Multicell Uplink Spectral Efficiency of Coded DS-CDMA With Random Signatures

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***Abstract*—A simple multicell uplink communication model is suggested and analyzed for optimally coded randomly spread direct sequence code-division multiple access (DS-CDMA). The model adheres to Wyner's (1994) infinite linear cell-array model, according to which only adjacent-cell interference is present, and characterized by a *single* parameter $0 \leq \alpha \leq 1$. The discussion is confined to asymptotic analysis where both the number of users and the processing gain go to infinity, while their ratio goes to some finite constant. Single cell-site processing is assumed and four multiuser detection strategies are considered: the matched-filter detector, "optimum" detection with adjacent-cell interference treated as Gaussian noise, the linear minimum mean square error (MMSE) detector, and a detector that performs MMSE-based successive interference cancellation for intracell users with linear MMSE processing of adjacent-cell interference. Spectral efficiency is evaluated under three power allocation policies: equal received powers (for all users), equal rates, and a maximal spectral efficiency policy. Comparative results demonstrate how performance is affected by the introduction of intercell interference, and what is the penalty associated with the randomly spread coded DS-CDMA strategy. Finally, the effect of intercell time-sharing protocols as suggested by Shamai and Wyner (1997) is also examined, and a significant system performance enhancement is observed.**

***Index Terms*—Capacity, cellular communication, code-division multiple access, multiuser detection, random signatures, spectral efficiency.**

I. Introduction

INFORMATION theoretic analyzes of direct sequence code division multiple access (DS-CDMA) systems have gained much attention in recent years, as a result of the rapid development of commercial cellular systems employing this multi-access strategy. Results for a *single cell* DS-CDMA system were recently presented in [1]–[4] (see also references therein). These works explicitly relate to CDMA systems with *random* spreading sequences, and the limiting scenario is examined, where both the number of users and the processing gain go to infinity, while their ratio goes to some finite constant. This ratio is commonly referred to as the *"system load."*

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The above asymptotic assumptions are particularly appealing since in such case performance measures of interest, such as output signal-to-interference-plus-noise ratio (SINR) or spectral efficiency, converge to deterministic values, that can be *analytically* expressed. These performance measures are functions of the empirical eigenvalue distribution of random matrices of particular structures, which is known to converge, when dimensions go to infinity, to some limiting distribution (determined in the case considered here through its Stieltjes transform, see [5] and Appendix). In contrast, the problem becomes analytically intractable even for moderate *finite* system sizes (except for some particular random matrix structures). Furthermore, previous results indicate a rather fast convergence to the asymptotic limits with system size, demonstrating the practical importance of these limits. The reader is referred to [3], [6], and [7] for some numerical examples and an analysis of the rate of convergence.

Assuming equal received powers and nonfading channels, four multiuser detection strategies are compared and analyzed in [1], in terms of spectral efficiency. The authors examine optimum decoding, the matched filter detector, the decorrelating detector, and the linear minimum mean squared error (MMSE) detector. In addition to explicit *analytical* expressions for the spectral efficiencies of the four detection strategies, asymptotic analysis for cases in which the system load or E_b/N_0 go to either zero or infinity is provided, and also comparison to the spectral efficiency without (the constraint of) spreading, and the spectral efficiency obtained with orthogonal (deterministic) spreading sequences. It should be noted here that it is well known [8] that the optimum spectral efficiency (without spreading) can be achieved with *orthogonal* spreading sequences when the system load equals unity. It is also known [9] that even when the system load is higher than unity, there exist spreading codes that incur no loss in capacity relative to multi-access with no spreading. The results obtained in [1] extend also to the case of homogeneous fading where each of the spreading sequences' chips is assumed to be affected by independent identically distributed (i.i.d.) fading coefficients with unit variance. The impact of flat (nonhomogeneous) fading on the four multiuser detection strategies considered in [1] is analyzed in [2]. SINRs at the output of linear detectors (the matched-filter detector, the linear MMSE detector, and the decorrelator) are presented in [3], and an extension of these results to fading channels with multi-antenna reception can be found in [10].

In this paper, *multicell* systems are addressed using the attractive cellular model suggested by Wyner in [11]. This simple model allows for analytical tractability on the one hand, while giving insight to practical systems on the other. Accordingly, the system's cells compose an infinite *linear* array, where the received signal at each cell site is the sum of the signals received from intracell users, plus a factor α $(0 \leq \alpha \leq 1)$ times the sum of the signals of users in the two adjacent cells, as received at their cell sites. Nonadjacent cell users are assumed to produce no interference. The received signal is embedded in ambient Gaussian noise. The multicell effect on performance is, thus, specified by a *single* parameter (α).

Nonfading channels are assumed, as in [11], and the spectral efficiency obtained by employing *optimally* coded *randomly* spread DS-CDMA with multiuser detection is analyzed. As in [1] and [3], the limiting case is considered in which, denoting by K the number of intracell users (assumed constant and equal in all cells), and by N the spreading factor (processing gain), $K, N \rightarrow \infty$, while $K/N \rightarrow \beta < \infty$ (i.e., the factor β denotes the system load). Assuming *single cell-site processing*, four types of multiuser detection strategies are considered.

1) The *"conventional" matched-filter detector* that treats *all* interference (either intracell or intercell) as additive white Gaussian noise (AWGN);
2) A *single-cell optimum (SCO)* detector that "optimally" detects the transmissions of *intracell* users, while treating *intercell* interference as AWGN;
3) The *linear MMSE detector* that knows the signature sequences of all interfering users (both intracell and in adjacent cells) and mitigates their interference by means of a linear MMSE filter;
4) A detector that employs *MMSE-based successive interference cancellation (MMSE-SC)* to decode transmissions of intracell users, while *intercell* interference is mitigated by means of a linear MMSE filter. To distinguish between this detector and a detector that performs MMSE based successive interference cancellation over *all* received transmissions, the latter shall be referred to as the *full MMSE-SC detector* (this detector is equivalent to a decision-feedback receiver, see Section III-D for a more detailed description of the MMSE-SC scheme).

It is emphasized that neither the linear MMSE detector, nor the MMSE-SC detector, try to *decode* the transmissions of adjacent cell users (which might be prohibitive if α is small). In fact, the cell-site detector may actually be ignorant regarding codebooks or code-mask sequences employed in other cells, but is aware, as usually is the case in practice, of the signature sequences of all users in adjacent cells. In addition to the above, it is also assumed that all detectors are provided with the required knowledge regarding the received powers of the interfering signals.

The above four multiuser detection strategies were chosen to demonstrate the effect of information on interfering signals on system performance. The matched-filter detector and the MMSE-SC detector represent the extreme cases in the analysis, the first being the least informed, and the latter the most informed (in fact, the MMSE-SC detector is optimum in terms of spectral efficiency in the setting considered in this paper, as shall be explained in Section III-D). The SCO detector and the linear MMSE detector reflect a tradeoff between additional information and detection complexity with respect to intracell users, versus having more information on adjacent-cell interference while using simpler interference mitigation techniques for intracell users. In this particular case, the SCO detector is fully informed regarding intracell transmissions and performs *optimum joint demodulation and decoding* while treating adjacent cell interference as additive Gaussian noise. In contrast, the linear MMSE detector (per each user) is only aware of the signature sequences and received signal-to-noise ratios (SNRs), but this holds for *all* interferers (both intracell and in adjacent cells), and the detector employs *linear* (suboptimum) interference mitigation followed by *single user decoding*. The explicit effect of the above tradeoff is of great practical interest, and this paper provides an *analytical* performance comparison of these two approaches as well as the matched-filter and MMSE-SC detectors.

Identifying the spectral efficiency as the fundamental measure of system performance for coded systems (see Section III), the spectral efficiency of all four detection strategies is comparatively examined under three power allocation policies, corresponding to three possible practical system design goals. First considered is the equal-powers policy that assigns equal *received* powers to all users. Next, the equal-rates policy that employs a power assignment function such that all users attain equal rates (in the sense of information theoretic capacity). The last policy to be considered is the one that maximizes the spectral efficiency. Particularizing to equal received powers, the penalty in system performance due to random spreading is also examined, by comparison (following [12]) to the spectral efficiency of the SCO detector, and that of a detector equivalent to the matched-filter detector, when no spreading is employed. Another detector considered in this respect is the *adjacent-cell decoder* (ACD) that also knows the *codebooks* of users in adjacent cells, and chooses either to decode their transmissions or treats them as an additive Gaussian noise, whichever is preferable in terms of spectral efficiency (see Section IV for more details). Finally, the effect on system performance of intercell time-sharing (ICTS) protocols, as suggested in [12], is also analyzed.

This paper is organized as follows. Section II presents the system model. Section III includes general equations through which the spectral efficiency of the four detectors is obtained. Sections IV–VI discuss the three power allocation policies mentioned above. Section VII is devoted to the examination of ICTS protocols, and finally, Section VIII ends the paper with a summary and some concluding remarks.

II. System Model

Following [11] and [12], the uplink of a fully synchronous cellular CDMA system is considered, whose cells are ordered in an *infinite* linear array, as depicted in Fig. 1. Using the standard discrete time equivalent channel representation, the signal vector received at an arbitrary cell-site at the discrete time related to the transmission of the ith symbol is given by

$$\boldsymbol{y}_i = \boldsymbol{S}_i \boldsymbol{x}_i + \alpha \boldsymbol{S}_i^- \boldsymbol{x}_i^- + \alpha \boldsymbol{S}_i^+ \boldsymbol{x}_i^+ + \boldsymbol{n}_i. \qquad (2\text{-}1)$$

The vector $\boldsymbol{x}_i = [x_{1,i}, \ldots, x_{K,i}]^{\mathrm{T}}$ in (2-1) comprises the K code symbols originated from intracell users at the ith discrete

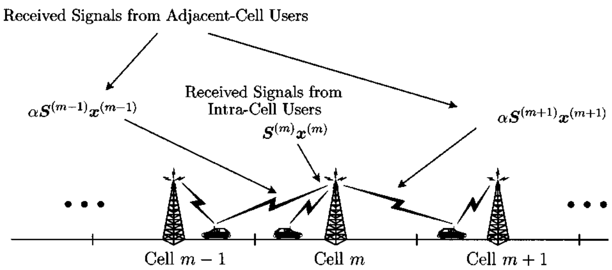

Fig. 1. Linear cell-array model.

time. The vectors $\boldsymbol{x}_i^{\pm} = [x_{1,i}^{\pm}, \ldots, x_{K,i}^{\pm}]^{\mathrm{T}}$ denote the vectors of symbols originated from users operating in adjacent cells. These symbols are assumed to be i.i.d., Gaussian (which conforms with the capacity achieving statistics), with $E\{x_{k,j}\} = 0$ and $E\{x_{k,j}^2\} = P_k \, \forall \, k, j$, where P_k is the received power of the kth user according to the applied power allocation policy. This model is justified by assuming that the codebooks of all users are chosen randomly, governed by an underlying i.i.d. Gaussian distribution per symbol, and *independently* for each message transmission (see [12]). The matrices $\boldsymbol{S}_i$ and $\boldsymbol{S}_i^{\pm}$ are $N \times K$ matrices, whose columns are the N-chip long random spreading (signature) sequeces of the K users in the considered cell and in its adjacent cells, respectively. The entries of the above matrices are treated as i.i.d. zero-mean random variables with variance $1/N$. The vector $\boldsymbol{n}_i$ represents a zero-mean white Gaussian noise vector with $E\{\boldsymbol{n}_i\boldsymbol{n}_i^{\dagger}\} = \boldsymbol{I}, \forall \, i$. Without loss of generality, all received powers are, thus, normalized with respect to the noise spectral level and represent, in fact, the SNRs at the input to the multiuser detectors.

As mentioned in Section I, the effect of power allocation policies on the overall system performance is analyzed. The "power allocation policy" refers here to a system-defined centrally controlled function, that assigns a *received* power level to each of the intracell users (it is assumed that the users adjust their transmission power so as to be received at the prescribed level). According to the system model considered, exactly the same power allocation policy is applied to all cells. In addition, a limiting continuous argument assumption is imposed, according to which it is assumed that as the number of users grows to infinity, the discrete power (SNR) assignment function $\{P_k\}_{k=1}^K$ converges to a limit given by a function of a continuous argument, denoted henceforth by $f(x)(0 < x \leq 1)$, i.e.

$$\begin{aligned} P_k =& f\left(\frac{k}{K}\right) \underset{K\to\infty}{\longrightarrow} f(x), \qquad s.t. \, \frac{k}{K} \underset{K\to\infty}{\longrightarrow} x \in (0,1] \\ k =& 1, 2, \ldots, K. \end{aligned} \tag{2-2}$$

The above power assignment function is assumed to be subject to an average power constraint, given by

$$\frac{1}{K}\sum_{k=1}^{K} P_k \underset{K\to\infty}{\longrightarrow} \int_0^1 f(x)dx = \bar{P} \tag{2-3}$$

where the definition of the Riemann integral has been used to arrive at the integral notation.

It is emphasized that in the nonfading regime considered in this paper, $f(x)$ is to be fully specified by the system designer according to the chosen system design goal. This is in contrast to the common practice in *fading* channels, where the power control law is a function of the instantaneous channel gain of each user. It is also noted that the roles of different users (which correspond to the user enumeration parameter x) may be switched from frame to frame, in order to achieve a more homogeneous/fair performance for different users.

III. Spectral Efficiency of the Multiuser Detectors

The fundamental figure of merit for system performance is the per cell *spectral efficiency* [1], defined as the total number of bits per chip that can be transmitted arbitrarily reliably in each cell. Denoting by g_k the SINR at the output of a linear detector for user k, and following central limit results showing that the interference at the output of each of the two linear detectors, i.e., the matched-filter detector and the linear MMSE detector, is well approximated by a Gaussian noise (see [1], [13], and [14] for justification of this Gaussian approximation), the spectral efficiency of these detectors is given by

$$\tilde{C} = \lim_{\substack{K,N\to\infty,\\ \frac{K}{N}\to\beta<\infty}} \frac{1}{N}\sum_{k=1}^{K}\frac{1}{2}\log(1+gk) = \frac{\beta}{2}\int_0^1 \log(1+g(x))dx \tag{3-1}$$

using the integral representation as in (2-3), and denoting $g_k = g(k/K) \longrightarrow_{K\to\infty} g(x), \, x \in (0,1]$. It is, however, convenient in some cases to express the spectral efficiency in terms of the *multiuser efficiency* (see [15]) of the detector, defined as the ratio between the detector's output SINR and the SNR, which in the continuous argument function notation becomes $\eta(x) \triangleq g(x)/f(x)$. Hence the spectral efficiency is equivalently expressed by

$$\tilde{C} = \frac{\beta}{2}\int_0^1 \log(1+\eta(x)f(x))dx. \tag{3-2}$$

It is noted here that (3-1) and (3-2) also apply to the *nonlinear* MMSE-SC detector since at any stage of the interference cancellation process, the detector employs *linear* MMSE processing, but with the number of interfering users getting smaller at each stage (see Section III-D). The spectral efficiency of the (nonlinear) SCO detector is most conveniently evaluated using interrelations between the spectral efficiency of the optimum multiuser detector and that of the *linear* MMSE detector as shall be explained in Section III-C. The reader is referred to [16] for an alternative derivation based on the interrelations between the spectral efficiency of the optimum detector and that of the full MMSE-SC detector.

The following generally describes how the spectral efficiency of all four multiuser detectors is obtained in terms of the power-assignment function. The notation of $(\cdot)_{\mathrm{mf}}$, $(\cdot)_{\mathrm{sco}}$, $(\cdot)_{\mathrm{ms}}$, and $(\cdot)_{\mathrm{m-sc}}$ is used to designate entries related to the matched-filter detector, the SCO detector, the linear MMSE detector, and the MMSE-SC detector, respectively. It is noted that when different systems are to be compared (with possibly different spreading gains and data rates), it is useful to express the spectral efficiency in terms of E_b/N_0, which is done through the relation $\bar{P} = (2/\beta)\tilde{C}(E_b/N_0)$ (see [1]). However, for simplicity of notation, equations are expressed in terms of the received power (which is in fact the SNR, following the normalization with respect to the noise spectral level).

A. Matched-Filter Detector

The matched-filter detector simply passes the received signal through a filter matched to the signature sequence of the user of interest, while treating all interfering signals as a pure AWGN. In the limiting scenario considered here, the following result [3] on the convergence of the multiuser efficiency of the matched-filter detector in a *single-cell* system holds.

Lemma 3.1 (Tse–Hanly [3]): Let the empirical distribution of the received powers of all users converge almost surely (a.s.) as $K, N \to \infty$, $K/N \to \beta < \infty$, to some nonrandom limit $\mathcal{H}(P)$. Then, the multiuser efficiency of the matched-filter detector converges in probability to a nonrandom limit η_{mf}, equal for all users, given by

$$\eta_{\mathrm{mf}} = \frac{1}{1 + \beta E_{\mathcal{H}}\{P\}} \tag{3-3}$$

where $E_{\mathcal{H}}\{\cdot\}$ represents expectation with respect to $\mathcal{H}(P)$.

Turning to the particular multicell model considered here, the matched-filter detector effectively operates in an *equivalent single-cell* system of $3K$ users, one-third of which (the intracell users) are received at (nonrandom) powers as given by $f(x)$, $x \in (0,1]$, while the remaining two-thirds (the adjacent-cell users) are received at powers $\alpha^2 f(x \bmod 1)$, $x \in (1,3]$. Hence, applying Lemma 3.1, while considering the average power constraint of (2-3), it is straightforward to see that the multiuser efficiency of the matched-filter detector is given by

$$\eta_{\mathrm{mf}} = \frac{1}{1 + \beta(1 + 2\alpha^2)\bar{P}} \tag{3-4}$$

and its spectral efficiency satisfies

$$\tilde{C}_{\mathrm{mf}} = \frac{\beta}{2}\int_0^1 \log\left(1 + \frac{f(x)}{1 + \beta(1 + 2\alpha^2)\bar{P}}\right) dx. \tag{3-5}$$

B. Linear MMSE Detector

The linear MMSE detector passes the received signal through a linear filter that minimizes the mean squared error between the channel input [the vector $\boldsymbol{x}$ in (2-1)] and the filter's output. Following is a result from [3], as it is formulated in [2], that applies to single-cell systems with flat fading channels.

Lemma 3.2 (Tse-Hanly [3]): Let the empirical distribution of the received powers of all users converge a.s. as $K, N \to \infty$, $K/N \to \beta$, to some nonrandom limit $\mathcal{H}(\mathcal{P})$. Then, the multiuser efficiency of the linear MMSE detector converges a.s. to a nonrandom limit η_{ms}, equal for all users, given by the unique positive solution to the implicit equation

$$\eta_{\mathrm{ms}} + \beta E_{\mathcal{H}}\left\{\left[\frac{\eta_{\mathrm{ms}} P}{1 + \eta_{\mathrm{ms}} P}\right]\right\} = 1. \tag{3-6}$$

As with the matched-filter detector in the considered multicell *nonfading* model, the linear MMSE detector also operates effectively in an equivalent single-cell system of $3K$ users with power distribution, as described in Section III-A above. Hence, applying Lemma 3.2, it follows that the multiuser efficiency of the linear MMSE detector is given by the unique positive solution to the implicit equation

$$\eta_{\mathrm{ms}} + \beta \int_0^1 \left[\frac{\eta_{\mathrm{ms}} f(\zeta)}{1 + \eta_{\mathrm{ms}} f(\zeta)} + 2\frac{\alpha^2 \eta_{\mathrm{ms}} f(\zeta)}{1 + \alpha^2 \eta_{\mathrm{ms}} f(\zeta)}\right] d\zeta = 1 \tag{3-7}$$

and the resulting spectral efficiency is given by [cf. (3-2)]

$$\tilde{C}_{\mathrm{ms}} = \frac{\beta}{2}\int_0^1 \log(1 + \eta_{\mathrm{ms}} f(x)) dx. \tag{3-8}$$

C. SCO Detector

As mentioned at the beginning of this section, the spectral efficiency of the SCO detector is most conveniently obtained using interrelations between the spectral efficiency of an optimum multiuser detector and that of the linear MMSE detector. The following Lemma is a result obtained in [2] with respect to single-cell systems and flat-fading channels (a factor of $1/2$ was added to account for real channels, as considered here).

Lemma 3.3 (Shamai–Verdú [2] Theorem IV.1): Let the empirical distribution of the received powers of all users converge as in Lemma 3.2 to some nonrandom limit $\mathcal{H}(\mathcal{P})$. Then, the spectral efficiency of the optimum multiuser detector is given by

$$\tilde{C}_{\mathrm{sco}} = \tilde{C}_{\mathrm{ms}}^{s-c} + \frac{1}{2}\log\frac{1}{\eta_{\mathrm{ms}}^{s-c}} + \frac{(\eta_{\mathrm{ms}}^{s-c} - 1)}{2}\log e \tag{3-9}$$

where η_{ms}^{s-c} is the limiting multiuser efficiency of the corresponding linear MMSE detector, as given by Lemma 3.2 ((3-6)), and

$$\tilde{C}_{\mathrm{ms}}^{s-c} = \frac{\beta}{2} E_{\mathcal{H}}\{\log(1 + \eta_{\mathrm{ms}}^{s-c} P)\} \tag{3-10}$$

is the spectral efficiency of this detector.

In order to apply the above Lemma to the multicell model considered here, the following observation is required. By definition, the SCO detector treats adjacent-cell interference as an AWGN, which tacitly implies that joint nearest neighbor decoding is employed for the detection of intracell transmissions. However, the additive interference originating from adjacent-cell users is, in fact, a *nonwhite* Gaussian noise, which puts us in the framework of the mismatched decoding problem, as analyzed in [17]. According to [17], adding a mild restriction that the additive adjacent cell interference is ergodic of second moment, and under the assumption, adhered to in this analysis, that all codebooks are Gaussian, the spectral efficiency of the detector depends on the actual noise plus interference distribution only via its power, and thus coincides with the spectral efficiency in a *white* Gaussian noise channel with signal and noise powers equal to those of the original channel. Following the above result, it may, therefore, be concluded that in terms of spectral efficiency the SCO detector is equivalent to an optimum detector in a *single-cell* system, where the additive white Gaussian background noise process has spectral level given by $1 + 2\beta\alpha^2\bar{P}$. This implies that the power assignment function $f(x)$ in the equivalent single-cell setting should be replaced by

$$f_{\rm eq}(x) \triangleq \frac{f(x)}{1+2\beta\alpha^2\bar{P}}. \qquad (3\text{-}11)$$

Following similar arguments to those used for deriving (3-7) and applying Lemma 3.3, the spectral efficiency of the SCO detector is finally given by

$$\tilde{C}_{\rm sco} = \frac{\beta}{2}\int_0^1 \log(1+\eta_{\rm sco} f_{\rm eq}(x))dx + \frac{1}{2}\log\frac{1}{\eta_{\rm sco}} + \frac{1}{2}(\eta_{\rm sco}-1)\log e \qquad (3\text{-}12)$$

where $\eta_{\rm sco}$ is the multiuser efficiency of the corresponding linear MMSE detector, given by the unique positive solution to the implicit equation

$$\eta_{\rm sco} + \beta\int_0^1 \frac{\eta_{\rm sco} f_{\rm eq}(\zeta)}{1+\eta_{\rm sco} f_{\rm eq}(\zeta)} d\zeta = 1. \qquad (3\text{-}13)$$

D. MMSE-SC Detector

The MMSE-SC detector uses a successive interference cancellation scheme, with linear MMSE processing at each stage (essentially as described in Section III-B). Starting from the first intracell user, the detector uses linear MMSE processing to mitigate the interference generated by $3K-1$ users ($K-1$ intracell, and $2K$ at adjacent cells), as experienced by user 1. After decoding, the signal generated by user 1 is reconstructed, subtracted from the total received signal, and the result is passed on to the detector for the second intracell user, now experiencing interference generated by $3K-2$ users ($K-2$ intracell, and $2K$ at adjacent cells). The procedure is repeated until the last intracell user (user K), which due to the cancellation process experiences only interference generated by the $2K$ adjacent-cell users. Since by the underlying assumption the cell-site detector is unaware of the codebooks of adjacent-cell users, no decoding and cancellation are performed with respect to the signals generated by these users.

One approach for obtaining the spectral efficiency of the MMSE-SC detector is first evaluating its multiuser efficiency (continuous argument) function $\eta_{\rm m-sc}(x)$, and then using (3-2). In contrast to the *linear* MMSE detector, the multiuser efficiency of the MMSE-SC detector is not equal for all users, as they experience interference from a decreasing number of users as the successive cancellation process progresses. This naturally holds also for the full MMSE-SC detector.

Applying Lemma 3.3 to each user (with the appropriate interpretation as in Section III-B), the multiuser efficiency function of the MMSE-SC detector is given by the unique positive solution (for each value of x) to the implicit equation

$$\eta_{\rm m-sc}(x) + \beta\left[\int_x^1 \frac{\eta_{\rm m-sc}(x) f(\zeta)}{1+\eta_{\rm m-sc}(x) f(\zeta)} d\zeta + 2\int_0^1 \frac{\alpha^2\eta_{\rm m-sc}(x) f(\zeta)}{1+\alpha^2\eta_{\rm m-sc}(x) f(\zeta)} d\zeta\right] = 1. \qquad (3\text{-}14)$$

The spectral efficiency of the MMSE-SC detector is given by

$$\tilde{C}_{\rm m-sc} = \frac{\beta}{2}\int_0^1 \log(1+\eta_{\rm m-sc}(x) f(x))dx. \qquad (3\text{-}15)$$

In cases in which only the spectral efficiency (sum rate) is of interest, and not the individual user rates, an alternative computationally simpler approach for evaluating the spectral efficiency of the MMSE-SC detector may be taken. The key tool in the derivation is the equivalence, in terms of spectral efficiency, between the (full) MMSE-SC detector and the optimum detector in a *single-cell environment*, i.e., when *all* received transmissions are to be decoded at the receiver (e.g., see [1],[18], and [19]).

The first step is observing that the spectral efficiency expression of (3-15) equals the sum of the maximum attainable rates by each of the intracell users, normalized by the processing gain, and written in the continuous argument function form (considering the limiting scenario of $K, N \to \infty$, $K/N \to \beta < \infty$). Let (3-15) be rewritten in the following way

$$\tilde{C}_{\rm m-sc} = \frac{\beta}{2}\int_0^1 \log(1+\eta_{\rm m-sc}(x) f(x))dx + \mathcal{S} - \mathcal{S} \quad (3\text{-}16)$$

where $\mathcal{S}$ denotes the normalized sum of the maximum rates that would have been attained by users $K+1\ldots 3K$ in a *single-cell* scenario with $3K$ users, employing *full* MMSE-SC detection, with the first K users received at powers $f(x)$, $x \in (0,1]$ and the remaining $2K$ users received at powers $\alpha^2 f(x \bmod 1)$, $x \in (1,3]$ (which are the received powers of the adjacent-cell users). Here the reader must be cautious as not to confuse the additional rates (those summing up to $\mathcal{S}$) with the *actual* attained rates of the users in adjacent cells, as these must be identical to the rates of the *first* K users (with corresponding indices), that represent the true rates of the intracell users in the multicell scenario, as all cells are assumed to be identical from all aspects. Rather, these fictitious single-cell scenario rates were introduced here for mathematical purposes only, as they lead to an equivalent expression for the spectral efficiency that is simpler to compute. Returning to the single-cell interpretation, the following is observed. The sum of the first two elements in the r.h.s. of (3-16) can be interpreted as the spectral efficiency of the full MMSE-SC detector in a single (isolated) cell with $3K$ users and with received powers as explained above. This spectral efficiency is denoted by $\mathcal{C}$. In addition, the term $\mathcal{S}$ in (3-16) can also be interpreted as the spectral efficiency of the full MMSE-SC detector in a single (isolated) cell with *only* $2K$ users, where *all* users are received with powers $\alpha^2 f(x \bmod 1)$, $x \in (0,2]$, and (3-16) may be rewritten as

$$\tilde{C}_{\rm m-sc} = \mathcal{C} - \mathcal{S}. \qquad (3\text{-}17)$$

The validity of this alternative interpretation follows from the fact that within the MMSE-SC detection scheme the first K users (out of $3K$) have no effect on the rates attained by the remaining $2K$ users, as they are cancelled out in the successive interference cancellation process.

The above single-cell interpretations allow to express the desired spectral efficiency as a difference of spectral efficiencies of full MMSE-SC detectors in two different *single-cell* scenarios. Toward this end, the equivalence in terms of spectral efficiency of the optimum detector and the full MMSE-SC detector in a single-cell scenario can be used, and Lemma 3.3 can be applied to express $\mathcal{C}$ and $\mathcal{S}$ as the spectral efficiencies of the corresponding optimum detectors. Accordingly, the spectral efficiency of the MMSE-SC detector (in the considered *multicell* scenario) is given by

$$\begin{aligned}\tilde{C}_{\mathrm{m-sc}} =& \frac{\beta}{2}\int_0^1 \log(1+\eta_{\mathcal{C}} f(x))dx \\ &+\beta\int_0^1 \log(1+\alpha^2\eta_{\mathcal{C}} f(x))dx \\ &+\frac{1}{2}\log\frac{1}{\eta_{\mathcal{C}}}+\frac{1}{2}(\eta_{\mathcal{C}}-1)\log e \\ &-\left[\beta\int_0^1 \log(1+\alpha^2\eta_{\mathcal{S}} f(x))dx\right. \\ &\left.+\frac{1}{2}\log\frac{1}{\eta_{\mathcal{S}}}+\frac{1}{2}(\eta_{\mathcal{S}}-1)\log e\right] \quad (3\text{-}18)\end{aligned}$$

where $\eta_{\mathcal{C}}$ is the unique positive solution to

$$\eta_{\mathcal{C}}+\beta\left[\int_0^1\frac{\eta_{\mathcal{C}} f(\zeta)}{1+\eta_{\mathcal{C}} f(\zeta)}d\zeta+2\int_0^1\frac{\alpha^2\eta_{\mathcal{C}} f(\zeta)}{1+\alpha^2\eta_{\mathcal{C}} f(\zeta)}d\zeta\right]=1 \quad (3\text{-}19)$$

and $\eta_{\mathcal{S}}$ is the unique positive solution to

$$\eta_{\mathcal{S}}+2\beta\int_0^1\frac{\alpha^2\eta_{\mathcal{S}} f(\zeta)}{1+\alpha^2\eta_{\mathcal{S}} f(\zeta)}d\zeta=1. \quad (3\text{-}20)$$

After deriving the spectral efficiency expressions, it is useful now to shed some light on an interesting property of the MMSE-SC detector. A careful observation shows, that in terms of spectral efficiency the MMSE-SC detector is in fact the *optimum multiuser detector*, under the assumption of single cell-site processing, and the assumption that the receiver has *no knowledge of the codebooks used in the adjacent cells*, and those codebooks are randomly selected per message (as is indeed assumed in the system model considered, see Section II). This is evident by noticing, that the matched filtering over all signatures sequences (both of intracell and adjacent-cell users) constitutes sufficient statistics, and the fact that, in the Gaussian regime, the MMSE estimator extracts all the desired information in the mutual information sense. By exactly the same arguments used in [19] for the the full MMSE-SC detector in a single-cell scenario, the sum of rates attained in the present model by the successive cancellation process, can be shown to correspond to the chain decomposition rule for mutual information. Hence, the MMSE-SC detector achieves the maximum mutual information between the channel output [as represented by the vector $\boldsymbol{y}$ in (2-1)] and the channel input due to intracell users only [as represented the vector $\boldsymbol{x}$ in (2-1)], with the above restrictions on information available with respect to adjacent-cell interference. It is noted, however, that in the case in which the codebooks of adjacent-cell users *are* known or chosen once for good, the MMSE-SC is *no longer* the optimum receiver, as the problem falls within the difficult framework of joint multiple-access/interference channels (see also Section IV).

IV. Equal Powers

Satisfying the average power constraint of (2-3), the equal-powers policy simply implies setting $f(x)=\bar{P}, \forall\, 0<x\leq 1$. Substituting the above power assignment function in (3-5), the spectral efficiency of the matched-filter detector is given by

$$\tilde{C}_{\mathrm{mf}}=\frac{\beta}{2}\log\left[1+\frac{\bar{P}}{1+\beta(1+2\alpha^2)\bar{P}}\right]. \quad (4\text{-}1)$$

As can be seen, the above result reaches a limit as the average power (or E_b/N_0) grows without bound, which is in accordance with the well-known interference-limited behavior of the matched-filter detector. The same behavior is observed taking $\beta\rightarrow\infty$, which is also optimum in terms of spectral efficiency [1].

The spectral efficiency of the SCO detector is evaluated by substituting $f(x)=\bar{P}$ into (3-12) and (3-13). Fortunately, the resulting spectral efficiency with equal received powers admits a closed explicit form expression, as observed for the single-cell scenario in [1]. Accordingly, the spectral efficiency of the SCO detector is given by

$$\begin{aligned}\tilde{C}_{\mathrm{SCO}} =& \frac{\beta}{2}\log\left[1+\bar{P}_{\mathrm{eq}}-\frac{1}{4}\mathcal{F}(\bar{P}_{\mathrm{eq}},\beta)\right] \\ &+\frac{1}{2}\log\left[1+\bar{P}_{\mathrm{eq}}\beta-\frac{1}{4}\mathcal{F}(\bar{P}_{\mathrm{eq}},\beta)\right] \\ &-\frac{\log e}{8\bar{P}_{\mathrm{eq}}}\mathcal{F}(\bar{P}_{\mathrm{eq}},\beta) \qquad (4\text{-}2)\end{aligned}$$

where $\bar{P}_{\mathrm{eq}} \triangleq \bar{P}/(1+2\beta\alpha^2\bar{P})$ [see (3-11)], and $\mathcal{F}(x,z) \triangleq \left(\sqrt{x(1+\sqrt{z})^2+1}-\sqrt{x(1-\sqrt{z})^2+1}\right)^2$. As with the matched-filter detector, it is observed that the above spectral efficiency reaches a limit letting $E_b/N_o\rightarrow\infty$ (note that in such case $\bar{P}_{\mathrm{eq}}\rightarrow 1/2\beta\alpha^2$). This interference-limited behavior of the SCO detector emanates from the fact that *intercell* interference is treated as AWGN. Again, $\beta\rightarrow\infty$ is optimum in terms of spectral efficiency.

The SINR at the output of the linear MMSE detector is given by the positive root of the following cubic equation obtained by substituting $f(x)=\bar{P}$ and $\eta_{\mathrm{ms}}=g_{\mathrm{ms}}/\bar{P}$ into (3-7)

$$\begin{aligned}g_{\mathrm{ms}}^3 &+ \frac{\left[1+(3\beta-1)\bar{P}\right]\alpha^2+1}{\alpha^2}g_{\mathrm{ms}}^2 \\ &+\frac{(2\beta-1)\bar{P}\alpha^2+(\beta-1)\bar{P}+1}{\alpha^2}g_{\mathrm{ms}}-\frac{\bar{P}}{\alpha^2}=0. \qquad (4\text{-}3)\end{aligned}$$

Clearly, with equal powers the output SINR for the linear MMSE detector is identical for all users, and thus, the spectral efficiency of the linear MMSE detector is simply given by

$$\tilde{C}_{\mathrm{ms}}=\frac{\beta}{2}\log(1+g_{\mathrm{ms}}) \quad (4\text{-}4)$$

where g_{ms} is the positive root of (4-3) [see (3-8)]

In contrast to the matched-filter and the SCO detector, the linear MMSE detector is not interference limited if the system load β is appropriately chosen. By examining (3-7), substituting

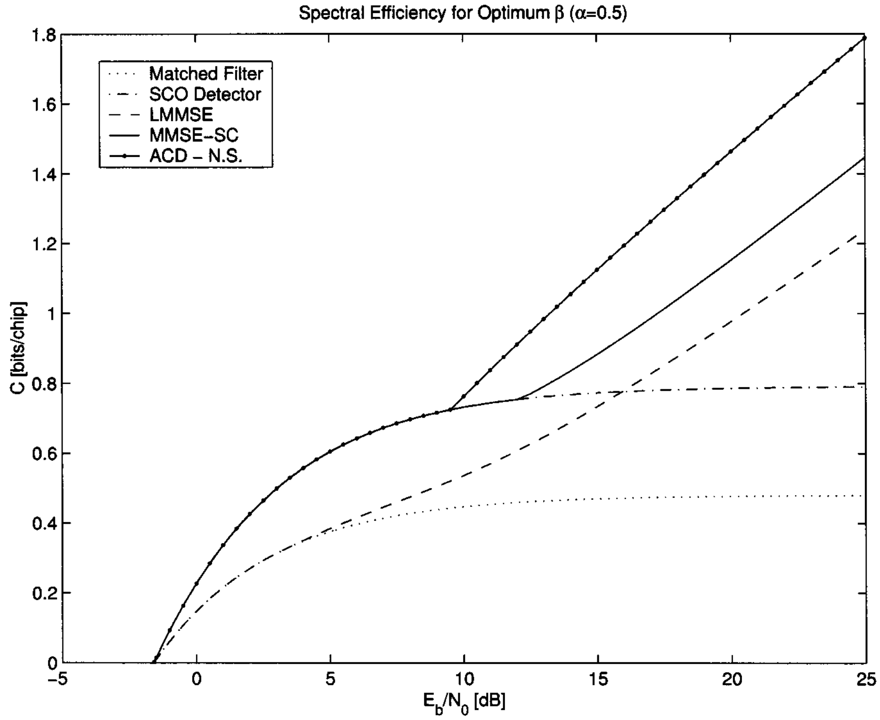

Fig. 2. Spectral efficiency with equal powers, $\alpha = 1/2$, and optimum system load β (N.S.: No spreading).

$\eta_{\rm ms} = g_{\rm ms}/\bar{P}$, it can be shown that the spectral efficiency of the linear MMSE detector grows without bound as $E_b/N_0 \to \infty$, provided that $\beta \leq 1/3$. It is also seen that the optimum choice for β is lower than $1/3$ for large E_b/N_0. Taking $\beta \to \infty$, the spectral efficiency of the linear MMSE detector coincides with that of the matched-filter detector, as expected from the single-cell results of [1].

Finally, substituting $\eta_{\rm m-sc}(x) = g_{\rm m-sc}(x)/\bar{P}$ (and $f(x) = \bar{P}$) into (3-14) results again in a cubic equation

$$g^3_{\rm m-sc}(x) + \frac{1+\alpha^2\left[1+\bar{P}((3-x)\beta-1)\right]}{\alpha^2} g^2_{\rm m-sc}(x) + \frac{1+\bar{P}\left[(2\beta-1)\alpha^2+\beta(1-x)-1\right]}{\alpha^2} g_{\rm m-sc}(x) - \frac{\bar{P}}{\alpha^2} = 0 \qquad (4\text{-}5)$$

whose positive root determines the value of the output SINR function of the MMSE-SC detector: $g_{\rm m-sc}(x)$, $\forall\, x \in (0,1]$ (note that here the SINR function is not a constant although all users are received with equal powers). Obtaining the SINR function, the resulting spectral efficiency can be evaluated through [see (3-15)]

$$\tilde{C}_{\rm m-sc} = \frac{\beta}{2}\int_0^1 \log(1+g_{\rm m-sc}(x))dx. \qquad (4\text{-}6)$$

As mentioned in Section III-D, the spectral efficiency may be alternatively evaluated through (3-18)–(3-20), however (3-14) lends itself more easily to asymptotic analysis, whose results are described next.

Considering again (3-14), substituting $\eta_{\rm m-sc}(x) = g_{\rm m-sc}(x)/\bar{P}$, the spectral efficiency of the MMSE-SC detector can also be shown to grow without bound as $E_b/N_o \to \infty$, if β is appropriately chosen (again the optimum choice here is setting $\beta < 1/3$ for large E_b/N_0). Taking $\beta \to \infty$ it is observed that the spectral efficiency of the MMSE-SC detector coincides with that of the SCO detector, which is in agreement with the behavior of the linear MMSE detector for $\beta \to \infty$.

Fig. 2 compares the spectral efficiencies of the four detectors discussed above with $\alpha = 1/2$ and the optimum choice of β. The value of $\alpha = 1/2$ has been chosen to mimic the case in which the average intercell interference power equals one half of the average power of intracell transmissions ($2\alpha^2 = 1/2$) which is in agreement with the early reports on IS-95 systems. The spectral efficiencies were evaluated at the *optimum* system load β (which for the two MMSE based detectors is a function of E_b/N_0) in view of its cardinal effect on system performance, making it a major system design parameter (as can be observed from the preceding discussion, and also indicated in [1]). Therefore, it is only reasonable to assume that for each multiuser detection scheme the system is optimally designed, and to compare the detectors on the basis of best achievable performance to make the comparison fair.

Accordingly, for low E_b/N_0 where it is optimum to choose $\beta \to \infty$, the spectral efficiencies of the linear MMSE and the MMSE-SC detectors coincide with those of the matched-filter detector and the SCO detector, respectively. However as E_b/N_0 increases beyond some critical value, the optimum choice of β for the two MMSE based detectors decreases, eventually becoming lower than $1/3$, and the spectral efficiency of these detectors grows without bound with E_b/N_0. The decrease in optimum β is the reason for the "knee effect" observed in the spectral efficiency curves of these two detectors (it is noted that the phenomena is not observed when the spectral efficiency is plotted for a *fixed* value of β). The slope of the spectral efficiency curve of the MMSE-SC detector is however steeper than that of the linear MMSE detector. One can also notice that beyond some critical value of E_b/N_0 (around 16 dB in Fig. 2), the spectral efficiency of the linear MMSE detector surpasses that of the SCO detector.

Another interesting issue is the penalty due to random spreading. Following [12], the spectral efficiency (sum rate) without spreading of the detector equivalent to the matched-filter detector[1] is given for any number of users K by

$$\tilde{C}_{\rm mf}^{n.s.} = \frac{K}{2}\log\left(1+\frac{\bar{P}}{1+(K-1)\bar{P}+2K\alpha^2\bar{P}}\right) \tag{4-7}$$

and the spectral efficiency of the SCO detector is given by

$$\tilde{C}_{\rm sco}^{n.s.} = \frac{1}{2}\log\left(1+\frac{K\bar{P}}{1+2K\alpha^2\bar{P}}\right). \tag{4-8}$$

Since in (4-7) and (4-8) $K=\beta$, one can check that the spectral efficiencies (with spreading) of both the matched-filter detector and the SCO detector [cf. (4-1) and (4-2)], are equivalent to (4-7) and (4-8) while taking $\beta\to\infty$ [which is the optimum choice (see Fig. 4)].

Still in the nonspreading framework, an interesting comparison is with respect to a detector that also knows the *codebooks* of users in adjacent cells, and either decodes their transmissions as well, or treats them as additive Gaussian noise whichever is preferable in terms of spectral efficiency. This detector is referred to as the adjacent-cell decoder (ACD), and its spectral efficiency is given by (see [12])

$$\tilde{C}_{\rm ACD}^{n.s.} = \max\left[\frac{1}{2}\log\left(1+\frac{K\bar{P}}{1+2K\alpha^2\bar{P}}\right),\ \min\left(\frac{1}{6}\log(1+(1+2\alpha^2)K\bar{P}),\ \frac{1}{4}\log(1+2\alpha^2K\bar{P})\right)\right]. \tag{4-9}$$

As can be observed from Fig. 2, for low E_b/N_0 the spectral efficiency of the ACD equals that of the MMSE-SC detector and the SCO detector since it is preferable to treat adjacent cell interference as noise. However, beyond some critical value of E_b/N_0 , where decoding is preferable, the curves depart and the spectral efficiency of the ACD grows quite rapidly without bound with E_b/N_0 (in comparison to the other detectors).

V. Equal-Rate Power Assignment

The SCO detector provides equal rates to all users by assigning equal received powers to all. With the remaining three detectors, guaranteeing equal rates to all users requires that the SINR function $g(x)$ is a constant. As can be seen from Section IV, the above requirement is satisfied for the two linear detectors, i.e, the matched-filter detector and the linear MMSE detector, by assigning *equal received powers* to all users. This leaves us with the examination of the MMSE-SC detector. Also considered, for the sake of comparison, is the MMSE-SC detector that treats adjacent cell interference as additive Gaussian noise. This detector is equivalent to the SCO detector in terms of (overall) spectral efficiency (see Section III-D and [16]).

[1]Shamai and Wyner consider in [12] *wideband* communications, where all bandwidth expansion is due to coding, and this detector is referred to in [12] as a *"single-user decoder,"* as it treats all other-user interference as AWGN. It is in that sense that this detector is equivalent to the matched-filter detector in DS spread systems.

The desired power assignment function that attains equal rates can be numerically determined by substituting $\eta(x) = \mathcal{W}/f(x)$, for some constant $\mathcal{W}$, into either (3-14) or

$$\eta_{\rm m-sc}(x)+\beta\int_x^1\frac{\eta_{\rm m-sc}(x)f_{\rm eq}(\zeta)}{1+\eta_{\rm m-sc}(x)f_{\rm eq}(\zeta)}d\zeta=1 \tag{5-1}$$

which is the corresponding implicit equation defining the multiuser efficiency of the detector that treats adjacent-cell interference as noise, and $f_{\rm eq}(x)$ is defined by (3-11). These equations can then be solved so that the power constraint of (2-3) is satisfied, by subdividing the interval $[0,1]$ into M equal intervals, approximating integrals through sums, and using successive approximation methods (the following results are based on $M=25$).

The numerical results show that in terms of (overall) spectral efficiency, evaluated at the optimum choice for β, the equal-rates policy is almost as good as the equal-powers policy, at least for E_b/N_0 values of up to 20 dB. For example (and within the accuracy of the numerical approximations as specified above), considering the detector that mitigates adjacent-cell interference by means of a linear MMSE filter, the spectral efficiency achieved with the equal-rates policy turned out to be about 98% of that with the equal-powers policy, at $E_b/N_0 = 20$ dB and with $\alpha=1/2$. The results obtained for the detector that treats adjacent-cell interference as noise were even closer in this case (about 99:8%). For $\alpha=0$ (and $E_b/N_0=20$ dB) the spectral efficiency with the equal-rates policy was about 97:6% of the spectral efficiency with equal powers (both detectors are equivalent in this case). The spectral efficiency difference was observed to slowly increase with E_b/N_0.

The equal-rates policy may, however, be preferable in terms of a more equitable rate allocation, as can be seen from the example described by Fig. 3. This figure shows the ratio between the rate per user while using the MMSE-SC detectors with equal powers, and the rate attained by the equal-rates policy (given by $\log(1+g(x))/\log(1+\mathcal{W})$), for $E_b/N_0=15$ dB, $\alpha=0$ and $1/2$, and optimum choice of β (the latter is very similar for both policies with interference suppression, less than 1% difference, and identical when interference is treated as noise, thus making the comparison valid). Taking the rate under the equal-rates policy as representing a minimum rate requirement, identical to all users, one can see that with equal powers a large proportion of the users fails to meet the requirement. The unbalanced rate distribution is, in particular, striking for $\alpha=0$, i.e., in a single-cell environment (which should have been expected). For example, with $E_b/N_0=15$ dB around 76% (!) of the users fall under the rate obtained by the equal-rates policy with $\alpha=0$. The nonuniformity in rate distribution under the equal-powers policy becomes moderate as the level of adjacent-cell interference is increased, however even for $\alpha=1/2$ around 40%–60% of the users fall under the rate attained with the equal-rates policy (depending on the chosen detector). Results for $E_b/N_0=20$ dB, are of similar nature. It may therefore be concluded that the equal-rates

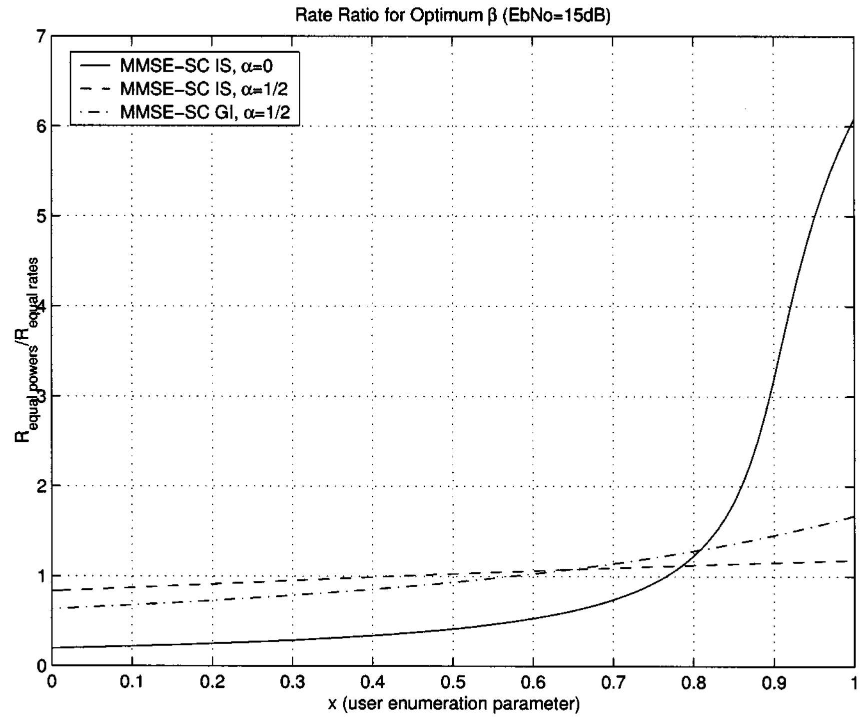

Fig. 3. Ratios of the rates attained with the equal-powers policy and those attained with the equal-rates policy, for $E_b/N_0 = 15$ dB and optimum system load β ($\beta \to \infty$ is represented by $\beta = 10$). IS: Adjacent-cell interference suppression. GI: Interference treated as noise (both are equivalent for $\alpha = 0$).

policy is preferable over the equal-powers policy, in terms service fairness, when all users are to be equally serviced. Furthermore, when the E_b/N_0 at which the system operates is not too high, choosing the equal-rates policy incures (almost) no loss in overall spectral efficiency. With this respect it is noted that the difference between the spectral efficiency with equal powers and that with equal rates was *analytically* obtained in [20], considering a single-cell scenario and the decorrelator based successive canceller, which is asymptotic to the MMSE-SC detector at the high SNR region. The difference in spectral efficiency was found to be increasing proportionally to the logarithm of the spectral efficiency, which is indeed quite slow.

VI. Maximum Spectral Efficieny

Observing (3-5) it is quite straightforward to show, using Jensen's inequality and the concavity of the $\log(1+x)$ function, that the spectral efficiency of the matched-filter detector is maximized by assigning equal powers to all users. The same result holds for the spectral efficiency of the SCO detector, as shown in the Appendix.

Solving the general optimization problem with respect to the spectral efficiency of both the linear MMSE detector and the MMSE-SC detector is quite complex, as can be seen from (3-7), (3-8), (3-14), and (3-15). Hence, a lower bound on the maximum spectral efficiency is pursued. Consider the power assignment function $f_\rho(x)$ defined as

$$f_\rho = \begin{cases} \frac{\bar{P}}{\rho} & 0 < x \le \rho, \\ 0 & \text{otherwise}, \end{cases} \qquad 0 < \rho \le 1. \tag{6-1}$$

Clearly, $f_\rho(x)$ satisfies the average power constraint of (2-3). The rationale behind $f_\rho(x)$ is that the power assignment function provides a mean for applying a "population control" mechanism (see [2]) to enhance system performance. Given a system, with some fixed E_b/N_0, interference factor α, and system load β, one might gain in spectral efficiency by effectively "shutting down" part of the users' transmissions by means of the power assignment function, and assigning more power to the remaining users while satisfying the average power constraint. It is noted however, that using the function $f_\rho(x)$ still confines us to equal power assignment to all effectively active users. Obviously, assigning all power to a single user ($\rho \to 0$) is useless since this drives the system's spectral efficiency to zero. The power assignment function $f_\rho(x)$ comes into effect if there exists some value of $\rho = \rho^* < 1$ for which the spectral efficiency is maximized. In the particular case in which $\rho^* = 1$, $f_\rho(x)$ simply reduces to the equal-powers policy discussed in Section IV. The above "population control" mechanism is in fact equivalent to the optimization of the spectral efficiency with respect to β under the equal-powers policy, for fixed E_b/N_0, if the initially set value of β is higher than the optimum value (note that $f_\rho(x)$ cannot *increase* the effective load). Hence, for a given system, at a fixed working point specified by E_b/N_0, α, and β, the spectral efficiency, maximized over the set of power assignment functions, is lower bounded by the spectral efficiency obtained by substituting $f_\rho(x)$ into (3-7) and (3-14) (or alternatively (3-18)), and by evaluating the spectral efficiency at $\rho = \rho^*$.

It is noted that the "population control" mechanism, based on the power assignment function $f_\rho(x)$, has also a natural practical implementation interpretation, in the form of a random-

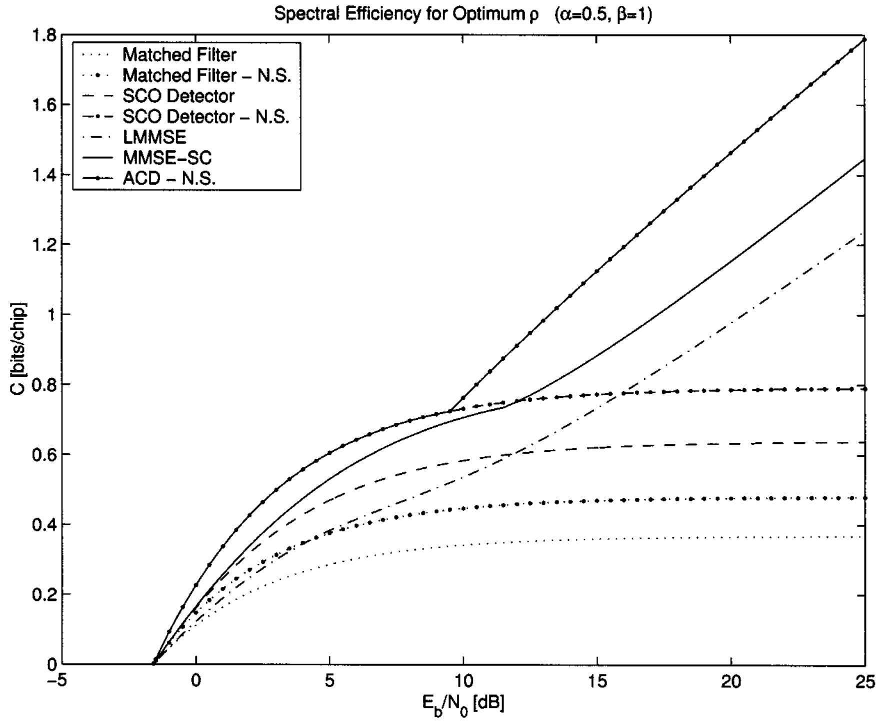

Fig. 4. Spectral efficiency with $f_\rho(x)|_{\rho=\rho^*}$, $\alpha = 1/2$, and system load $\beta = 1$.

ized time division multiple access (TDMA) scheme. Due to the major role of the load β in determining system performance (as also emphasized by Fig. 4 to be discussed below), the system designer can employ a TDMA scheme in which the number of simultaneously active users in each cell is chosen to yield an optimum load according to the desired system working point, and the active users are randomly selected every time frame so as to offer a uniform service quality to all users.

Some comparative results, demonstrating the effect of maximizing the spectral efficiency via the power assignment function are demonstrated in Fig. 4, where the spectral efficiency of all four detectors is plotted for $\alpha = 1/2$ and $\beta = 1$. The spectral efficiency of the linear MMSE and MMSE-SC detector is evaluated using the power assignment function $f_\rho(x)$, with $\rho = \rho^*$. For the matched-filter detector and the SCO detector the spectral efficiency has been evaluated with the equal power assignment (which attains the maximum). Also included in Fig. 4 are the spectral efficiencies without spreading of the three detectors mentioned in Section IV (denoted by "N.S."), as given by (4-7)–(4-9). From Fig. 4 ($\beta = 1$), it can be seen that for low E_b/N_0, the spectral efficiencies of the linear MMSE detector and the MMSE-SC detector are lower than the spectral efficiencies without spreading of the matched-filter equivalent and SCO detectors, respectively (equality is attained for $\beta \to \infty$). This is also the case with the spectral efficiency of the matched-filter, and the SCO detector, for *all* values of E_b/N_0. However, beyond some critical values of E_b/N_0 the optimum β for the linear MMSE and the MMSE-SC detectors decreases below unity, and thus using the power assignment function $f_\rho(x)$ (with $\rho = \rho^*$) brings the spectral efficiency of both detectors to its maximum value with the equal power assignment (cf. Fig. 2).

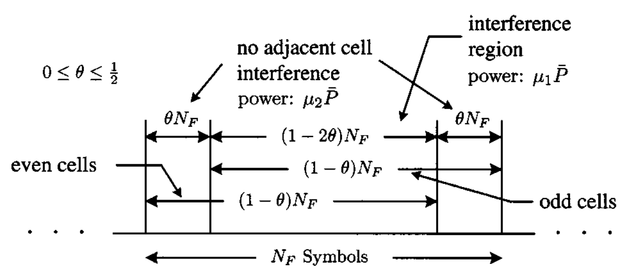

Fig. 5. ICTS scheme.

VII. Intercell Time-Sharing (ICTS)

Following Shamai and Wyner [12], it is also of interest to examine the effect of *intercell time-sharing* (ICTS) protocols between adjacent cells within the scenario considered here. The time-sharing protocol suggested in [12] is described in Fig. 5. It is assumed that the transmission time is divided into frames of N_F channel symbols per frame. The users in each of the cells use only a fraction of $1 - \theta$ of the frame ($0 \le \theta \le 1/2$): the users in even-numbered cells modulate the initial $(1 - \theta)N_F$ symbols of the frame, and those in odd-numbered cells modulate the ending $(1 - \theta)N_F$ symbols of the frame. Thus, the users in each cell are interfered by adjacent-cell users for a fraction $1 - 2\theta$ of the frame, and experience no adjacent-cell interference for a fraction θ of the frame. For simplicity, the analysis is restricted to the equal-powers policy, however it is assumed that the received power level is optimized according to the channel state (i.e., with/without adjacent-cell interference), while satisfying the per-user average power constraint. Denoting by $\mu_1 \bar{P}$ the (equal) received power allocated to users

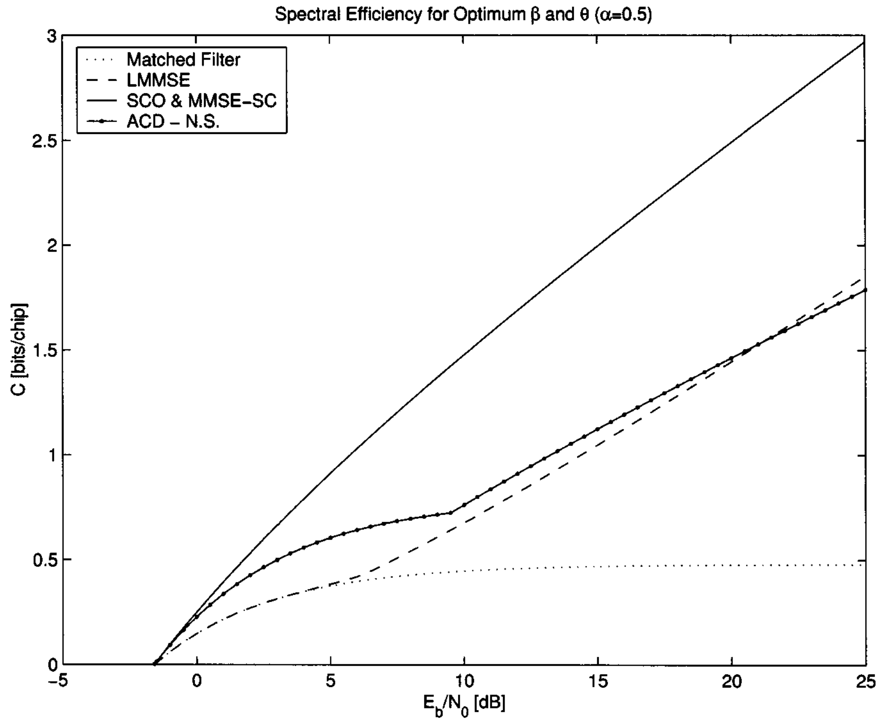

Fig. 6. Spectral efficiency with optimum ICTS, $\alpha = 1/2$, and optimum system load β.

when adjacent-cell interference is present, and by $\mu_2\bar{P}$ the received power when it is not present, the latter requirement yields $\mu_2 = (1-(1-2\theta)\mu_1)/\theta$, $\forall\,\theta \in (0, 1/2)$ (obviously $\mu_1 = 1$ for $\theta = 0$, and $\mu_2 = 2$ for $\theta = 1/2$). Further denoting by $\tilde{C}(\bar{P}, \alpha, \beta)$ the spectral efficiency expression obtained without time-sharing and with equal powers, given an average power constraint $\bar{P}$, interference level α and system load β, as in Section IV, the resulting spectral efficiency *with* ICTS is obtained as a function of E_b/N_0 by solving the following equation [substituting $\bar{P} = \frac{2}{\beta}\tilde{C}\frac{E_b}{N_0}$]

$$\tilde{C} = (1-2\theta)\tilde{C}(\mu_1\bar{P}, \alpha, \beta) + \theta\tilde{C}(\mu_2\bar{P}, 0, \beta). \qquad (7\text{-}1)$$

The above spectral efficiency can then be further optimized with respect to the time-sharing parameter θ and the system load β.

Comparative results with such ICTS protocol are plotted in Fig. 6, again with $\alpha = 1/2$ (it is noted that the results are highly dependent on the value of α). The results were obtained by first solving numerically the equation $d\tilde{C}/d\mu_1 = 0$ (according to (7-1)) in order to obtain the optimum power distribution given θ, and whenever relevant also given β (recall that for the matched-filter detector and SCO detector it is always optimum to take $\beta \to \infty$). Next, standard numerical optimization tools were used to determine the optimum value for θ (given β, unless $\beta \to \infty$). Finally, the same standard numerical optimization tools were also used to determine the optimum system load β for the linear MMSE and MMSE-SC detectors. For the sake of comparison, the spectral efficiency of the ACD (with no spreading and no time sharing), as given by (4-9), has also been included in the figure.

The results show that for all E_b/N_0 values no-ICTS is optimal for the matched-filter detector, while full ICTS is optimal for the SCO and MMSE-SC detectors. Thus, the spectral efficiencies of the two latter detectors coincide (recall the equivalence in terms of spectral efficiency of the MMSE-SC and the optimum detector for Gaussian inputs, as mentioned in Section III). For the linear MMSE detector, no ICTS is optimal up to $E_b/N_0 \approx 6$ dB, while full ICTS is optimal for higher E_b/N_0 (a *short* transition interval where partial ICTS is optimal is observed in between). As can be seen, in particular by comparing Figs. 2 and 6, the introduction of ICTS protocols considerably improves the spectral efficiency of all multiuser detectors considered, except for the matched-filter detector. The dramatic enhancement in system performance is most clearly observed while taking as reference the spectral efficiency curve of the ACD. As can be seen from Fig. 2, without ICTS the ACD is superior to all other detectors. However, with the introduction of ICTS, the spectral efficiencies of all detectors but the matched-filter surpass beyond some critical E_b/N_0 the spectral efficiency of the ACD (in fact the MMSE-SC and the SCO detector exhibit superior performance for all E_b/N_0 values). The matched-filter detector is interference limited *regardless* of whether adjacent cell interference is present or not, and thus no improvement in spectral efficiency can be obtained with ICTS for $\alpha \leq 1/\sqrt{2}$. The performance enhancement is at most striking for the SCO detector as it is *interference limited* without ICTS.

VIII. Conclusion

This paper has analyzed the spectral efficiency of four multiuser detectors that differ by the amount and type of information made available to the detector with regard to interference (both intracell and intercell). The assumptions on available information were made in view of the common practice in practical CDMA systems, and, in particular, those based on the IS-95 standard [21], as described below. Clearly, the cell-site receiver (in any system) must be aware of the exact signature sequences and codebooks of all intra-cell users, as otherwise no detection is possible even with the "conventional" matched-filter detector. Furthermore, when soft-handoff is employed (see [21]) each cell site must also be aware of the signature sequences of users in adjacent cells (which for example in the IS-95 standard are in essence phase-shifted versions of the same long pseudo-noise (PN) sequence), and the option exists of detecting their signals as well. In order to avoid excessive complexity at the receiver, it was assumed that the receiver is unaware of the *codebooks* of adjacent-cell users. It is noted that the use of adaptive MMSE multiuser receivers is an attractive option in this respect (e.g., see [22]) as those do not distinguish between intracell and other-cell interferers.

The results obtained in this paper explicitly demonstrate the dramatic effect of information about interfering signals on system performance. The effect is most clearly seen by comparing the linear MMSE detector and the SCO detector, which as mentioned in Section I represent a tradeoff between intracell transmissions processing complexity, and additional information on adjacent-cell interference. It was shown that one can gain even without trying to decode the transmissions of the interfering users in adjacent cells (which enables interference *cancellation*), or treating them optimally in the setting of an interference channel [12]. The gain emerges by the very fact that the linear MMSE filter will account for the reduction of interference, provided that the signatures of interfering users are known not only at the intended cell-site but at those cell-sites where they cause interference. It was shown that when E_b/N_0 and the interference factor α are not too low, the relatively simple *linear* MMSE detector, which is more informed regarding adjacent cell interference, is preferable over the interference limited SCO detector that employs nonlinear detection of intracell users, while treating intercell interference as noise. It may, therefore, be concluded that for high-data rates, inherently demanding high E_b/N_0, it is advantageous to mitigate out-of-cell interference through linear MMSE processing. The penalty due to random spreading was also demonstrated via comparison to corresponding detectors suggested for nonspreading systems in [12]. Finally, it was shown that a dramatic increase in spectral efficiency can be achieved by employing intercell time-sharing protocols for all detectors but the matched-filter detector.

Power allocation policy comparison shows that the equal-powers policy obtains equal rates with the matched-filter detector, the SCO detector, and the linear MMSE detector, and it attains the maximum spectral efficiency with the matched-filter and the SCO detectors. This policy may also provide a lower bound to the maximum spectral efficiency of the linear MMSE and the MMSE-SC detectors. The obtained results show that applying the equal-rates policy to the MMSE-SC detector results in almost no loss in spectral efficiency, in comparison to the equal-powers policy (when E_b/N_0 is not too high). Conditions at which the equal-rates policy is preferable were also discussed.

This paper considers a simple linear cell array model which has now gained recognition as a tractable model amenable to analytical analyzes. It is emphasized, however, that extension of the results to the popular hexagonal (two dimensional) multicell model is quite straightforward, using again the equivalent single-cell interpretation as in Section III above, but now considering a single-cell system of $7K$ users, where K users are received with powers according to $f(x)$ and the rest $6K$ users are received with powers according to $\alpha^2 f(x)$. The results can also be extended rather straightforwardly (using Lemmas 3.1–3.3) to a multicell model with more than just adjacent-cell interference, by defining multiple interference factors $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_n$. The complexity of the equations, though, increases accordingly. Another simplification, assumed for analytical tractability, is the the *full* synchronism of the received signals (this model has been used in a number of previous works, see [11], [12], and [23]). A more realistic asynchronous model poses *considerable* difficulties and analytical results for an asymptotic system model with random spreading, as considered here, are not yet available. An attempt in this direction can be found for example in [6], where a *single-cell* asynchronous but chip-synchronous system (as opposed to a *fully* asynchronous system) is analyzed. In general, the results indicate that synchronous system performance provides a rather tight upper bound to the performance of finite horizon linear receivers for analogous asynchronous systems. The approach taken in [6] can be used, in principle, to extend the results obtained here to the asynchronous case. However it is emphasized that the general nature of the results, the effect of knowledge on the structure of interfering signals on system performance, the cardinal role of the system load β, and the relative performance of the different detectors should be essentially retained in an asynchronous system as well.

Finally, it is noted that the random signature model analyzed here can also be used to model *frequency-selective* fading, for example by considering the dual to DS-spreading: multicarrier CDMA (see [15]). Extension of the multicell results obtained in this paper to flat-fading channels (*with* DS-spreading) is under current study, as well as the analysis of the optimum *multicell site processing* detector [11], and the characterization of the *optimum* power allocation policy, in terms of spectral efficiency, for the linear MMSE and MMSE-SC detectors.

APPENDIX
EQUAL-POWERS POLICY IS OPTIMAL FOR THE SCO DETECTOR

In the following, the optimality of the equal-powers policy for the SCO detector, in terms of spectral efficiency, is shown. First, observe that since all adjacent-cell interference is treated by the receiver as an additive Gaussian noise, the discrete time equivalent channel of (2-1) can be replaced by

$$\boldsymbol{y}_i = \boldsymbol{S}_i \boldsymbol{x}_i + \tilde{\boldsymbol{n}}_i \tag{A-1}$$

where $\tilde{\boldsymbol{n}}_i$ represents an equivalent zero mean Gaussian noise vector with distribution $\mathcal{N}(0, (1+2\beta\alpha^2\bar{P})\boldsymbol{I})$ (see justification in Section III-C). Now, the spectral efficiency of the above channel (expressed as the normalized input-output mutual information) converges to

$$\begin{aligned}\widetilde{C}_{\rm sco} &= \lim_{\substack{K,N\to\infty,\\ \frac{K}{N}\to\beta<\infty}} \frac{1}{2N} E\left\{\log\det\left\{\boldsymbol{I} + \bar{P}_{\rm eq}\boldsymbol{S}_i^{\dagger}\boldsymbol{S}_i\boldsymbol{A}\right)\right\} \\ &= \frac{\beta}{2} \lim_{\substack{K,N\to\infty,\\ \frac{K}{N}\to\beta<\infty}} E\left\{\frac{1}{K}\sum_{k=1}^{K}\log(1+\bar{P}_{\rm eq}\lambda_k)\right\} \\ &= \frac{\beta}{2} \lim_{\substack{K,N\to\infty,\\ \frac{K}{N}\to\beta<\infty}} E\left\{\int_0^{\infty}\log(1+\bar{P}_{\rm eq}\nu)dF_{\lambda}^{(K)}(\nu)\right\}\end{aligned}$$

(A-2)

where $\bar{P}_{\rm eq} \triangleq \bar{P}/(1+2\beta\alpha^2 P)$ (as in Section IV), $\boldsymbol{A}$ is the $K \times K$ diagonal matrix with entries equal to the received powers of the K intracell users, normalized with respect to their average power $\bar{P}$, $\{\lambda_k\}_{k=1}^K$ are the eigenvalues of the matrix $\boldsymbol{S}_i^\dagger \boldsymbol{S}_i \boldsymbol{A}$, and $F_\lambda^{(K)}(\nu)$ is their empirical distribution. It is also assumed that the limiting empirical distribution of the normalized received powers converges almost surely to a distribution $\mathcal{H}$, satisfying

$$\int_o^\infty \xi d\mathcal{H}(\xi) = 1. \tag{A-3}$$

Following [5], the empirical eigenvalue distribution $F_\lambda^{(K)}(\nu)$ converges almost surely in distribution to $F_\lambda(\nu)$ specified by its Stieltjes transform

$$m_F(\zeta) = \int_0^\infty \frac{1}{\xi(1-\beta-\beta\zeta m_F(\zeta)) - \zeta} d\mathcal{H}(\xi) \tag{A-4}$$

and the last equality in (A-2) can be rewritten in terms of the *limiting* eigenvalue distribution as

$$\tilde{C}_{\rm sco} = \frac{\beta}{2}\int_0^\infty \log(1+\bar{P}_{\rm eq}\nu) dF_\lambda(\nu). \tag{A-5}$$

The Stieltjes transform is defined (assuming nonnegative eigenvalues) for any distribution $F_\lambda(\nu)$ as

$$m_F(\zeta) = \int_0^\infty \frac{1}{\nu - \zeta} dF_\lambda(\nu), \quad \forall\, \zeta \in \mathbb{C}^+. \tag{A-6}$$

Hence, it follows from (A-5) that there is a simple relation between the Stieltjes transform of the limiting eigenvalue distribution and the derivative of the spectral efficiency with respect to $\bar{P}_{\rm eq}$ given by

$$\frac{d\tilde{C}_{\rm sco}(\bar{P}_{\rm eq})}{d\bar{P}_{\rm eq}} = \frac{\beta}{2}\left[\frac{1}{\bar{P}_{\rm eq}} - \frac{1}{\bar{P}_{\rm eq}^2} m_F\left(-\frac{1}{\bar{P}_{\rm eq}}\right)\right]. \tag{A-7}$$

The spectral efficiency can now be expressed as

$$\tilde{C}_{\rm sco}(\bar{P}_{\rm eq}) = \int_0^{\bar{P}_{eq}} \frac{d\tilde{C}_{\rm sco}(s)}{ds} ds. \tag{A-8}$$

By observing (A-7), it is clear that the power distribution that minimizes the Stieltjes transform, maximizes $d\bar{C}_{\rm sco}(\bar{P}_{\rm eq})/d\bar{P}_{\rm eq}$ and, thus, the spectral efficiency through (A-8). However, from the convexity of $1/x$, Jensen's inequality, (A-4), and (A-3), it follows that

$$m_F(\zeta) \geq \frac{1}{1-\beta-\beta\zeta m_F(\zeta) - \zeta} \tag{A-9}$$

and the above lower bound is attained by letting

$$\mathcal{H}(\xi) = \mathcal{U}(\xi - 1) \tag{A-10}$$

where $\mathcal{U}(\xi)$ is the unit step function. It has thus been shown that the spectral efficiency is maximized by the equal-powers policy.

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