Spectral Efficiency of MC-CDMA: Linear and Non-Linear Receivers*

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Abstract

This paper analyzes the spectral efficiency (total channel capacity per subband) of randomly-spread synchronous multicarrier code-division multiple-access (MC-CDMA) channel subject to frequency-selective fading in the asymptotic regime of number of users and bandwidth going to infinity with a constant ratio. We analyze the spectral efficiency for uplink and downlink conditioned on subcarrier frequency-selective fading. The following receivers are analyzed: a) jointly optimum receiver, b) linear MMSE receiver, c) decorrelator, and d) single-user matched filter.

1 Introduction

The spectral efficiency of a CDMA system is the total number of bits/s/Hz that can be transmitted arbitrarily reliably. For multicarrier CDMA (MC-CDMA), it is also the aggregate capacity per subband supported by the system. Results on the eigenvalue distribution of large random matrices enable the study of the asymptotic behavior of randomly-spread CDMA systems [2, 8, 12]. The spectral efficiency for randomly-spread non-fading as well as flat-fading direct-sequence CDMA (DS-CDMA) is studied in [7, 12] in the wideband limit and large number of users for the jointly optimum receiver as well as linear receivers.

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Instead of the direct-sequence spread spectrum model analyzed in previous works, in this paper we analyze MC-CDMA. Moreover, in contrast to previous random-matrix analyses we analyze the impact of frequency-selective fading. The advantages of MC-CDMA include: a) the requirement of having a continuous band of frequency for transmission as in DS-CDMA is dropped; b) for equally spaced subcarriers, IFFT and FFT can be used to implement the modulation/demodulation. In MC-CDMA systems there are N subcarriers shared by all the users. The chips of a user's spreading sequence modulated by the information symbols are transmitted in parallel through the subcarriers. A more detailed account of MC-CDMA can be found in [3].

In this paper our analysis is focused in the asymptotic regime, where K (the number of users) and N both go to infinity with their ratio $\frac{K}{N} = \beta$ held constant, which is referred to as the system load. We analyze four receivers: a) jointly optimum receiver, b) linear MMSE receiver, c) decorrelator, and d) single-user matched filter. We find analytically the spectral efficiency of the four receiving schemes for both uplink and downlink MC-CDMA channels *conditioned* on fading. The effect of multicarrier transmission on the spectral efficiency of CDMA systems is examined and several operating conditions, such as low $\frac{E_b}{N_0}$, high $\frac{E_b}{N_0}$, and high load, receive particular attention. Our analysis for the scenario where only statistical knowledge of the subcarrier fading is available (referred to as the *unconditioned* uplink and downlink) can be found in [5].

2 System Model

2.1 Uplink MC-CDMA Conditioned on Fading

For MC-CDMA the received spreading sequences are transformed from the original sequences by subband fading. Denote the complex instantaneous fading coefficient at the i-th subcarrier of user k by H_k^i $(1 \leq k \leq K, 1 \leq i \leq N)$. The received spreading sequence of user k is $\tilde{\mathbf{s}}_k = \mathbf{H}_k \mathbf{s}_k$, where $\mathbf{H}_k = \mathrm{diag}\{H_k^1, \ldots, H_k^N\}$.

In this case the spectral efficiency is calculated conditioned on the fading coefficients. This is advantageous because in this way we can then find the effect of an arbitrary frequency-selective fading distribution on capacity, and possibly further average capacity with respect to an ensemble of those fading distributions.

2.2 Downlink MC-CDMA Conditioned on Fading

For the downlink all users experience the same fading. Denote the subcarrier fading coefficients by $H_1, \ldots, H_N$. User k's received sequence is $\tilde{\mathbf{s}}_k = \mathbf{H}\mathbf{s}_k$, where $\mathbf{H} = \mathrm{diag}\{H_1, \ldots, H_N\}$. We define the fading power at the i-th subband as $C_i^{\mathsf{DCon}} \triangleq |H_i|^2$.

2.3 K-user Synchronous Channel

Assuming synchronous users, the subband-matched filter output vector is

$$\mathbf{r} = \sum_{k=1}^{K} A_k b_k \tilde{\mathbf{s}}_k + \mathbf{n} = \tilde{\mathbf{S}}\mathbf{A}\mathbf{b} + \mathbf{n}, \tag{1}$$

where $A_k > 0$ is the transmitted amplitude of user k, b_k is the transmitted symbol of user k which is assumed to be i.i.d. across users and to have unit energy, $\mathbf{n}$ is the noise vector distributed as $\mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$, $\tilde{\mathbf{S}} = (\tilde{\mathbf{s}}_1, \ldots, \tilde{\mathbf{s}}_K)$, $\mathbf{A} = \mathrm{diag}\{A_1, \ldots, A_K\}$, and $\mathbf{b} = (b_1, \ldots, b_K)^T$. Here we assume that $\mathbf{s}_k$'s are randomly and independently chosen and consist of binary $\{+\frac{1}{\sqrt{N}}, -\frac{1}{\sqrt{N}}\}$ chips. However, using the methods in [9] it is possible to show that the results in this paper are actually insensitive to the chip distribution.

In practical implementations, in order to simplify receiver design it is common to insert cyclic (or empty) prefixes. As in the case of nonideal chip waveform in direct-sequence, this is a source of loss of spectral efficiency which is easily incorporated as a penalty factor on the results obtained with the ideal model.

2.4 Allocation Function

Definition 2.1 *For an $N \times K$ matrix $U_{i,j}$ $(1 \le i \le N, 1 \le j \le K)$, the one-dimensional allocation function $\rho_{j,N}(x)$ and the two-dimensional allocation function $\rho_N(x, y)$ defined on $[0, 1]$ and $[0, 1] \times [0, \frac{K}{N}]$ respectively are:*

$$\rho_{j,N}(x) \triangleq \begin{cases} U_{i,j} & \text{for } \frac{i-1}{N} \le x \le \frac{i}{N} \\ 0 & \text{otherwise} \end{cases} \quad \rho_N(x,y) \triangleq \begin{cases} U_{i,j} & \text{for } \frac{i-1}{N} \le x \le \frac{i}{N}, \frac{j-1}{N} \le y \le \frac{j}{N} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

For the vectors and matrices of interest in the sequel the above allocation functions converge as the dimensions go to infinity ($K, N \to \infty$, $\frac{K}{N} = \beta$). The limits are denoted by $\rho_j(x)$ and $\rho(x, y)$ respectively.

3 Linear MMSE Receiver

In this section we consider the capacity of a bank of linear MMSE receivers followed by single-user decoders.

3.1 Uplink Capacity Conditioned on Subcarrier Fading

Let $\rho(x, y)$ $(0 \le x \le 1, 0 \le y \le \beta)$ be the two-dimensional asymptotic allocation function of $|H_k^i|^2 A_k^2$, and $\rho_k(x)$ $(0 \le x \le 1)$ be the one-dimensional asymptotic allocation function of $|H_k^i|^2 A_k^2$ for fixed k. Let us denote the average received energy among users by Q. It is readily shown by definition that $Q = \frac{1}{\beta}\int_0^\beta dy \int_0^1 \rho(x, y)dx$. We also define the average received signal-to-noise ratio (SNR) as $\mathsf{snr} \triangleq \frac{Q}{\sigma^2}$. Q can be interpreted as the power constraint of the users' codewords, and snr is the per-symbol SNR constraint of the users. In a system that achieves spectral efficiency C, the energy per bit per noise level is given by $\frac{E_b}{N_0} = \frac{\beta\,\mathsf{snr}}{\mathsf{C}}$ [7].

The following Theorems 3.1 and 3.2 give our results on the asymptotic (in K, N) MMSE multiuser efficiency as well as spectral efficiency for the uplink MC-CDMA channel conditioned on fading. The multiuser efficiency is the ratio of the output signal-to-interference-to-noise ratio (SINR) to the SNR without multiaccess interference (MAI) [10]. For the proofs of the theorems, the readers are referred to [5].

Theorem 3.1 *Conditioned on the fading coefficients, the MMSE multiuser efficiency of user k converges almost surely as $K, N \to \infty$ with $\frac{K}{N} = \beta$ to*

$$\eta_k = \frac{\int_0^1 \rho_k(x) v(x, \mathsf{snr})dx}{\int_0^1 \rho_k(x)dx}, \qquad \textit{where } v(x, \mathsf{snr}) \textit{ satisfies} \tag{3}$$

$$v(x, \mathsf{snr}) = \frac{1}{1 + \mathsf{snr} \int_0^\beta \frac{\rho(x,y)}{Q + \mathsf{snr} \int_0^1 v(w,\mathsf{snr})\rho(w,y)dw} dy}. \quad (4)$$

Theorem 3.2 *Conditioned on the fading coefficients, the spectral efficiency of the MMSE receiver converges almost surely as K, $N \to \infty$ with $\frac{K}{N} = \beta$ to*

$$\mathsf{C}^{\mathsf{mmse}} = \int_0^\beta \log_2 \left(1 + \frac{\mathsf{snr}}{Q} \mu(y', \mathsf{snr})\right) dy', \qquad \text{where } \mu(y', \mathsf{snr}) \text{ satisfies} \quad (5)$$

$$\mu(y', \mathsf{snr}) = \int_0^1 \frac{\rho(x, y')dx}{1 + \mathsf{snr} \int_0^\beta \frac{\rho(x,y)}{Q + \mathsf{snr}\,\mu(y,\mathsf{snr})} dy}. \quad (6)$$

Several properties of μ as a function of snr, which have been proven in [5], are useful in our analysis presented below for the behavior of the spectral efficiency under various operating conditions. From Theorem 3.2 we calculate the minimum energy per bit requirement for reliable communication: $\frac{E_b}{N_0}_{\min} = \log_e 2 = -1.59$ dB, which is the same as that in [7] for single carrier DS-CDMA. We also calculate from (5) the slope of $\mathsf{C}^{\mathsf{mmse}}$ vs. $\frac{E_b}{N_0}$ in 3-dB units at $\frac{E_b}{N_0}_{\min}$ (denoted by S_0^{mmse}) [11]:

$$S_0^{\mathsf{mmse}} = \frac{2\beta^2 Q^2}{2\int_0^1 \left(\int_0^\beta \rho(x,y)\,dy\right)^2 dx + \int_0^\beta \left(\int_0^1 \rho(x,y)dx\right)^2 dy}. \quad (7)$$

In addition, the high-SNR slope of $\mathsf{C}^{\mathsf{mmse}}$ vs. $\frac{E_b}{N_0}$ in 3-dB units (denoted by S_∞^{mmse}) is

$$S_\infty^{\mathsf{mmse}} = \begin{cases} Z_Y, & \frac{Z_Y}{Z_X} < 1; \\ \frac{Z_Y}{2}, & \frac{Z_Y}{Z_X} = 1; \\ 0, & \frac{Z_Y}{Z_X} > 1, \end{cases} \quad (8)$$

$$\text{where} \quad Z_X \triangleq \int_0^1 1\left\{\int_0^\beta \rho(x,y)dy \neq 0\right\} dx, \quad Z_Y \triangleq \int_0^\beta 1\left\{\int_0^1 \rho(x,y)dx \neq 0\right\} dy. \quad (9)$$

Z_X represents the proportion of active subcarriers, and Z_Y represents β times the proportion of simultaneously active users. Thus, $\frac{Z_Y}{Z_X}$ can be viewed as the effective load of the system.

3.2 Downlink Capacity Conditioned on Subcarrier Fading

To gain more engineering insight, it is worthwhile to study the downlink where the involved expressions simplify significantly. Next we give our result on the MMSE multiuser efficiency, which first appeared in [4,9].

Theorem 3.3 *Let $D(C)$ be the asymptotic empirical distribution of $C_1^{\mathsf{DCon}}, \ldots, C_N^{\mathsf{DCon}}$ as $N \to \infty$, and $F(P)$ be the asymptotic empirical distribution of $P_1 \triangleq A_1^2, \ldots, P_K \triangleq A_K^2$ as $K \to \infty$. The MMSE multiuser efficiency (the same for all users) converges almost surely as $K, N \to \infty$ with $\frac{K}{N} = \beta$ to the solution of the following fixed point equation*

$$\eta = \mathrm{E}_{C'}\left[\frac{C'}{1+\beta C'\,\mathrm{E}_{P'}\left[\frac{\mathsf{snr}\,P'}{1+\mathsf{snr}\,\eta\,P'}\right]}\right] \tag{10}$$

with $P' = \frac{P}{\mathrm{E}_P[P]}, C' = \frac{C}{\mathrm{E}_C[C]}$ *and* $Q = \mathrm{E}_P[P]\cdot\mathrm{E}_C[C]$.

Theorem 3.3, first shown in [4] with a different method, can be obtained as a special case of Theorem 3.1. To this end it is enough to invoke: (a) the factorization of $\rho(x,y)$ as the product of two one-dimensional allocation functions of C_i and A_k^2, and (b) the integral relations between allocation functions and empirical distributions (refer to [5] for details). From (10) and applying Jensen's inequality, it follows that the multicarrier frequency-selective fading reduces the MMSE multiuser efficiency. As a result, the decorrelator multiuser efficiency is also reduced from the fact that it equals the MMSE multiuser efficiency as $\sigma^2 \to 0$. The following shorthand notation is useful

$$P^+ = \Pr[P>0], \qquad C^+ = \Pr[C>0]. \tag{11}$$

For the downlink the effective load $\frac{Z_Y}{Z_X}$ particularizes to $\frac{\beta P^+}{C^+}$. Using (10), we obtain the following result for the spectral efficiency of the conditioned downlink.

Theorem 3.4 *For the conditioned downlink MC-CDMA, as* $K, N \to \infty$ *with* $\frac{K}{N} = \beta$*, the linear MMSE spectral efficiency converges to*

$$\mathsf{C}^{\mathsf{mmse}} = \beta\,\mathrm{E}_{P'}\left[\log_2\left(1+\mathsf{snr}\,\eta P'\right)\right], \tag{12}$$

where η *is the MMSE multiuser efficiency satisfying (10).*

From (12) the reduction of η by subcarrier fading in turn decreases $\mathsf{C}^{\mathsf{mmse}}$. $\frac{E_b}{N_0}_{\min}$ for reliable communication is the same as that for DS-CDMA: $\frac{E_b}{N_0}_{\min} = -1.59$ dB. Also from (12) the slope of $\mathsf{C}^{\mathsf{mmse}}$ in 3-dB units at $\frac{E_b}{N_0}_{\min}$ is

$$S_0^{\mathsf{mmse}} = \frac{2\beta}{2\beta\kappa(|H|)+\kappa(A)}. \tag{13}$$

In (13), $\kappa(|H|)$ (respectively $\kappa(A)$) indicates the *kurtosis* of the empirical distribution of $\sqrt{C_1^{\mathsf{DCon}}}, \ldots, \sqrt{C_N^{\mathsf{DCon}}}$ (respectively $A_1, \ldots, A_K$). The kurtosis of a non-negative random variable X is defined as $\kappa(X) \triangleq \frac{\mathrm{E}[X^4]}{\mathrm{E}^2[X^2]}$. It is always greater than or equal to 1 [7], with equality only if X is deterministic. Compared with the result in [7] which corresponds to $\kappa(|H|) = 1$, it follows that S_0^{mmse} is reduced by the frequency-selective fading.

Our result for the slope of the MMSE spectral efficiency at high $\frac{E_b}{N_0}$ is

$$S_\infty^{\mathsf{mmse}} = \begin{cases} \beta P^+, & \frac{\beta P^+}{C^+} < 1; \\ \frac{1}{2}\beta P^+, & \frac{\beta P^+}{C^+} = 1; \\ 0, & \frac{\beta P^+}{C^+} > 1. \end{cases} \tag{14}$$

Comparing (14) with the result in [7], we can see that S_∞^{mmse} is not affected by multicarrier fading if $C^+ = 1$. From (14) it is also clear that $\mathsf{C}^{\mathsf{mmse}}$ grows without bound as $\frac{E_b}{N_0} \to \infty$ if $\frac{\beta P^+}{C^+} \leq 1$. If $\frac{\beta P^+}{C^+} > 1$, we can show that the MMSE receiver is interference limited and $\mathsf{C}^{\mathsf{mmse}}$ goes to the following limit as $\frac{E_b}{N_0} \to \infty$ (see [5])

$$\lim_{\mathsf{snr}\to\infty} \mathsf{C}^{\mathsf{mmse}} = \beta\, \mathrm{E}_{P'}\left[\log_2\left(1+\eta_1 P'\right)\right], \quad \text{where} \quad \frac{\beta}{C^+}\mathrm{E}_{P'}\left[\frac{\eta_1 P'}{1+\eta_1 P'}\right] = 1. \tag{15}$$

Comparing it with the result for DS-CDMA in [7], we observe an interesting phenomenon: if $C^+ = 1$ and $\beta P^+ > C^+$, the spectral efficiency gap between DS-CDMA and MC-CDMA is bridged as $\frac{E_b}{N_0} \to \infty$. However, if $C^+ < 1$, the spectral efficiency of MC-CDMA is strictly smaller than that of DS-CDMA even as $\frac{E_b}{N_0} \to \infty$. This nonzero loss has a straightforward explanation: since $C^+ \triangleq \Pr[C > 0] < 1$, some subbands are so deeply faded that virtually all the energy we put into them is wasted.

Next we consider the situation where the number of users per subband grows without bound. The reader is referred to [5] for a proof.

Theorem 3.5 *Fix* $\frac{E_b}{N_0}$ *and denote* $\mathsf{C}^{\mathsf{mmse}}_\infty \triangleq \lim_{\beta\to\infty} \mathsf{C}^{\mathsf{mmse}}\left(\frac{E_b}{N_0}\right)$. *Then* $\mathsf{C}^{\mathsf{mmse}}_\infty$ *(in bits/s/Hz) is the solution to*

$$\mathrm{E}_{C'}\left[\frac{\frac{E_b}{N_0} C'}{1+\mathsf{C}^{\mathsf{mmse}}_\infty \frac{E_b}{N_0} C'}\right] = \log_e 2. \tag{16}$$

Upon application of Jensen's inequality to (16), we conclude that multicarrier frequency-selective fading reduces the MMSE spectral efficiency even as $\beta \to \infty$. This capacity loss originates from Jensen's inequality, being different from the loss incurred by $C^+ < 1$. The non-uniformity of the subband fading powers has the overall effect of reducing the number of transmitting subbands.

Figure 1 shows the MMSE spectral efficiency vs. $\frac{E_b}{N_0}$ for MC-CDMA and single carrier DS-CDMA for $\beta = 2.5$. In the figure we use the Rayleigh distribution for the amplitude profile $U(A)$ (that is, exponential distribution for the power profile $F(P)$). And a two-value distribution is used for the subcarrier fading profile $D(C)$. This corresponds to a situation where there are two types of subband channels: "good" channels and "bad" channels. The channel gains are chosen such that the kurtosis of the subcarrier fading distribution is 1.25. From Figure 1 we can see that the MMSE spectral efficiency is bounded for $\frac{\beta P^+}{C^+} > 1$, and that the capacity loss due to subcarrier fading vanishes as $\frac{E_b}{N_0} \to \infty$ if $\beta P^+ > C^+ = 1$.

4 Optimum Receiver

Let us now consider the total capacity of an MC-CDMA system with jointly optimum (non linear) processing.

4.1 Uplink Capacity Conditioned on Subcarrier Fading

In the asymptotic regime, we obtain an interesting closed form relation between the MMSE spectral efficiency and the optimum spectral efficiency. It is one of the main results in this paper. The proof of the theorem has been included in [5].

Theorem 4.1 *Conditioned on the subcarrier fading coefficients, as* $K, N \to \infty$ *with* $\frac{K}{N} = \beta$, *the optimum spectral efficiency can be expressed in terms of the MMSE spectral efficiency as*

$$\mathsf{C}^{\mathsf{opt}} = \mathsf{C}^{\mathsf{mmse}} - \log_2 e \cdot \int_0^{\beta} \mu(y', \mathsf{snr})\pi(y', \mathsf{snr})dy' + \int_0^1 \log_2 \left(1 + \int_0^{\beta} \rho(x, y')\pi(y', \mathsf{snr})dy'\right) dx, \quad (17)$$

$$\textit{where} \quad \pi(y', \mathsf{snr}) \triangleq \frac{\mathsf{snr}}{Q + \mathsf{snr}\,\mu(y', \mathsf{snr})}, \quad (18)$$

$\mu(y', \mathsf{snr})$ *is the solution to (6), and* $\rho(x, y)$ *is defined in Section 3.1.*

The additional capacity achieved by going from linear to non linear processing for frequency-flat fading DS-CDMA channels was found in [7]. As in [7], we find here that the capacity gain attained by optimum non linear processing depends only on the linear uncoded performance measure $\mu(y', \mathsf{snr})$, which is proportional to the output SINR of the $\lfloor y'N \rfloor$-th user.

4.2 Downlink Capacity Conditioned on Subcarrier Fading

For the conditioned downlink, Theorem 4.1 particularizes to

Theorem 4.2 *As* $K, N \to \infty$ *with* $\frac{K}{N} = \beta$*, let the MMSE multiuser efficiency be denoted by* η*, which satisfies (10), the optimum spectral efficiency converges to*

$$\mathsf{C}^{\mathsf{opt}} = \mathsf{C}^{\mathsf{mmse}} - \beta\, \mathrm{E}_{P'} \left[\frac{\mathsf{snr}\,\eta P'}{1 + \mathsf{snr}\,\eta P'}\right] \log_2 e + \mathrm{E}_{C'} \left[\log_2 \left(1 + \beta\, \mathrm{E}_{P'} \left[\frac{\mathsf{snr}\, P'C'}{1 + \mathsf{snr}\,\eta P'}\right]\right)\right]. \quad (19)$$

Then it follows that $\frac{E_b}{N_0}_{\min} = -1.59$ dB, which is the same as that of DS-CDMA [7]. The slope of $\mathsf{C}^{\mathsf{opt}}$ in 3-dB units at $\frac{E_b}{N_0}_{\min}$ is derived as

$$S_0^{\mathsf{opt}} = \frac{2\beta}{\beta\kappa(|H|) + \kappa(A)}. \quad (20)$$

We also obtain the slope of $\mathsf{C}^{\mathsf{opt}}$ vs. $\frac{E_b}{N_0}$ in 3-dB units at high SNR as $S_\infty^{\mathsf{opt}} = \min\{\beta P^+, C^+\}$. Therefore, from S_∞^{opt}, $\mathsf{C}^{\mathsf{opt}}$ grows without bound as $\frac{E_b}{N_0} \to \infty$ if $P^+ > 0$ and $C^+ > 0$, which is always true in practical situations. Also, by comparison of S_∞^{opt} with the result for DS-CDMA in [7], the high-SNR slope is not affected by multicarrier fading if $C^+ = 1$. Because of lack of space, our result for the high-load scenario is omitted (see [5] for details) and our conclusion is that the high-load $\mathsf{C}^{\mathsf{opt}}$ is reduced by the subband number reduction effect discussed in previous sections. Figure 2 shows the optimum spectral efficiency vs. $\frac{E_b}{N_0}$ of MC-CDMA and DS-CDMA for $\beta = 2.5$.

5 Decorrelator

5.1 Uplink Capacity Conditioned on Subcarrier Fading

From (5) and letting $\mathsf{snr} \to \infty$, we obtain the decorrelator spectral efficiency as follows.

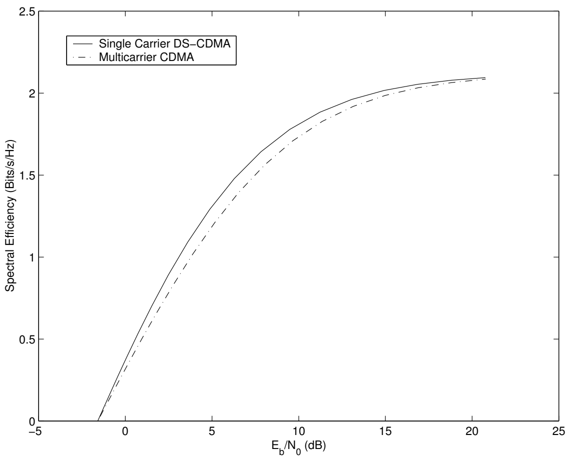

Figure 1: $\mathsf{C}^{\mathsf{mmse}}$ for $\beta = 2.5$.

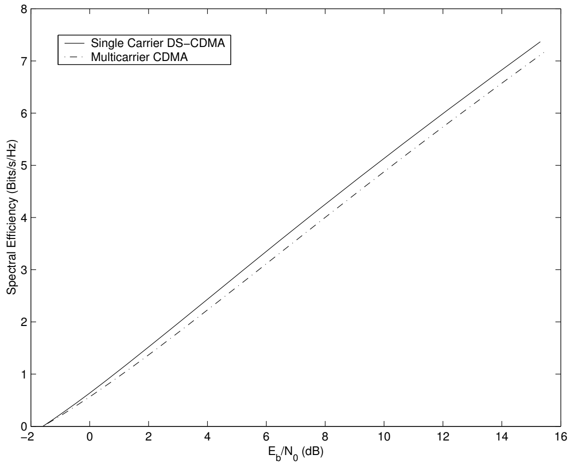

Figure 2: $\mathsf{C}^{\mathsf{opt}}$ for $\beta = 2.5$.

Theorem 5.1 *Conditioned on the fading coefficients, the spectral efficiency of the decorrelator converges almost surely as* $K, N \to \infty$ *with* $\frac{K}{N} = \beta$ *to*

$$\mathsf{C}^{\mathsf{deco}} = \int_0^\beta \log_2 \left[1 + \frac{\mathsf{snr}}{Q}\mu_0(y')\right] dy', \quad \textit{where} \quad \mu_0(y') = \lim_{\mathsf{snr}\to\infty} \mu(y', \mathsf{snr}), \tag{21}$$

and $\mu_0(y') = 0$ *if* $\frac{Z_Y}{Z_X} \geq 1$*. If* $\frac{Z_Y}{Z_X} < 1$*,* $\mu_0(y')$ *satisfies*

$$\mu_0(y') = \int_0^1 \frac{\rho(x, y')}{1 + \displaystyle\int_0^\beta \frac{\rho(x, y)}{\mu_0(y)} dy} dx. \tag{22}$$

5.2 Downlink Capacity Conditioned on Subcarrier Fading

Specializing Theorem 5.1 to the downlink we obtain the following results: $\frac{E_b}{N_0}_{\min} = -1.59 + 10\log_{10}\frac{1}{\eta_0}$ dB where η_0 is the decorrelator multiuser efficiency. Therefore, the reduction of η_0 by the multicarrier frequency-selective fading shown in Section 3 translates into a stricter $\frac{E_b}{N_0}_{\min}$ requirement. The low-SNR and high-SNR slopes of $\mathsf{C}^{\mathsf{deco}}$ are the same as those in [7] for DS-CDMA. Figure 3 shows $\mathsf{C}^{\mathsf{deco}}$ as a function of $\frac{E_b}{N_0}$.

6 Single-user Matched Filter

6.1 Uplink Capacity Conditioned on Subcarrier Fading

Theorem 6.1 *Conditioned on the fading coefficients, the spectral efficiency of the matched filter converges almost surely as* $K, N \to \infty$ *with* $\frac{K}{N} = \beta$ *to*

$$\mathsf{C}^{\mathsf{sumf}} = \int_0^\beta \log_2 \left(1 + \frac{\left[\int_0^1 \rho(x, y')dx\right]^2}{\int_0^1 dx \int_0^\beta \rho(x, y)\rho(x, y')dy + \sigma^2 \int_0^1 \rho(x, y')dx}\right) dy', \tag{23}$$

where $\rho(x, y)$ *is defined in Section 3.1.*

6.2 Downlink Capacity Conditioned on Subcarrier Fading

Particularizing (23) to the downlink, our main results are as follows. $\frac{E_b}{N_0}_{\min} = -1.59$ dB is unaffected by subcarrier fading. The low-SNR slope S_0^{sumf} coincides with that obtained for the MMSE receiver in (13). $S_\infty^{\mathsf{sumf}} = 0$ because the matched filter is interference limited. Figure 4 shows $\mathsf{C}^{\mathsf{sumf}}$ as a function of $\frac{E_b}{N_0}$. Comparing the figures with those of the other receivers, we conclude that the matched filter is much more sensitive to the subcarrier fading than the other three receivers.

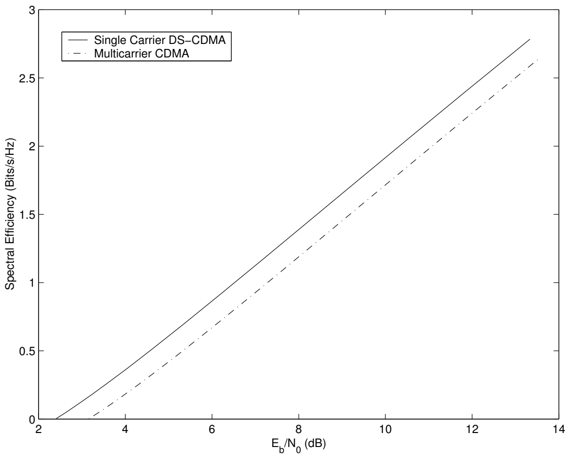

Figure 3: $\mathsf{C}^{\mathsf{deco}}$ for $\beta = 0.6$.

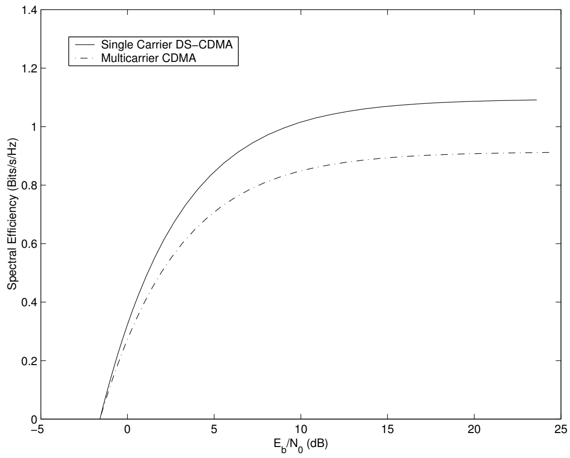

Figure 4: $\mathsf{C}^{\mathsf{sumf}}$ for $\beta = 2.5$.

7 Conclusions

The spectral efficiency of several receivers is analyzed in this paper for MC-CDMA channels subject to multicarrier frequency-selective fading. We analyze both the uplink and the downlink conditioned on subcarrier fading. The conditioned capacity converges asymptotically to an expression that depends, in general —when no assumption on the ergodicity of the channel is made— on the empirical instantaneous power profile of the subcarriers. This is a versatile result whose use transcends wireless channels subject to Rayleigh fading and may be of interest to other potential applications of MC-CDMA to wireline communication. One of our main results is the expression characterizing the extra capacity attained going from optimum linear to optimum nonlinear processing as a function of the uncoded linear MMSE performance measure.

The effect of multicarrier frequency-selective fading on the capacity of several multiuser receivers is also studied. Our results show that there is generally a capacity loss incurred by subcarrier fading. There are two main causes for the loss. The first cause happens when some subbands are so deeply faded that virtually all the energy that was put into them is wasted. This loss can be more readily avoided in practice. The second cause, reflected by Jensen's inequality, can be explained by the nonuniformity of the subband fading powers reducing the effective number of transmitting subbands. The impact of subcarrier fading is more significant for the single-user matched filter than for the optimum receiver, MMSE receiver, and decorrelator. Therefore exploiting the structure of MAI in receiver design for MC-CDMA is beneficial, not only to increase channel capacity, but to desensitize against the impact of subband frequency-selective fading.

Technically, the development of asymptotic results for MC-CDMA and DS-CDMA uses different tools. While in the latter, the Marcenko-Pastur [6] framework was shown to be sufficient in [7], here the more general framework by Girko [1] proves to be instrumental in the key technical results [9].

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