

Strongest-Users-Only Detectors for Randomly Spread CDMA

Shlomo Shamai (Shitz)
The Dept. of Electrical Engineering,
Technion - IIT, Haifa 32000, Israel.
E-mail: sshlomo@ee.technion.ac.il

Benjamin M. Zaidel
The Dept. of Electrical Engineering,
Technion - IIT, Haifa 32000, Israel.
E-mail: bennyzz@inter.net.il

Sergio Verdú
The Dept. of Electrical Engineering,
Princeton Univ. USA 08544.
E-mail: verdu@ee.princeton.edu

Abstract — A randomly spread direct sequence code division multiple access (DS-SS) system operating in flat fading is considered. Users are assumed to employ equal rates and transmit powers. The receiver ranks the users according to their received powers, and decodes only the subset that can be reliably decoded. Confining the discussion to asymptotic analysis (in terms of number of users and processing gain), we study the matched-filter detector, the linear minimum mean squared error (MMSE) detector, and the “optimum detector” under the above strongest-users decoding scheme. The total capacities under outage constraint, derived as functions of the fraction of users that cannot be reliably decoded (equivalent to the “outage probability”), are analyzed and compared.

I. INTRODUCTION

Denoting by K the number of users, and by N the processing gain, the maximum achievable throughput, or “spectral efficiency”, of single-cell randomly spread DS-SS systems with flat-fading channels, has been thoroughly analyzed in [1], while focusing on the limiting scenario, in which $K, N \rightarrow \infty$, while $K/N \rightarrow \beta < \infty$ (referred to as the “system load”). Tacit in the spectral efficiency analysis is the assumption that all active users are decoded *regardless of their received powers*, and that the users adjust their rates (and possibly their powers) as a function of the channel fading level they experience (via feedback from the receiver). Unfortunately, in practice, such feedback and ideal tuning of the users’ transmissions can hardly be accomplished if the fading varies too fast.

In this paper it is assumed that all users transmit at equal rates and powers regardless of the individual fade levels. Therefore, due to fading, the receiver at the cell-site can no longer guarantee reliable decoding of all active users. Both the limiting scenario mentioned above and independent flat-fading channels are assumed, as in [1], and system performance is analyzed while employing *optimally* coded randomly spread DS-SS with multiuser detection. It is assumed that the cell-site receiver ranks all active users by their received powers, and then only decodes the transmissions of a *subset* of these users, equal in number to the largest integer for which decoding is successful [2]. The matched-filter detector, the linear MMSE detector, and the “optimum detector” under the strongest-users decoding scheme are analyzed and compared in terms of their total achievable rate sum over all *decodable* users, to be referred to henceforth as the “outage constrained capacity”. The underlying assumption is that the system designer sets the transmission rate of all users in order to achieve a target maximum fraction of *undecodable* users (FUU), denoted by Q (equivalent to the “outage probability”). Since different users experience independent fades, as the number of users grows, the percentage of undecodable users converges to a deterministic constant. Optimization of the overall throughput in this

regime, by choosing the optimum Q , is also considered. The rationale behind such optimization is the tradeoff between the (fixed and equal) rate to be employed by each individual user, and the FUU. System performance can be further optimized if the system load β is a degree of freedom.

II. SUMMARY OF RESULTS

Considering the outage constrained capacity of the detectors, we have shown that the strongest-users decoding scheme induces a penalty factor of $[(1-Q)\mathcal{F}_v^{-1}(Q)]^{-1}$ in the minimum E_b/N_0 allowing reliable communications (with \mathcal{F}_v denoting the fade level distribution), as compared to the case where *all* users are decoded (for which $E_b/N_{0,\min} = \ln 2$). This penalty is explained by the power “wasted” when users fail to be decoded. The minimum penalty for Rayleigh fading is $10 \log_{10} e \approx 4.34$ dB, and it is attained by setting the FUU to $Q = 1 - 1/e$, the optimum choice at the low SNR regime. The same degradation has been identified in the broadcast strategy of communicating over fading channels with no rate feedback to the transmitter [3]. Both linear detectors suffer a significant performance degradation for all E_b/N_0 values, as compared to their spectral efficiency (specifying the ultimate performance [1]). In contrast, the outage constrained capacity of the “optimum detector” is shown to asymptotically approach the optimum spectral efficiency in the high E_b/N_0 region, when the system load is made large ($\beta \rightarrow \infty$), and when the FUU vanishes at an appropriate rate as $E_b/N_0 \rightarrow \infty$. This asymptotic approach emphasizes the crucial role of *rate adjustment feedback* at the low SNR regime, where the strongest-users decoding scheme induces a *severe degradation* in system performance. An interesting observation is that when the FUU is fixed, the “optimum detector” becomes *interference limited* in the large system load region. In such case, the use of some amount of spreading ($\beta < \infty$) turns out beneficial beyond some critical E_b/N_0 , and the outage constrained capacity of the detector grows without bound with E_b/N_0 for an appropriately chosen system load. This behavior is due to the inherent suboptimality of the strongest-users decoding scheme. The reader is referred to [4] for a detailed presentation of the analysis.

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