Spectral Efficiency of Randomly Spread DS-CDMA in a Multi-Cell Model

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Abstract
A simple multi-cell uplink communication model is suggested and analyzed for optimally coded randomly spread direct sequence code division multiple access (DS-CDMA) with linear detection strategies. We adhere to Wyner's (1994) infinite linear cell array model, according to which only adjacent cell interference is present, and characterized by a single parameter $0 \leq \alpha \leq 1$. We confine our discussion to asymptotic analysis where both the number of users and the processing gain go to infinity, while their ratio is kept fixed and finite. We focus on single cell site processing and consider the matched-filter detector, "optimum" detection with adjacent cell interference treated as Gaussian noise, the linear MMSE detector, and a Decision-Feedback MMSE detector with linear MMSE processing of adjacent cell interference. Spectral efficiency is evaluated under chosen power control strategies: equal received power (for all users), equal rates, and a maximal spectral efficiency policy. Comparative results demonstrate how performance of the diffuse detector is affected by the introduction of inter-cell interference, and what is the penalty associated with the randomly spread coded DS-CDMA strategy.

1 Introduction
Information theoretic analysis of direct sequence code division multiple access (DS-CDMA) systems is of great interest in recent years, due to the rapid development of commercial cellular systems employing this multi-accessing strategy. Results and observations of major importance, for a single cell DS-CDMA system, were recently published in [1], and [2] (see also references therein). These works explicitly relate to CDMA systems with random spreading sequences, and the limiting scenario is examined, where both the number of users and the processing gain go to infinity, while their ratio is kept
fixed and finite. Assuming equal powers, expressions are presented in [1] for spectral efficiencies of the optimum detector, the matched filter detector, the deconvolutor, and the linear minimum mean squared error (MMSE) detector. Signal to interference plus noise ratio at the output of linear detectors (the matched filter detector, the linear MMSE detector, and the deconvolution detector) are presented in [2], and an extension of these results to fading channels with multi-antenna reception can be found in [3].

Our approach uses multi-cell cellular systems using the attractive cellular model suggested by Wyner in [4]. This simple model allows for analytical tractability on the one hand, while giving insight to "real life" systems on the other. Accordingly, the system's cells compose an infinite linear array, where the received signal at each cell site is the sum of the signals received from intra-cell users, plus a factor $\alpha (0 \leq \alpha \leq 1)$ times the sum of the signals of users in the two adjacent cells, as received at their cell outer. This signal is accompanied by an ambient Gaussian noise. The multi-cell effect on performance is thus specified by a single parameter $\alpha$.

We assume, as in [4], non-fading channels, and analyze the spectral efficiency obtained by employing optimally coded randomly spread DS-CDMA multi-user detection. As in [1] and [2], we consider the limiting case in which, denoting by $K$ the number of users (assumed constant and equal in all cells), and by $N$ the spreading factor (processing gain), $K, N \to \infty$, where $\beta = \theta$ remains constant. Assuming single-cell user processing, four types of multiuser detection strategies are considered. We first consider the conventional matched filter detector which treats all interference (either intra-cell or inter-cell) as additive white Gaussian noise. The second detector, optimally, detects the transmitted sequences of intra-cell users, while treating inter-cell interference as additive white Gaussian noise. This detector will be henceforth referred to as the single-cell optimum detector (SCD). Next we consider the linear MMSE detector which makes use of knowing the signature sequences of interfering users (both intra-cell and at adjacent cells) to suppress their interference by means of a linear MMSE filter. The last detector to be considered is a decision feedback (DF) MMSE detector with decision feedback detection performed over transmissions of intra-cell users, while inter-cell interference is suppressed by means of a linear MMSE filter. We emphasize that neither the linear MMSE detector, nor the DF-MMSE detector, try to decode the transmissions of users from adjacent cells (which might be impossible if $\alpha$ is small). In fact, the cell-site detector may actually be ignorant regarding codebooks or code-book sequences employed in other cells, but is aware, as usually is the case in practice, of the signature sequences of all users in adjacent cells. In addition to the above it is also assumed that all detections are provided with the required knowledge regarding the received powers of the interfering signals.

The above detectors are decomposed under three power control strategies. First we consider the equal powers strategy, which assigns equal received power to all users. Next, we consider the equal rates strategy which employs a power assignment function such that all users maintain equal rate (in the sense of information theoretic capacity). We call the strategies which are based on equal rates strategy the strategy which attains the maximum spectral efficiency. Finally, we consider the systems with different powers for the different users, which are based on the equal rates strategy.

This paper is organized as follows. In section 2 we present the system model, and the expressions through which the spectral efficiency of the four detectors is obtained. The following sections 3, 4, and 5, discuss the three power control strategies mentioned above. Section 6 ends the paper with a summary and some concluding remarks.

We transform from cells further away as assumed.

The use of adaptive MMSE detection may be attractive with this respect, as it makes no distinction between intra-cell and adjacent-cell interference.

2 System Model and Performance Analysis

2.1 Multi-Cell System Model

Following [4] and [7], we consider the system whose cells are ordered in a time equivalent channel representation at the discrete time related to the transmits

$$y_n = X_n \bar{s}_n + \nu_n$$

The vector $x_n = [x_1, \ldots, x_n]$ is $N$-intra-cell users at the $n$th discrete time, while $s_n$ is a matrix of symbols received at adjacent cells. These symbols are assumed to be $X_n \in \mathbb{C}^N$, where $\mathbb{C}$ is the power control, and $S_n$ are $N \times K$ matrices, whose columns are the sequences of the $K$ users in the cell. According to our assumptions, the entries are mean random variables, with variance Gaussian noise vector, with $\mathbb{E}\{n_n\} = 0$.

We analyze the effect of power control by "power control strategy" we mean which assigns a received power level for each cell by a function of a continuous argument, which can be replaced by integral and subject to an average power constraint $\bar{C}$.

$\bar{C}$ is justified by assuming that the cells are independently random distributions per cell [8].

$\bar{C}$ is assumed that the users adjust this level.

$\bar{C}$ is the best rate of alleviation of the constraint argument, and follows the interference at the output of each of noise, the spectral efficiency is given by the detector.

$\bar{C} = \frac{\beta}{2}$

This fundamental figure of merit is the second-order (i.e. Signal to Information Ratio (SIR) function at the output of a multi-user detector, which is the ratio of the spectral efficiency the fourth-detection is obtained. The following sections 3, 4, and 5, discuss the three power control strategies mentioned above. Section 6 ends the paper with a summary and some concluding remarks.

$\{\bar{C}_n\} = \mathbb{E}\{n_n\}$, $\bar{C}_n = \beta/2, \bar{C} = \beta/2$

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2 System Model and Multiuser Detection Strategies

2.1 Multi-Cell System Model

Following [4] and [5], we consider the uplink of a fully synchronous cellular CDMA system whose cells are ordered in an infinite linear array. Using the standard discrete time equivalent channel representation, the signal vector received at an arbitrary cell site, at the discrete time related to the transmission of the ith symbol, is given by

\[ \mathbf{y}_i = \mathbf{S}_i \mathbf{x}_i + \mathbf{n}_i \]  

(2-1)

where \( \mathbf{y}_i \) is an \( N \times 1 \) vector comprised of the samples received at all cells in cell \( i \) during the symbol interval \( T \). \( \mathbf{S}_i \) is an \( N \times N \) matrix whose elements are the tap gains, \( \mathbf{x}_i \) is a \( N \times 1 \) vector of the symbols transmitted from all cells in cell \( i \) during the symbol interval \( T \), and \( \mathbf{n}_i \) is an \( N \times 1 \) vector of the noise samples received at all cells in cell \( i \) during the symbol interval \( T \).

The vector \( \mathbf{x} = [x_1, \ldots, x_L]^T \) in (2-1) comprises the \( K \) code symbols received from intra-cell users at the \( i \)th discrete time. The vectors \( \mathbf{x}_j = [x_{j1}, \ldots, x_{jK}]^T \) denote the vector of symbols received at adjacent cell sites from the users operating in these adjacent cells. These symbols are assumed to be i.i.d. Gaussian, with \( E[\mathbf{x}_j] = 0 \) and \( E[\mathbf{x}_j]\mathbf{x}_j^H = \mathbf{P}_j \mathbf{I} \), where \( \mathbf{P}_j \) is the power control received power of the \( j \)th user. The matrices \( \mathbf{S}_i \) and \( \mathbf{S}_j^H \) are \( N \times N \) matrices, whose columns are the chip duration spreading (signature) sequences of the \( K \) users in the considered cell and in its adjacent cells, respectively. According to our assumptions the entries of the above matrices are treated as i.i.d. zero mean random variables, with variance \( 1/N \).

The vector \( \mathbf{n}_i \) represents a zero mean white Gaussian noise vector, with \( E[\mathbf{n}_i] = 0 \) and \( E[\mathbf{n}_i]\mathbf{n}_i^H = \mathbf{I} \).

We analyze the effect of power control strategies on the overall system performance. By "power control strategy" we mean a system defined centrally controlled function, which assigns a power level to each cell in the cell array. According to our model exactly the same power control strategy is applied to all cells. Denoting the normalized power (SNR) of the \( j \)th user by \( P_j \), it is assumed that the number of users grows to infinity: the discrete power control function \( P_j \) goes to a limit given by a function of a continuous argument, denoted henceforth by \( f(x) \) \((0 < x < 1)\), and sums can thus be replaced by integrals. The above power control function is assumed to be subject to an average power constraint, which is expressed in the continuous form by

\[ \int_0^1 f(x) \, dx = \bar{P} \]  

(2-2)

The fundamental figure of merit for system performance is the per cell spectral efficiency (see [1]), defined as the total number of bits per cell that can be transmitted arbitrarily reliably in a single cell. Denoting the signal to interference plus noise ratio (SINR) function at the output of a detector by \( g(z) \) (according to our limiting continuous argument assumption), and following central limit results which have shown that the interference at the output of each of the detectors is well approximated by a Gaussian noise, the spectral efficiency is given by (see next subsection for applicability to the SCD detector)

\[ \bar{E}_s = \frac{\bar{P}}{\frac{1}{2} \int_0^1 \log(1 + g(z)) \, dz} \]  

(2-3)

This is justified by assuming that the codewords of all users are chosen randomly, governed by an underlying i.i.d. Gaussian distribution for each symbol, and independently for each symbol transmission (see [6]).

It is assumed that the user adjust their transmission power so as to be received at the prescribed level.

This is that, \( P_j = \frac{1}{x_j} \mathbf{I}, x_j = 1, \ldots, K \), with \( x = \frac{1}{x_j} \).

The authors of [2], [6], and [7] for justification of this Gaussian approximation.
In the following subsection we generally describe how the spectral efficiency of all four multiuser detectors is obtained in terms of the power control function. We use the notation of \( \xi, \xi_m, \xi_s, \) and \( \xi_f \) to designate entities related to the matched filter detector, the SCO detector, the linear MMSE detector, and the DF-MMSE detector, respectively.

2.2 Spectral Efficiency of the Multiuser Detectors

The matched filter detector simply passes the received signal through a filter matched to the signature sequence of the user of interest, thus treating all interfering signals as a pure additive white Gaussian noise. It is hence straightforward to show that the spectral efficiency of the matched filter detector is given by

\[
C_\mu = \frac{\beta}{2} \int_\mathbb{R} \log \left( 1 + \frac{f(x)}{1 + \beta (1 + 2\sigma^2)P} \right) dx.
\]  

(2-4)

The spectral efficiency of the SCO detector is more conveniently obtained using the well-known observation (e.g. see [6] and [8]) that the DF-MMSE detector achieves the optimum spectral efficiency, assuming Gaussian input symbols. Following [2], the SIR function at the output of the above equivalent DF-MMSE detector is determined by solving the following implicit equation

\[
g_m(x) = \frac{f(x)}{1 + 2\sigma^2 P + \beta \int f(t) \frac{d^2 f(t)}{dt^2} dt}.
\]  

(2-5)

The spectral efficiency of the SCO detector can be evaluated through (2-3). Similarly, the SIRs at the output of the linear MMSE detector and the DF-MMSE detector are obtained, respectively, via

\[
g_m(x) = \frac{f(x)}{1 + \beta \int f(t) \frac{d^2 f(t)}{dt^2} dt} + \beta \int \frac{f(t) d^2 f(t)}{dt^2} dt
\]  

(2-6)

and

\[
g_m(x) = \frac{f(x)}{1 + \beta \int f(t) \frac{d^2 f(t)}{dt^2} dt} + \beta \int \frac{f(t) d^2 f(t)}{dt^2} dt
\]  

(2-7)

Again we use (2-3) to evaluate the spectral efficiency of both detectors.

We note that when different systems (with possibly different spreading gains and data rates) are to be compared, it is imperative to express the spectral efficiency in terms of \( \xi_f \), which is done through the relation \( P = \frac{f}{C_{\xi_f}^{[1]}(\xi_f)} \) (see [1]). However, for simplicity of notations we express equations in terms the normalized power. For details of the derivation of the above equations see [8].

3 Equal Powers

Satisfying the above power constraint of (2-2), the equal powers strategy simply implies setting \( f(x) = P, \forall 0 < x \leq 1 \). Substituting the above power control function in (2-4), the spectral efficiency of the matched filter detector is given by

\[
C_\mu = \frac{\beta}{2} \log \left[ 1 + \frac{P}{1 + \beta (1 + 2\sigma^2)P} \right].
\]  

(5-1)

As can be seen, the above result remains without bound, which is in accordance with matched filter detector. The same holds for the optimum in terms of spectral efficiency of the two.

\[
C_{\xi_f} = \frac{\beta}{2} \log \left[ 1 + \frac{1}{1 + \beta + 2\sigma^2} \right] + \frac{1}{4} \int \frac{P}{1 + \beta + 2\sigma^2} dP
\]  

where \( P(x) = (\sqrt{2}(1 + \sqrt{2})^2 + 1 - x^2) \) detector, it is observed that the above \( \xi_f \) (note that \( P_{\xi_f} \rightarrow \frac{P}{2} \xi_f \) as \( \xi_f \rightarrow \infty \)) detector-emancipates from the fact that Gaussian noise. Again, \( \beta \rightarrow \infty \) as \( \xi_f \rightarrow \infty \).

Taking \( \beta \rightarrow \infty \), the spectral efficiency of the matched filter, as expected.

Substituting \( f(x) = P \) into (2-7) does not determine the value of the SIR function is not a constant). Obtaining can be evaluated through (2-3). As with the DF-MMSE detector, the linear MMSE detector can be shown appropriately chosen (again the optimum is observed that the spectral efficiency the SCO detector, which is in agreement for \( \beta \rightarrow \infty \)).

Fig. 1 compares the spectral efficiency \( \alpha = \frac{1}{2} \) and optimum choice of \( \beta \). As with the MMSE and the DF-MMSE detectors, the and the SCO detector, for low \( \xi_f \) where increases, beyond some critical values of based detectors decreases, eventually beyond which the detector gives without bound in the spectral efficiency curves (so not spectral efficiency is plotted for a fixed of the DF-MMSE detector is however. One can also notice that beyond some spectral efficiency of the linear MMSE.

Another interesting issue is the power to observe that the spectral efficiency
It is seen that the spectral efficiency of all the power control function. We use the same entries related to the matched filter detector, and the DF-MMSE detector.

**Itzner Detectors**

Received signals through a filter matched thus treating all interfering signals as a straightforward to show that the spectral by

\[
    f(x) = \frac{1}{1 + 2n\sigma^2} P
\]

is most conveniently obtained using the at the DF-MMSE detector achieves the input symbols. Following [2], the SIR DF-MMSE detector is determined by

\[
    \frac{\partial C_{\text{MMSE}}}{\partial \beta} = \frac{1}{1 + 2n\sigma^2} \frac{\partial}{\partial \beta} \left( \frac{1}{1 + 2n\sigma^2} \right) = \frac{1}{1 + 2n\sigma^2}
\]

where the spectral efficiency of the MMSE detector and the DF-MMSE detector is determined by

\[
    \frac{\partial C_{\text{MMSE}}}{\partial \beta} = \frac{1}{1 + 2n\sigma^2} \frac{\partial}{\partial \beta} \left( \frac{1}{1 + 2n\sigma^2} \right) = \frac{1}{1 + 2n\sigma^2}
\]

and of both detectors.

Equation of the same powers strategy simply implies above power result function in (3-4), that is given by

\[
    P = 1 + 2n\sigma^2 \frac{1}{P}
\]

As can be seen, the above result reaches a limit as the average power (or \( P \)) grows without bound, which is in accordance with the interference limited behavior of the matched filter detector. The same behavior is observed as we take \( \beta \to \infty \), which is also optimum in terms of spectral efficiency.

Denoting \( P_{\text{opt}} = \frac{C_{\text{MMSE}}}{C_{\text{DF}}} \), and substituting \( f(x) = P \) into (2.5), it can be shown that the spectral efficiency of the SIR detector is given by

\[
    C_{\text{DF}} = \frac{3}{2} \log \left[ 1 + P_{\text{opt}} \right] + \frac{3}{2} \log \left[ 1 + P_{\text{opt}} \right] - \frac{3}{2} \log \left[ 1 + P_{\text{opt}} \right] - \frac{3}{2} \log \left[ 1 + P_{\text{opt}} \right] - \frac{3}{2} \log \left[ 1 + P_{\text{opt}} \right]
\]

(2.6)

where \( f(x, \beta) = \sqrt{\beta} (1 + \sqrt{\beta}) + 1 - \sqrt{\beta} (1 + \sqrt{\beta}) \). As with the matched filter detector, it is observed that the above spectral efficiency reaches a limit as we set \( P_{\text{opt}} \to \infty \) (note that \( P_{\text{opt}} \to \frac{C_{\text{MMSE}}}{C_{\text{DF}}} \to \infty \)). The interference limited behavior of the SIR detector emanates from the fact that jamming interference is treated as white Gaussian noise. Again, \( \beta \to \infty \) is optimum in terms of spectral efficiency.

The SIR at the output of the linear MMSE detector is given by the positive root of the cubic equation obtained by substituting \( f(x) = P \) into (3.4). As can be seen, the resulting SIR function is a constant, and thus the spectral efficiency of the linear MMSE detector is simply given by \( C_{\text{MMSE}} = \frac{3}{2} \log (1 + \alpha \nu) \). In contrast to the two previous detectors, this linear MMSE filter is interference limited if \( \beta \) is appropriately chosen. It can be shown that the spectral efficiency of the linear MMSE detector grows without bound as we take \( \beta \to \infty \), as long as we keep \( \beta \leq \frac{3}{2} \) (optimum choice is \( \beta = \frac{3}{2} \) for large \( \alpha \)).

Taking \( \beta \to \infty \), the spectral efficiency of the linear MMSE detector coincides with that of the matched filter, as expected from the single-cell results of [1].

Substituting \( f(x) = P \) into (2.7) results again in a cubic equation, whose positive root determines the value of the SIR function \( x(P, \beta) \), \( \beta \in [0, \frac{3}{2}] \) (note that the SIR function is not a constant). Obtaining the SIR function, the resulting spectral efficiency can be evaluated through (2.3). With the linear MMSE detector, the spectral efficiency of the DF-MMSE detector can be shown to grow without bound as \( \beta \to \infty \), if \( \beta \) is appropriately chosen (again the optimum choice is \( \beta = \frac{3}{2} \) for large \( \alpha \)). Taking \( \beta \to \infty \) it is observed that the spectral efficiency of the DF-MMSE detector coincides with that of the SIR detector, which is in agreement with the behavior of the linear MMSE detector for \( \beta \to \infty \).

Fig. 1 compares the spectral efficiencies of the four detectors discussed above with \( \alpha = 1 \) and optimum choice of \( \beta \). As can be seen the spectral efficiencies of the linear MMSE and the DF-MMSE detectors, coincide with those of the matched filter detector and the SIR detector, for low values of \( \beta \) where it is optimum to choose \( \beta \to \infty \). However as \( \beta \) increases, beyond some critical values of \( \beta \) the optimum choice of \( \beta \) for the two MMSE based detectors decreases, eventually becoming lower than \( \frac{3}{2} \), and the spectral efficiency of these detectors grows without bound with \( \beta \). This explains the "bend effect" observed in the spectral efficiency curves (we note that the "bend effect" is not observed when the spectral efficiency is plotted for a fixed value of \( \beta \)). The slope of the spectral efficiency of the DF-MMSE detector is however smaller than that of the linear MMSE detector. One can also notice that beyond some critical value of \( \beta \) (around \( 16 \) in Fig. 1) the spectral efficiency of the linear MMSE detector surpasses that of the SIR detector.

Another interesting issue is the penalty due to random spreading. Following [5], one can observe that the spectral efficiencies of the matched filter detector and the SIR
4 Equal Rates

The SCO detector provides equal rates to all users by assigning equal received powers to all. With the remaining three detectors, guaranteeing equal rates to all users requires that the SIR function $g(x)$ is a constant. As can be seen from section 3, the above requirement is satisfied for the two linear detectors, i.e., the matched filter detector and the linear MMSE detector, by assigning (again) equal powers to all users. This leaves us with the examination of the DF-MMSE detector. We also consider, for the sake of comparison, the DF-MMSE detector which treats adjacent cell interference as additive Gaussian noise (this detector is equivalent to the SCO detector in terms of (overall) spectral efficiency, as mentioned above).

The desired power control function can be numerically obtained by substituting $g(x) = W$, for some constant $W$, into either (3-3) or (2-2), and solving these equations so that the power constraint of (2-2) is satisfied.

Results show that in terms of (overall) spectral efficiency evaluated at the optimum

\[\text{choice for } \beta, \text{ the equal rate strategy is least for } \frac{S}{N}, \text{ values of up to } 20\text{dB (assuming the Gaussian interference assumption, with difference is increasing with } \frac{S}{N}.\]

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powers for $n = 1$, and an optimum

Figure 2: Base ratio of the equal-power and equal-rate strategies for optimum $\delta / \gamma = \infty$ is represented by $\delta = 10$ and $\delta = 1565$ (SE) designates adjacent-cell interference suppression. "GS" designates interference treated as noise (both are equivalent for $n = 0$).

5 Maximum Spectral Efficiency

Choosing $\delta > 0$, it is quite straightforward to show, using Jensen's inequality and the convexity of the $\log(1 + z)$ function, that the spectral efficiency of the matched filter
detector is maximized by assigning equal powers to all users.

The same result holds for the spectral efficiency of the SCO detector. This is observed by direct evaluation of the spectral efficiency of the SCO detector through the channel's input-output mutual information (without resorting to the equivalent DF-MMSE detector). The derivative of the spectral efficiency with respect to \( P_k \) (taken as an argument), as defined in section 3, is thus seen to be a decreasing function of the Stieltjes transform of the eigenvalue distribution of matrices of the form \( S_i S_i^H A \), where \( S_i \) is the signatures matrix as defined in section 2 above, and \( A \) is a diagonal matrix with the normalized powers of the users as its entries. The above Stieltjes transform (see [10]) is minimized by the equal power distribution, thus maximizing the spectral efficiency of the SCO detector.

Solving the general optimization problem with respect to the spectral efficiency of both the linear MMSE detector and the DF-MMSE detector is quite complex, as can be seen from (2.4) and (2.7). We therefore turn to the derivation of a lower bound on the maximum spectral efficiency. Consider the power control function \( f^e(x) \) defined as

\[
  f^e(x) = \begin{cases} 
  0 & 0 < x \leq \rho^* \\
  0 & 0 < x \leq 1 
  \end{cases}
\]

Clearly, \( f^e(x) \) satisfies the average power constraint (see (2.3)). The rationale behind \( f^e(x) \) is that given a system, with some fixed \( \beta \), interference factor \( \alpha \), and system load \( \beta \), one might gain in spectral efficiency by effectively "shutting down" part of the users' transmissions by means of the power control function, and assigning more power to the remaining users while satisfying the average power constraint \( \alpha \). Obviously, assigning all power to a single user (\( \rho = 0 \)) is useless since this drives the system's spectral efficiency to zero. The power control function \( f^e(x) \) comes into effect if there exists some value of \( \rho = \rho^* < 1 \) for which the spectral efficiency is maximized. In the case in which \( \rho^* = 1 \), \( f^e(x) \) simply reduces to the equal power strategy discussed in section 2. The above optimization is in fact equivalent to the optimization of the spectral efficiency with respect to \( \beta \) under the equal power strategy, for fixed \( \rho^* \), if the initially set value of \( \beta \) is higher than the optimum value (note that \( f^e(x) \) cannot increase the effective load). Hence, for a given system, at a fixed working point specified by \( \rho^* \), \( \alpha \), and \( \beta \), the maximum spectral efficiency, where maximization is taken over the set of power control functions, is lower bounded by the spectral efficiency obtained by substituting \( f^e(x) \) into (2.4) and (2.7), and evaluating the spectral efficiency as \( \rho = \rho^* \).

Some comparative results, demonstrating the effect of maximizing the spectral efficiency via the power control function are demonstrated in Fig. 3. Here the spectral efficiency of all four detectors is plotted for \( \alpha = 1.5 \) and \( \beta = 1 \). The spectral efficiency of the linear MMSE and DF-MMSE detector is evaluated assigning the power control function \( f^e(x) \), with \( \rho = \rho^* \). For the matched filter detector and the SCO detector we evaluated the spectral efficiency with the equal power assignment (which attains the maximum). Also included in Fig. 3 are the spectral efficiencies without random spreading of the three detectors mentioned in section 3 (denoted by "SS") and a curve labeled by the spectral efficiency for \( \beta = 1 \). This is also the case with the spectral efficiency of the matched filter, and the SCO detector, for all values of \( \beta \). However, beyond some MMSE and the DF-MMSE detector, control function \( f^e(x) \) (with \( \rho = \rho^* \)) produces a maximum value with the equal power assignment.

### 6 Summary and Conclusion

In this paper we analyzed the spectral efficiency of the system by the amount and type of information to interference (both intra-cell and inter-cell) and the effect of information about interfering users (both linear MMSE detector and the SCO detector) to the overall system performance. We showed the potential of the interference-aware scheduling scheme for decreasing the interference to the users of the interference-aware scheduling scheme, which is also shown to be superior to the linear MMSE detector. The interference-aware scheduling scheme is shown to be able to reduce the interference to the users of the interference-aware scheduling scheme, which is also shown to be superior to the linear MMSE detector. The interference-aware scheduling scheme is shown to be able to reduce the interference to the users of the interference-aware scheduling scheme, which is also shown to be superior to the linear MMSE detector.
to all users.

cy of the SCO detector. This is observed the SCO detector through the channel’s time to the equivalent DF-MMSE denote-
Fourth respect to $F_0$, taken as an argument, time function of the Staircase transform form $S[A]$, where $S$ is the signature
a diagonal matrix with the normalized
 manipulated
ups-transform [see (10)] is minimized by
spectral efficiency of the SCO detector.
respect to the spectral efficiency of
4SE detector is quite complex, we can be
the derivation of a lower bound on the
or control function, $f_{\alpha}(x)$ defined as

$$f_{\alpha}(x) = \frac{x^\alpha}{\Gamma(\alpha+1)}.$$  \hspace{1cm} \text{(6)}$$

trained (see (2-8)). The rationale behind
$\hat{\theta}$, interfering factor $\sigma$, and system load
levy “shutting down” part of the users
function, and assigning more power to the
over constraint.\footnote{Actually, assigning all
this drive the system’s spectral efficiency
by $\beta$, if the initially we value
that $f_{\alpha}(x)$ obtains increase the effective
loading points specified by $\hat{\theta}$, $\sigma$, and $\beta$ the
$\beta$ is taken over the set of power control
values obtained by substituting $f_{\alpha}(x)$ into
constraint $\beta = \alpha$.

The spectral efficiency is evaluated assigning the power control
able detector and the SCO detector for
over assignment (which attains the max-
max-efficiencies without random spreading of
ated by $\text{SNR}$.\footnote{For the parameters where $\beta = 0$}. This is
matched filter, and the SCO detector, for
confirm us to equal power assignment in all

valid values of $\hat{\theta}$. However, beyond some critical value of $\hat{\theta}$, the optimum $\beta$ for the linear

Figure 3: Spectral efficiency comparison using $f_{\alpha}(x)$, for $\alpha = \frac{1}{2}$ and $\beta = 1$.

6. Summary and Conclusions

In this paper we analyzed the spectral efficiency of our miniature detectors, which differ by the amount and type of interference available to the detector, with regard to interference (both intra-cell and inter-cell). Our results demonstrate the dramatic effect of information about interfering signals, which is most clearly seen by comparing the linear MMSE detector and the SCO detector (the DF-MMSE detector is obviously superior to all, being the most informed, and the matched filter detector is inferior to all), being the least informed). We showed that one can gain, even without trying to decode the transmission of the interfering users in adjacent cells\footnote{Which enables interference mitigation}, or treating them optimally in the setting of an interference channel (see [5]), by the very fact that the linear MMSE filter will average for reducing the interference they cause, provided that their signatures are known. It was shown that when $\hat{\theta}$ and the interference factor are not too low, the

\textit{Which enables interference mitigation.}
spectral efficiency of the linear MMSE and the DF-MMSE detectors. Our results show that applying the equal rates power control strategy to the DF-MMSE detector results in almost no loss in spectral efficiency, in comparison to the equal power strategy (where $\alpha$ is not too high). Conditions at which the equal rate strategy is preferable were also discussed.

Analysis of the optimum multi cell site processing detector [4] is under current study, as well as characterization of the optimum power control strategy, in terms of spectral efficiency, for the linear MMSE and DF-MMSE detectors, and consideration of fading.

References


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