

Spectral Efficiency of Peak Power Limited Rician Block-Fading Channels

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In this paper, we study the capacity and spectral efficiency of peak power limited Rician block-fading channels when neither the receiver nor the transmitter knows the fading coefficients. Among fading channel models, it is of particular interest to consider the case in which the receiver and transmitter have imperfect knowledge of the fading which is a realistic assumption especially when the channel experiences fast fading. Verdú [1] has recently shown that when the receiver has imperfect fading side information, input signals with increasingly higher peak power is required to achieve the capacity as $\text{SNR} \rightarrow 0$. Abou-Faycal et al. [2] proved that the capacity-achieving input amplitude distribution of the average power limited memoryless unknown Rayleigh fading channel is discrete with a finite number of mass points.

We consider the following Rician m -block fading channel model

$$\mathbf{y}_i = h_i \mathbf{x}_i + \mathbf{n}_i \quad (1)$$

where $\{\mathbf{x}_i\}$ and $\{\mathbf{y}_i\}$ are sequences of m complex-dimensional channel input and output vectors respectively, $\{h_i\}$ is a sequence of independent and identically distributed (i.i.d.) proper complex Gaussian random variables with $E\{h_i\} = d$ and $\text{var}(h_i) = \gamma^2$, and $\{\mathbf{n}_i\}$ is a sequence of temporally and spatially white zero mean circular complex Gaussian random vectors with $E\{\|\mathbf{n}_i\|^2\} = mN_0$. We assume the *noncoherent scenario*, where neither the receiver nor the transmitter knows the fading coefficients $\{h_i\}$. We first investigate the structure of the capacity-achieving input distribution.

Theorem 1 For the noncoherent Rician m -block fading channel (1) where the input is subject to only a peak power limitation $\frac{1}{m}\|\mathbf{x}_i\|^2 \stackrel{\text{a.s.}}{\leq} P$, the capacity-achieving input vector can be written as $\mathbf{x}^* = r\mathbf{v}$, where the magnitude, r , is a non-negative real random variable that has a discrete distribution with a finite number of mass points, and \mathbf{v} is an independent isotropically distributed unit random vector.

Next we study the spectral-efficiency/bit-energy tradeoff in the low power regime. In particular, we analyze the energy per information bit required at zero spectral efficiency, and the wideband slope [1] which is defined as the slope of the spectral efficiency curve at zero spectral efficiency. We first consider the case where the transmitter peak-to-average power ratio is limited.

Theorem 2 For the noncoherent Rician m -block fading channel (1) where the input is subject to $\frac{1}{m}E\{\|\mathbf{x}_i\|^2\} \leq P_{av}$ and $\frac{1}{m}\|\mathbf{x}_i\|^2 \stackrel{\text{a.s.}}{\leq} \kappa P_{av}$ with $1 \leq \kappa < \infty$, the received bit energy required at zero spectral efficiency and wideband slope are

$$\frac{E_b^r}{N_0} \Big|_{C=0} = \left(1 + \frac{1}{K}\right) \log_e 2 \quad \text{and} \quad S_0 = \frac{2K^2}{(1+K)^2 - m\kappa}$$

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respectively, where $K = \frac{|d|^2}{\gamma^2}$ is the Rician factor. Moreover, the wideband slope is achieved by a two-mass-point discrete magnitude distribution with one mass at the origin and the other mass at $\|\mathbf{x}\| = \sqrt{m\kappa P_{av}}$ with probability $1/\kappa$.

Note that in the Rayleigh channel where $K = 0$, $\frac{E_b^r}{N_0} \Big|_{C=0} = \infty$, and thus reliable communication is not possible as $\text{SNR} \rightarrow 0$. In the Rician channel with $K > 0$, the bit energy required at zero spectral efficiency is finite. However, if $m\kappa > (1+K)^2$, then the wideband slope is negative and hence the minimum bit energy is achieved at a nonzero spectral efficiency, $C^* > 0$. In this case, one should avoid operating at $C < C^*$. Below we further consider the case in which the transmitter is limited in peak power, but there is no constraint on the peak-to-average power ratio.

Theorem 3 Consider the following more general noncoherent Rician m -block fading channel

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x}_i + \mathbf{n}_i \quad (2)$$

where, as the only difference from (1), \mathbf{H}_i is a diagonal matrix whose entries are jointly Gaussian complex random variables with a covariance matrix Σ . We assume that each component of \mathbf{H}_i has mean d and variance γ^2 . We further assume that the input is subject to $\frac{1}{m}E\{\|\mathbf{x}_i\|^2\} \leq P_{av}$ and $\frac{1}{m}\|\mathbf{x}_i\|^2 \stackrel{\text{a.s.}}{\leq} \nu$, where ν is a fixed peak limit that does not vary with P_{av} . Then, the minimum received bit energy, which is achieved at zero spectral efficiency, is

$$\frac{E_b^r}{N_0 \min} = \frac{\log_e 2}{1 - \frac{1}{K+1} \frac{\log_e(1+m\eta)}{m\eta}}$$

where $K = \frac{|d|^2}{\gamma^2}$ is the Rician factor and $\eta = \frac{\gamma^2}{N_0} \nu$.

Theorem 4 For the noncoherent Rician m -block fading channel (1) with input constraints $\frac{1}{m}E\{\|\mathbf{x}_i\|^2\} \leq P_{av} \leq \nu$ and $\frac{1}{m}\|\mathbf{x}_i\|^2 \stackrel{\text{a.s.}}{\leq} \nu$, the two-mass-point input magnitude distribution

$$F_{\|\mathbf{x}\|} = \left(1 - \frac{P_{av}}{\nu}\right) u(\|\mathbf{x}\|) + \frac{P_{av}}{\nu} u(\|\mathbf{x}\| - \sqrt{m\nu})$$

achieves the minimum bit energy and the optimal wideband slope.

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