = \int_0^\beta \log_2(1 + SIR(\beta'))\mathrm{d}\beta'$. Fig. 1 shows the spectral efficiency for fixed E_b/N_0 as a function of the load $\beta = K/N$. At low β simple linear receivers noticeably outperform the single-user matched filter. At high β simple nonlinear receivers noticeably outperform the exact MMSE receiver.

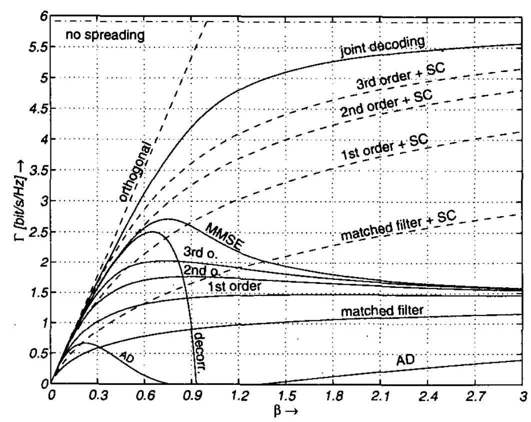

Fig. 1: Spectral efficiency vs. system load for several multiuser detectors and fixed $10\log_{10}(E_b/N_0) = 10$ dB.

III. Conclusion

Increasing spectral efficiency by multiuser detection need not involve significant increase in receiver complexity even with long spreading sequences.

References

[1] R. R. Müller. *Power and Bandwidth Efficiency of Multiuser Systems with Random Spreading.* Shaker, Aachen, 1999.

[2] S. Verdú and S. Shamai (Shitz). Spectral efficiency of CDMA with random spreading. *IEEE Trans. Inf. Th.*, 45(2):622–640, Mar. 1999.

[3] D. Tse and S. Hanly. Linear multiuser receivers: Effective interference, effective bandwidth and user capacity. *IEEE Trans. Inf. Th.*, 45(2):641–657, Mar. 1999.

[4] S. Verdú. *Multiuser Detection.* Cambridge University Press, New York, 1998.

[5] S. Moshavi, E. G. Kanterakis, and D. L. Schilling. Multistage linear receivers for DS–CDMA systems. *Int. J. Wireless Inf. Networks*, 3(1):1–17, Jan. 1996.

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