The Effect of Asynchronism on the Total Capacity of Gaussian Multiple-access Channels

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The Multiple-access channel is an information-theoretic model of communication systems where several independent users transmit simultaneously to a common receiver. Typical practical multiplexing strategies that fall into that category include TDMA, FDMA and CDMA (i.e., Time, Frequency and Code Division Multiple Access), depending on whether the users modulate pulse waveforms which are nonoverlapping in the time-domain, in the frequency-domain, or in neither domain, respectively.

A practically and theoretically important question is the degradation in achievable performance caused by the absence of synchronism among the transmitters. This has been investigated in [1, 2] for frame-asynchronous channels and in [3] for completely asynchronous channels (i.e., when asynchronism is not assumed at either the codeword or the symbol level). The latter work [3] found a formula for the capacity region of the asynchronous two-user white Gaussian channel for linear modulation of arbitrary pulse waveforms. Using those results, in this paper we quantify the effect of asynchronism on the total capacity (maximum achievable sum of transmission rates) of the white Gaussian multiple-access channel. This measure allows a succinct comparison of the overall system throughput with and without synchronism.

The comparison of capacities is carried out assuming that the transmitters and the receiver know whether the users maintain synchronism or not, and therefore the coding strategies can be chosen accordingly. Specifically, we do not investigate the effect of asynchronism on coding strategies designed for synchronous channels. Naturally, it is assumed that when choosing their codebooks the asynchronous transmitters have no knowledge of the offsets, or delays, between their data streams. Since reliable communication is required regardless of the actual offsets (in particular even in the case where the users happen to be perfectly aligned), it is clear that asynchronism cannot increase capacity. Notice that this conclusion would not be necessarily true, had we assumed that the transmitters have access to the value of the relative offsets prior to encoding their messages.

If all users modulate the same pulse, then it is known [3, corollary on p.745] that the synchronous and asynchronous capacities coincide and are given by the Cover-Wyner pentagon. Even though such a modulation choice affords the nice property of complete insensitivity to asynchronism, it also achieves the lowest capacity for given signal-to-noise ratios as it does not attempt to differentiate the users' pulses for multiplexing purposes.

A multiplexing strategy that does use different pulses for different users is TDMA. It is common that the pulse waveforms assigned to TDMA users are shifted nonoverlapping versions of a common pulse. In such case, we find that asynchronism degrades the total capacity of the equipower K-user TDMA channel by a factor of K. Moreover, we show that no other multiplexing strategy can suffer a higher degradation due to asynchronism. Naturally, symbol-asynchronism plays havoc with TDMA, a strategy predicated on the availability of a common clock for all transmitters; although nonzero transmission rates are indeed achievable thanks to the use of appropriate channel coding strategies.

We also investigate the effect of asynchronism in both high and low signal-to-noise ratio cases. We show that regardless of the multiplexing strategy, asynchronism causes vanishing relative degradation in total capacity as the signal-to-noise ratios go to zero. This is due to the fact that in such an asymptotic scenario, the overwhelmingly dominant factor limiting performance is the background Gaussian noise rather than the interference from other users. On the other hand, as the signal-to-noise ratios go to infinity, we show that the effect of asynchronism on the total capacity also approaches zero if the pulses $s_k(t)$ are linearly independent among all relative delays. This is because as long as the pulse waveforms are *independent* for all relative delays, the receiver can distinguish the users and the equivalent signal-to-noise ratio is an *non-zero* constant fraction of the actual signal-to-noise ratio. As the actual signal-to-noise ratios go to infinity, the effect of the non-zero constant factor vanishes since the capacity is proportional to the logarithm of the signal-to-noise ratio.

Ideally, the system designer would like to design a system that combines the advantages of the two aforementioned multiplexing methods: the insensitivity to asynchronism achieved by common-pulse signalling and the orthogonality of synchronous TDMA which leads to the simultaneous achievability of single-user capacities by all users. Even though it is not possible to design pulses that are orthogonal for all possible relative delays, it is indeed possible to design pulses that approach that utopian situation as closely as desired provided that bandwidth is unlimited. For example, in an FDMA system with sufficiently separated frequencies or in a Direct-Sequence Spread Spectrum system with very large spreading factors, pulses are quasi-orthogonal for all relative delays. Therefore, if bandwidth is unlimited, total capacity is equal to the sum of single-user capacities even if the channel is asynchronous.

More interesting is the situation where the effect of asynchronism is studied for signals designed under a bandwidth constraint. Since we assume that the transmitters and the receiver know whether the users are synchronized or not, different pulse waveforms can be designed accordingly. We define the Root-Mean-Square (RMS) bandlimited synchronous (asynchronous) capacity as the maximum synchronous (asynchronous) capacity over all time-limited pulse waveforms satisfying the RMS bandwidth constraint. The two-user RMS bandlimited synchronous capacity and the corresponding optimum pulse waveforms are found in [4]. It turns out that, in the two-user case, for a certain region of signal-to-noise ratios, the RMS bandlimited asynchronous capacity coincides with the RMS bandlimited synchronous capacity. Outside that region, the asynchronous capacity is at least 88% of the synchronous capacity.

We also find that the RMS bandlimited total capacity is insensitive to asynchronism in both asymptotically high and asymptotically low signal-to-noise ratios. In asymptotically low signal-to-noise ratio, since the effect of asynchronism vanishes regardless of the multiplexing strategy, it is clear that the RMS bandlimited asynchronous capacity approaches the RMS bandlimited synchronous capacity. However, in asymptotically high signal-to-noise ratios, it is not obvious that the effect of asynchronism also vanishes since the aforementioned appropriate pulses may not satisfy the RMS bandwidth constraint. We demonstrate the existence of some simple pulses which cause asymptotically small relative asynchronous degradation in high signal-to-noise ratios and satisfy the RMS bandwidth constraint.

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