Spectral Efficiency of Randomly Spread DS-CDMA in a Multi-Cell Model

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Abstract

A simple multi-cell uplink communication model is suggested and analyzed for optimally coded randomly spread direct sequence code division multiple access (DS-CDMA) multiuser detection strategies. We adhere to Wyner's (1994) infinite linear cell array model, according to which only adjacent cell interference is present, and characterized by a *single* parameter $0 \leq \alpha \leq 1$. We confine our discussion to asymptotic analysis where both the number of users and the processing gain go to infinity, while their ratio is kept fixed and finite. We focus on single cell-site processing and consider the matched filter detector, "optimum" detection with adjacent cell interference treated as Gaussian noise, the linear MMSE detector, and a Decision-Feedback MMSE detector with linear MMSE processing of adjacent cell interference. Spectral efficiency is evaluated under three power control strategies: equal received powers (for all users), equals rates, and a maximal spectral efficiency policy. Comparative results demonstrate how performance of the different detectors is affected by the introduction of inter-cell interference, and what is the penalty associated with the randomly spread coded DS-CDMA strategy.

1 Introduction

Information theoretic analysis of direct sequence code division multiple access (DS-CDMA) systems is of great interest in recent years, due to the rapid development of commercial cellular systems employing this multi-accessing strategy. Results and observations of major importance, for a *single cell* DS-CDMA system, were recently published in [1], and [2] (see also references therein). These works explicitly relate to CDMA systems with *random* spreading sequences, and the limiting scenario is examined, where both the number of users and the processing gain go to infinity, while their ratio is kept fixed and *finite*. Assuming equal powers, expressions are presented in [1] for spectral efficiencies of the optimum detector, the matched filter detector, the decorrelator, and the linear minimum mean squared error (MMSE) detector. Signal to interference plus noise ratios at the output of linear detectors (the matched filter detector, the linear MMSE detector, and the decorrelating detector) are presented in [2], and an extension of these results to fading channels with multi-antenna reception can be found in [3].

Our work addresses *multi-cell* cellular systems using the attractive cellular model suggested by Wyner in [4]. This simple model allows for analytical tractability on the one hand, while giving insight to "real life" systems on the other. Accordingly, the system's cells compose an infinite *linear* array, where the received signal at each cell site is the sum of the signals received from intra-cell users, plus a factor α ($0 \leq \alpha \leq 1$) times the sum of the signals of users in the two adjacent cells[1], as received at their cell sites. This signal is accompanied by an ambient Gaussian noise. The multi-cell effect on performance is thus specified by a *single* parameter (α).

We assume, as in [4], non-fading channels, and analyze the spectral efficiency obtained by employing *optimally* coded *randomly* spread DS-CDMA multi-user detection. As in [1] and [2], we consider the limiting case in which, denoting by K the number of users (assumed constant and equal in all cells), and by N the spreading factor (processing gain), $K, N \to \infty$, while $\frac{K}{N} = \beta$ remains constant. Assuming *single cell-site processing*, four types of multiuser detection strategies are considered. We first consider the *"conventional" matched filter detector* which treats *all* interference (either intra-cell or inter-cell) as additive white Gaussian noise. The second detector "optimally" detects the transmissions of *intra-cell* users, while treating *inter-cell* interference as additive white Gaussian noise. This detector shall be henceforth referred to as *the single-cell "optimum" detector* (SCO). Next we consider the *linear MMSE detector* which makes use of knowing the signature sequences of interfering users (both intra-cell and in adjacent cells) to suppress their interference by means of a linear MMSE filter. The last detector to be considered is a *decision feedback (DF) MMSE detector* with decision-feedback detection performed over transmissions of intra-cell users, while inter-cell interference is suppressed by means of a linear MMSE filter. We emphasize that neither the linear MMSE detector, nor the DF-MMSE detector, try to *decode* the transmissions of users from adjacent cells (which might be impossible if α is small). In fact, the cell site detector may actually be ignorant regarding codebooks or code-mask sequences employed in other cells, but is aware, as usually is the case in practice, of the signature sequences of all users in adjacent cells. In addition to the above it is also assumed that all detectors are provided with the required knowledge regarding the received powers of the interfering signals[2].

The above detectors are examined under three power control strategies. First we consider the equal powers strategy, which assigns equal *received* powers to all users. Next, we consider the equal rates strategy which employs a power assignment function such that all users attain equal rates (in the sense of information theoretic capacity). The last strategy we discuss is the strategy which attains the maximum spectral efficiency.

This paper is organized as follows. In section 2 we present the system model, and the equations through which the spectral efficiency of the four detectors is obtained. The following sections 3, 4, and 5, discuss the three power control strategies mentioned above. Section 6 ends the paper with a summary and some concluding remarks.

[1]No interference form cells further away is assumed.

[2]The use of adaptive MMSE detection may be attractive with this respect, as it makes no distinction between intra-cell and adjacent-cell interference.

2 System Model and Multiuser Detection Strategies

2.1 Multi-Cell System Model

Following [4] and [5], we consider the uplink of a fully synchronous cellular CDMA system whose cells are ordered in an *infinite* linear array. Using the standard discrete time equivalent channel representation, the signal vector received at an arbitrary cell site, at the discrete time related to the transmission of the ith symbol, is given by

$$\boldsymbol{y}_i = \boldsymbol{S}_i \boldsymbol{x}_i + \alpha \boldsymbol{S}_i^- \boldsymbol{x}_i^- + \alpha \boldsymbol{S}_i^+ \boldsymbol{x}_i^+ + \boldsymbol{n}_i \ . \tag{2-1}$$

The vector $\boldsymbol{x}_i = [x_{1,i}, \ldots, x_{K,i}]^T$ in (2-1) comprises the K code symbols received from intra-cell users at the ith discrete time. The vectors $\boldsymbol{x}_i^\pm = \left[x_{1,i}^\pm, \ldots, x_{K,i}^\pm\right]^T$ denote the vectors of symbols received at adjacent cell sites from the users operating in these adjacent cells. These symbols are assumed to be i.i.d., Gaussian[3], with $E\{x_{k,j}\} = 0$ and $E\{x_{k,j}^2\} = P_k \ \forall \, k, j$, where P_k is the power controlled received power of the kth user. The matrices $\boldsymbol{S}_i$ and $\boldsymbol{S}_i^\pm$ are $N \times K$ matrices, whose columns are the N-chip long spreading (signature) sequences of the K users in the considered cell and in its adjacent cells, respectively. According to our assumptions the entries of the above matrices are treated as i.i.d. zero mean random variables, with variance $1/N$. The vector $\boldsymbol{n}_i$ represents a zero mean white Gaussian noise vector, with $E\{\boldsymbol{n}_i \boldsymbol{n}_i^\dagger\} = \sigma^2 \boldsymbol{I}$, $\forall i$.

We analyze the effect of power control strategies on the overall system performance. By "power control strategy" we mean a system defined centrally controlled function, which assigns a *received*[4] power level to each of the intra-cell users. According to our model exactly the same power control strategy is applied to all cells. Denoting the normalized power (SNR) of the kth user by $\tilde{P}_k = \frac{P_k}{\sigma^2}$, it is assumed that as the number of users grows to infinity, the discrete power control function $\{\tilde{P}_k\}_{k=1}^K$ goes to a limit given by a function of a continuous argument, denoted henceforth by $f(x)$ $(0 < x \leq 1)$[5], and sums can thus be replaced by integrals. The above power control function is assumed to be subject to an average power constraint, which is expressed in the continuous form by

$$\int_0^1 f(x) \, dx = \bar{P} \ . \tag{2-2}$$

The fundamental figure of merit for system performance is the per cell *spectral efficiency* (see [1]), defined as the total number of bits per chip that can be transmitted arbitrarily reliably in a single cell. Denoting the signal to interference plus noise ratio (SIR) function at the the output of a detector by $g(x)$ (according to our limiting continuous argument assumption), and following central limit results which have shown that the interference at the output of each of the detectors is well approximated by a Gaussian noise[6], the spectral efficiency is given by (see next subsection for applicability to the SCO detector)

$$\tilde{C} = \frac{\beta}{2} \int_0^1 \log(1 + g(x)) \, dx \ . \tag{2-3}$$

[3]This is justified by assuming that the codebooks of all users are chosen randomly, governed by an underlying i.i.d. Gaussian distribution per symbol, and *independently* for each message transmission (see [5]).

[4]It is assumed that the users adjust their transmission power so as to be received at the prescribed level.

[5]That is, $\tilde{P}_k \xrightarrow[K\to\infty]{} f(\frac{k}{K})$, $k = 1, 2, \ldots, K$, with $x = \frac{k}{K}$.

[6]See [1], [6], and [7] for justification of this Gaussian approximation.

In the following subsection we generally describe how the spectral efficiency of all four multiuser detectors is obtained in terms of the power control function. We use the notation of $(\cdot)_{\mathrm{mf}}$, $(\cdot)_{\mathrm{sco}}$, $(\cdot)_{\mathrm{ms}}$, and $(\cdot)_{\mathrm{df}}$ to designate entries related to the matched filter detector, the SCO detector, the linear MMSE detector, and the DF-MMSE detector, respectively.

2.2 Spectral Efficiency of the Multiuser Detectors

The matched filter detector simply passes the received signal through a filter matched to the signature sequence of the user of interest, thus treating all interfering signals as a pure additive white Gaussian noise. It is hence straightforward to show that the spectral efficiency of the matched filter detector is given by

$$\tilde{C}_{\mathrm{mf}} = \frac{\beta}{2} \int_0^1 \log\left(1 + \frac{f(x)}{1 + \beta\,(1 + 2\alpha^2)\,\bar{P}}\right) dx \ . \tag{2-4}$$

The spectral efficiency of the SCO detector is most conveniently obtained using the well known observation (e.g. see [8], and [1]) that the DF-MMSE detector achieves the optimum spectral efficiency, assuming Gaussian input symbols. Following [2], the SIR function at the output of the above equivalent DF-MMSE detector is determined by solving the following implicit equation

$$g_{\mathrm{sco}}(x) = \frac{f(x)}{1 + 2\beta\alpha^2\bar{P} + \beta\int_x^1 \frac{f(x)f(\zeta)}{f(x)+f(\zeta)g_{\mathrm{sco}}(x)}\,d\zeta} \ . \tag{2-5}$$

The spectral efficiency of the SCO detector can be evaluated through (2-3).

Similarly, the SIRs at the output of the linear MMSE detector and the DF-MMSE detector are obtained, respectively, via

$$g_{\mathrm{ms}}(x) = \frac{f(x)}{1 + \beta\int_0^1 \left[\frac{f(x)f(\zeta)}{f(x)+f(\zeta)g_{\mathrm{ms}}(x)} + \frac{2\alpha^2 f(x)f(\zeta)}{f(x)+\alpha^2 f(\zeta)g_{\mathrm{ms}}(x)}\right] d\zeta} \ , \tag{2-6}$$

and

$$g_{\mathrm{df}}(x) = \frac{f(x)}{1 + \beta\left[\int_x^1 \frac{f(x)f(\zeta)}{f(x)+f(\zeta)g_{\mathrm{df}}(x)}\,d\zeta + \int_0^1 \frac{2\alpha^2 f(x)f(\zeta)}{f(x)+\alpha^2 f(\zeta)g_{\mathrm{df}}(x)}\,d\zeta\right]} \ . \tag{2-7}$$

Again we use (2-3) to evaluate the spectral efficiency of both detectors.

We note that when different systems (with possibly different spreading gains and data rates) are to be compared, it is imperative to express the spectral efficiency in terms of $\frac{E_b}{N_0}$, which is done through the relation $\bar{P} = \frac{2}{\beta}\,\tilde{C}\,\frac{E_b}{N_0}$ (see [1]). However, for simplicity of notations we express equations in terms the normalized power. For details of the derivation of the above equations see [9].

3 Equal Powers

Satisfying the average power constraint of (2-2), the equal powers strategy simply implies setting $f(x) = \bar{P}$, $\forall\ 0 < x \leq 1$. Substituting the above power control function in (2-4), the spectral efficiency of the matched filter detector is given by

$$\tilde{C}_{\mathrm{mf}} = \frac{\beta}{2} \log\left[1 + \frac{\bar{P}}{1 + \beta\,(1 + 2\alpha^2)\,\bar{P}}\right] \ . \tag{3-1}$$

As can be seen, the above result reaches a limit as the average power (or $\frac{E_b}{N_0}$) grows without bound, which is in accordance with the interference limited behavior of the matched filter detector. The same behavior is observed as we take $\beta \to \infty$, which is also optimum in terms of spectral efficiency.

Denoting $\bar{P}_{\text{eq}} \triangleq \frac{\bar{P}}{1+2\beta\alpha^2\bar{P}}$, and substituting $f(x) = \bar{P}$ into (2-5), it can be shown that the spectral efficiency of the SCO detector is given by

$$\tilde{C}_{\text{sco}} = \frac{\beta}{2}\log\left[1+\bar{P}_{\text{eq}} - \frac{1}{4}\mathcal{F}(\bar{P}_{\text{eq}},\beta)\right] + \frac{1}{2}\log\left[1+\bar{P}_{\text{eq}}\beta - \frac{1}{4}\mathcal{F}(\bar{P}_{\text{eq}},\beta)\right] - \frac{\log e}{8\bar{P}_{\text{eq}}}\mathcal{F}(\bar{P}_{\text{eq}},\beta)\,, \tag{3-2}$$

where $\mathcal{F}(x,z) \triangleq \left(\sqrt{x\,(1+\sqrt{z})^2+1} - \sqrt{x\,(1-\sqrt{z})^2+1}\,\right)^2$. As with the matched filter detector, it is observed that the above spectral efficiency reaches a limit as we let $\frac{E_b}{N_0} \to \infty$ (note that $\bar{P}_{\text{eq}} \to \frac{1}{2\beta\alpha^2}$ as $\frac{E_b}{N_0} \to \infty$). The interference limited behavior of the SCO detector emanates from the fact that inter-cell interference is treated as additive white Gaussian noise. Again, $\beta \to \infty$ is optimum in terms of spectral efficiency.

The SIR at the output of the linear MMSE detector is given by the positive root of the cubic equation obtained by substituting $f(x) = \bar{P}$ into (2-6). As can be seen the resulting SIR function is a constant, and thus the spectral efficiency of the linear MMSE detector is simply given by $\tilde{C}_{\text{ms}} = \frac{\beta}{2}\log(1+g_{\text{ms}})$. In contrast to the two previous detectors, the linear MMSE filter is not interference limited if β is appropriately chosen. It can be shown that the spectral efficiency of the linear MMSE detector grows without bound as we take $\frac{E_b}{N_0} \to \infty$, as long as we keep $\beta \le \frac{1}{3}$ (optimum choice is $\beta < \frac{1}{3}$ for large $\frac{E_b}{N_0}$). Taking $\beta \to \infty$, the spectral efficiency of the linear MMSE detector coincides with that of the matched filter, as expected from the single-cell results of [1].

Substituting $f(x) = \bar{P}$ into (2-7) results again in a cubic equation, whose positive root determines the value of the SIR function $g_{\text{df}}(x)$, $\forall\, x \in [0,1]$ (note that here the SIR function is not a constant). Obtaining the SIR function, the resulting spectral efficiency can be evaluated through (2-3). As with the linear MMSE detector, the spectral efficiency of the DF-MMSE detector can be shown to grow without bound as $\frac{E_b}{N_0} \to \infty$, if β is appropriately chosen (again the optimum choice is $\beta < \frac{1}{3}$ for large $\frac{E_b}{N_0}$). Taking $\beta \to \infty$ it is observed that the spectral efficiency of the DF-MMSE detector coincides with that of the SCO detector, which is in agreement with the behavior of the linear MMSE detector for $\beta \to \infty$.

Fig. 1 compares the spectral efficiencies of the four detectors discussed above with $\alpha = \frac{1}{2}$ and optimum choice of β. As can be seen the spectral efficiencies of the linear MMSE and the DF-MMSE detectors, coincide with those of the matched filter detector and the SCO detector, for low $\frac{E_b}{N_0}$ where it is optimum to choose $\beta \to \infty$. However as $\frac{E_b}{N_0}$ increases, beyond some critical values of $\frac{E_b}{N_0}$ the optimum choice of β for the two MMSE based detectors decreases, eventually becoming lower than $\frac{1}{3}$, and the spectral efficiency of these detectors grows without bound with $\frac{E_b}{N_0}$. This explains the "knee effect" observed in the spectral efficiency curves (we note that the "knee effect" is not observed when the spectral efficiency is plotted for a *fixed* value of β). The slope of the spectral efficiency of the DF-MMSE detector is however steeper than that of the linear MMSE detector. One can also notice that beyond some critical value of $\frac{E_b}{N_0}$ (around $16dB$ in Fig. 1) the spectral efficiency of the linear MMSE detector surpasses that of the SCO detector.

Another interesting issue is the penalty due to random spreading. Following [5], one can observe that the spectral efficiencies of the matched filter detector and the SCO

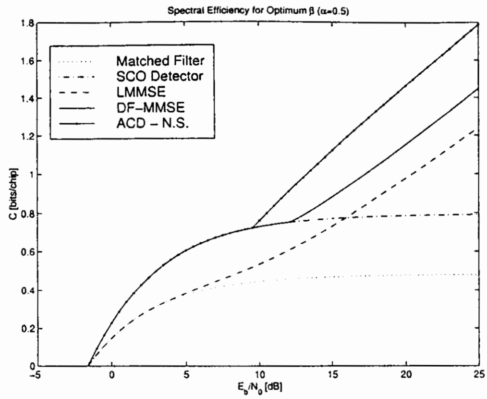

Figure 1: Spectral efficiency comparison with equal powers for $\alpha = \frac{1}{2}$, and an optimum choice of β ("N.S." denotes - No Spreading).

detector, coincide with those of the two corresponding detectors without the constraint of random spreading, when we take $\beta \to \infty$ (which is the optimum choice, c.f. Fig. 3). Another interesting comparison is with respect to a detector which knows the *codebooks* of users in adjacent cells, and either decodes their transmissions as well, or treats them as additive Gaussian noise (referred to as ACD - "adjacent-cell decoder"). As can be seen for low $\frac{E_b}{N_0}$ the spectral efficiency of the ACD is equal to that of the DF-MMSE detector and the SCO detector, since it is preferable to treat adjacent cell interference as noise. However beyond some critical value of $\frac{E_b}{N_0}$, where decoding is preferable, the curves depart and the spectral efficiency of the ACD grows quite rapidly with $\frac{E_b}{N_0}$ (in comparison to the other detectors).

4 Equal Rates

The SCO detector provides equal rates to all users by assigning *equal received powers* to all. With the remaining three detectors, guaranteeing equal rates to all users requires that the SIR function $g(x)$ is a constant. As can be seen from section 3, the above requirement is satisfied for the two linear detectors, i.e, the matched filter detector and the linear MMSE detector, by assigning (again) *equal powers* to all users. This leaves us with the examination of the DF-MMSE detector. We also consider, for the sake of comparison, the DF-MMSE detector which treats adjacent cell interference as additive Gaussian noise (this detector is equivalent to the SCO detector in terms of (overall) spectral efficiency, as mentioned above).

The desired power control function can be numerically obtained[7] by substituting $g(x) = \mathcal{W}$, for some constant $\mathcal{W}$, into either (2-5) or (2-7), and solving these equations so that the power constraint of (2-2) is satisfied.

Results show that in terms of (overall) spectral efficiency, evaluated at the optimum

[7]This is done by subdividing the interval $[0, 1]$ into N equal parts, approximating integrals through sums, and using successive approximation methods (our numerical results are based on $N = 25$).

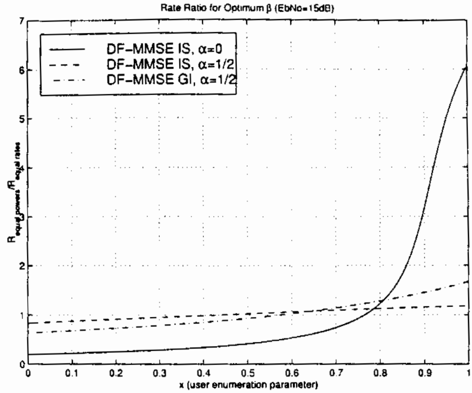

Figure 2: Rate ratio of the equal powers and equal rates strategies for optimum β ($\beta \to \infty$ is represented by $\beta = 10$) and $\frac{E_b}{N_0} = 15dB$. "IS" designates adjacent cell interference suppression, "GI" designates interference treated as noise (both are equivalent for $\alpha = 0$).

choice for β, the equal rates strategy is almost as good as the equal powers strategy, at least for $\frac{E_b}{N_0}$ values of up to $20dB$ (around 98% with interference suppression, closer with the Gaussian interference assumption, worst results for $\alpha = 0$). The spectral efficiency difference is increasing with $\frac{E_b}{N_0}$. However, the equal rates strategy may be preferable in terms of a more equitable rate allocation, as can be seen from the example described by Fig. 2. Fig. 2 shows the ratio between the rate per user while using the DF-MMSE detectors with equal powers, and the rate attained by the equal rates strategy (given by $\log(1 + g(x))/\log(1 + \mathcal{W})$), for $\frac{E_b}{N_0} = 15dB$, $\alpha = 0$, and $\frac{1}{2}$, and optimum choice of β (the latter is very similar for both strategies with interference suppression, less than 1% difference, and identical when interference is treated as noise, thus making the comparison valid). Taking the rate under the equal rates strategy as representing a minimum rate requirement, identical to all users, one can see that with equal *powers* a large proportion of the users fails to meet the requirement. The unbalanced rate distribution is in particular striking for $\alpha = 0$, i.e., in a single cell environment (which should have been expected). For example, with $\frac{E_b}{N_0} = 15dB$ around 76% of the users fall under the rate obtained by the equal rates strategy with $\alpha = 0$. The non-uniformity in rate distribution under the equal powers strategy becomes moderate as we increase the level of adjacent cell interference (results for $\frac{E_b}{N_0} = 20dB$, are of similar in nature). We may therefore conclude that the equal rates strategy is preferable over the equal powers strategy, in terms service quality, when all users are to be equally serviced (as is the case, e.g., with cellular voice communication in current systems). Furthermore, when the $\frac{E_b}{N_0}$ at which the system operates is not too high, choosing the equal rates strategy incures (almost) no loss in overall spectral efficiency.

5 Maximum Spectral Efficiency

Observing (2-4), it is quite straightforward to show, using Jensen's inequality and the convexity of the $\log(1 + x)$ function, that the spectral efficiency of the matched filter detector is maximized by assigning equal powers to all users.

The same result holds for the spectral efficiency of the SCO detector. This is observed by direct evaluation of the spectral efficiency of the SCO detector through the channel's input-output mutual information (without resorting to the equivalent DF-MMSE detector). The derivative of the spectral efficiency with respect to $\bar{P}_{\text{eq}}$ (taken as an argument), as defined in section 3, is thus seen to be a decreasing function of the Stieltjes transform of the eigenvalue distribution of matrices of the form $\boldsymbol{S}_i^{\dagger}\boldsymbol{S}_i\boldsymbol{A}$, where $\boldsymbol{S}_i$ is the signatures matrix as defined in section 2 above, and $\boldsymbol{A}$ is a diagonal matrix with the normalized powers of the users as its entries. The above Stieltjes transform (see [10]) is minimized by the equal power distribution, thus maximizing the spectral efficiency of the SCO detector.

Solving the general optimization problem with respect to the spectral efficiency of both the linear MMSE detector and the DF-MMSE detector is quite complex, as can be seen from (2-6) and (2-7). We therefore turn to the derivation of a lower bound on the maximum spectral efficiency. Consider the power control function $f_\rho(x)$ defined as

$$f_\rho(x) = \begin{cases} \frac{\bar{P}}{\rho} & 0 < x \le \rho \\ 0 & \text{otherwise} \end{cases}, \quad 0 < \rho \le 1 \,. \tag{5-1}$$

Clearly, $f_\rho(x)$ satisfies the average power constraint (see (2-2)). The rationale behind $f_\rho(x)$ is that given a system, with some fixed $\frac{E_b}{N_0}$, interference factor α, and system load β, one might gain in spectral efficiency by effectively "shutting down" part of the users' transmissions by means of the power control function, and assigning more power to the remaining users while satisfying the average power constraint[8]. Obviously, assigning all power to a single user ($\rho \to 0$) is useless since this drives the system's spectral efficiency to zero. The power control function $f_\rho(x)$ comes into effect if there exists some value of $\rho = \rho^* < 1$ for which the spectral efficiency is maximized. In the case in which $\rho^* = 1$, $f_\rho(x)$ simply reduces to the equal powers strategy discussed in section 3. The above optimization is in fact equivalent to the optimization of the spectral efficiency with respect to β under the equal powers strategy, for fixed $\frac{E_b}{N_0}$, if the initially set value of β is higher than the optimum value (note that $f_\rho(x)$ cannot *increase* the effective load). Hence, for a given system, at a fixed working point specified by $\frac{E_b}{N_0}$, α, and β, the maximum spectral efficiency, where maximization is taken over the set of power control functions, is lower bounded by the spectral efficiency obtained by substituting $f_\rho(x)$ into (2-6) and (2-7), and evaluating the spectral efficiency at $\rho = \rho^*$.

Some comparative results, demonstrating the effect of maximizing the spectral efficiency via the power control function are demonstrated in Fig. 3. Here the spectral efficiency of all four detectors is plotted for $\alpha = \frac{1}{2}$, and $\beta = 1$. The spectral efficiency of the linear MMSE and DF-MMSE detector is evaluated assigning the power control function $f_\rho(x)$, with $\rho = \rho^*$. For the matched filter detector and the SCO detector we evaluated the spectral efficiency with the equal power assignment (which attains the maximum). Also included in Fig. 3 are the spectral efficiencies without random spreading of the three detectors mentioned in section 3 (denoted by "N.S."). As can be seen, since $\beta = 1$, for low $\frac{E_b}{N_0}$ the spectral efficiencies of the linear MMSE detector and the DF-MMSE detector, are lower than the spectral efficiencies without random spreading of the matched filter and SCO detector, respectively (equality is attained for $\beta \to \infty$). This is also the case with the spectral efficiency of the matched filter, and the SCO detector, for

[8]Notice, however, that using the function $f_\rho(x)$ still confines us to equal power assignment to all effectively active users.

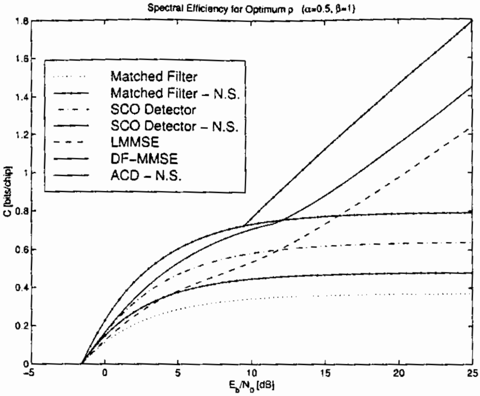

Figure 3: Spectral efficiency comparison using $f_\rho(x)\big|_{\rho=\rho^*}$, for $\alpha = \frac{1}{2}$, and $\beta = 1$.

all values of $\frac{E_b}{N_0}$. However, beyond some critical values of $\frac{E_b}{N_0}$ the optimum β for the linear MMSE and the DF-MMSE detectors decreases below unity, and thus using the power control function $f_\rho(x)$ (with $\rho = \rho^*$) brings the spectral efficiency of both detectors to its maximum value with the equal power assignment (c.f. Fig. 1).

6 Summary and Conclusions

In this paper we analyzed the spectral efficiency of four multiuser detectors, which differ by the amount and type of information made available to the detector, with regard to interference (both intra-cell and inter-cell). Our results demonstrate the dramatic effect of information about interfering signals, which is most clearly seen by comparing the linear MMSE detector and the SCO detector (the DF-MMSE detector is obviously superior to all, being the most informed, and the matched filter detector is inferior to all, being the least informed). We showed that one can gain, even without trying to decode the transmissions of the interfering users in adjacent cells[9], or treating them optimally in the setting of an interference channel (see [5]), by the very fact that the linear MMSE filter will account for reducing the interference they cause, provided that their signatures are known. It was shown that when $\frac{E_b}{N_0}$ and the interference factor are not too low, the relatively simple *linear* MMSE detector, which is more informed regarding adjacent cell interference, is preferable over the SCO detector which employs *"optimum"* detection of intra-cell users, while treating inter-cell interference as noise. We may therefore conclude that for high data rates, inherently demanding high $\frac{E_b}{N_0}$, the MMSE based detection strategies yield a pronounced gain over the other two strategies considered. The penalty due to random spreading was also demonstrated.

Comparing power control strategies we showed that the equal power assignment obtains equal rates with the matched filter detector, the SCO detector, and the linear MMSE detector, and it attains the maximum spectral efficiency with the matched filter and the SCO detectors. This strategy may also provide a lower bound to the maximum

[9]Which enables interference *cancellation.*

spectral efficiency of the linear MMSE and the DF-MMSE detectors. Our results show that applying the equal rates power control strategy to the DF-MMSE detector results in almost no loss in spectral efficiency, in comparison to the equal powers strategy (when $\frac{E_b}{N_0}$ is not too high). Conditions at which the equal rates strategy is preferable were also discussed.

Analysis of the optimum multi cell site processing detector [4] is under current study, as well as characterization of the *optimum* power control strategy, in terms of spectral efficiency, for the linear MMSE and DF-MMSE detectors, and consideration of fading.

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