Chapter 1

DECODING ONLY THE STRONGEST CDMA USERS

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Dedicated to George David Forney Jr.
a scholar, and a ripe and good one;
exceeding wise, fair-spoken, and persuading.

Abstract We consider the standard randomly spread CDMA channel where users are subject to independent flat fading. We analyze the outage probability of a joint receiver that decodes only the users that are received with the strongest power. All users are assumed to transmit at the same rate which is adjusted so that all users that are decoded by the receiver have vanishing block error probability. For an arbitrary number of users, we show that the capacity region achieved by this decoder is amenable to significant simplification. When the number of users grow without bound, we give asymptotic readily-computable outage formulas based on our previous application of the theory of asymptotic singular values of large matrices to CDMA systems.

1. INTRODUCTION

Ranging from the single-user receiver that neglects the presence of multiaccess interference to the maximum likelihood joint receiver, a wide variety of techniques have been proposed for demultiplexing digital streams that overlap in both time and frequency [1]. Linear and nonlinear receivers that take into account the presence of multiaccess interference can achieve substantial gains in spectral efficiency, as evidenced by the information theoretic analysis of the capacity of randomly spread CDMA [2, 3]. The analysis in [2, 3] obtains the maximum throughput (total rate) in a multiaccess channel where all active users are reliably decoded regardless of their received powers assuming that the transmitters can adjust their rates (and possibly also their powers) as a function of the channel fade levels they experience. An alternative strategy, motivated by practical applications, is to rule out even a modicum of feedback and have all users transmit at the same rate. Then, due to fading, the receiver can no longer guarantee with overwhelming probability that it will decode all active users. The appropriate performance measure in that scenario is the probability that a randomly selected user will not be successfully decoded, commonly referred to as the outage probability. References [4, 3] show how to compute the outage probability of linear CDMA receivers as a function of the rate, E_b/N_0, and fading distribution.

The receiver analyzed in this paper ranks all K active users by their received powers and then demodulates the J strongest users. J is chosen as the largest integer for which the demodulation is successful, a number that depends not only on the fade levels but on the noise realization. Thus, in general, the receiver may need to attempt to demodulate several population sizes until it ascertains the maximal decodable subset of users. However, since different users experience independent fades, as the number of users grows, the percentage of successfully decodable users converges to a deterministic constant. Thus, in many practical scenarios the receiver can be easily simplified to attempt the demodulation of a fixed percentage of users dictated by either complexity constraints or by a specified tolerable probability of joint decoding failure. Furthermore, if the rate is communicated to the users by the receiver, it can be adjusted so that all the decoded users are reliably decoded. If the fading evolves ergodically, then such feedback to the transmitters is not required. Underpinning the rationale for this receiver is the unsurprising property that for any $L = 1, \dots K$, the probability of decoding failure for an arbitrary subset of L users is greater or equal than the probability of decoding failure for the strongest L users. While the $K - J$ weakest users are not decoded, their presence is not ignored by the decoder; Indeed, it takes into account their spreading sequences and received powers (but not the structure of their error control codes, which are assumed to have empirical Gaussian statistics in accordance with well-known multiaccess channel capacity principles). With that information, the decoder then makes an optimum decision about the J strongest users. Clearly, the capabilities of such a receiver serve as an upper bound to those of less complex receivers such as the group detectors of [6] where the effect of the unwanted users is mitigated by means of a linear front end.

Following the above referenced papers on the capacity of randomly spread CDMA, our analysis of the strongest-subset decoder focuses on the asymptotic limit of $K \to \infty$ and $N \to \infty$ with $K/N = \beta$, where N is the spreading gain and β is a design parameter.

The analysis of outage probability assumes that all users are subject to the same flat-fading distribution and that the fade levels are independent from user to user, stay constant for the duration of the transmitted codeword, and are perfectly known at the receiver.

Power ranking has played a longstanding role in multiuser detection. For example, in successive cancellation, it is common (although not leading to minimum uncoded bit error rate) to demodulate users in decreasing order of received powers [1]. Recently proposed receivers that involve sorting of powers include [8] and [5], the latter of which analyzes decoding only the strongest users in an unspread narrowband channel.

2. MODEL

Let K and N denote the number of users and the spreading gain, respectively. Following the standard notation [1, 3] for the synchronous nondispersive CDMA channel, the receiver obtains an N-dimensional observable

$$\mathbf{y} = \mathbf{S}\mathbf{A}\mathbf{x} + \mathbf{m} \tag{1}$$

where the kth column of the matrix $\mathbf{S}$ represents the unit-norm kth signature sequence of the kth user, the diagonal matrix of received complex-valued fading coefficients is denoted by

$$\mathbf{A} = \mathsf{diag}\{\mathsf{A}_1, \dots \mathsf{A}_K\}, \tag{2}$$

$$\mathbf{x} = [x_1, \dots, x_K]^T \tag{3}$$

is the K-vector of information-carrying channel symbols, and $\mathbf{m}$ is a complex Gaussian vector all whose real and imaginary components have variance $1/2$ and are independent.

All transmitted codewords of blocklength n are subject to the average power constraint:

$$\frac{1}{n}\sum_{i=1}^{n} x_k^2[i] \le \mathsf{SNR}. \tag{4}$$

The number of bits transmitted in each symbol $x_k[i]$ is common to all users. We denote the number of bits per second per Hz transmitted by each user by R. If the codebook has M messages, then

$$R = \frac{\log_2 M}{nN}. \tag{5}$$

3. ORDERED DISTRIBUTION OF FADING POWERS

Let i_k denote the index of the kth strongest user at the receiver, i.e.,

$$|\mathsf{A}_{i_1}|^2 \geq |\mathsf{A}_{i_2}|^2 \geq \ldots \geq |\mathsf{A}_{i_K}|^2. \tag{6}$$

Although the definition of i_k is nonunique, it is immaterial how ties are broken. The (power) fade level experienced by the kth strongest user is denoted by

$$\nu_k = |\mathsf{A}_{i_k}|^2. \tag{7}$$

The set of the J strongest users is denoted by

$$\mathcal{D}_J = \{i_1, \ldots, i_J\}. \tag{8}$$

Furthermore, we use overbar/underbar to denote the subset of strong/weak users as in

$$\begin{aligned}
\bar{\mathbf{X}}_J &= \{X_{i_1}, \ldots, X_{i_J}\}, && (9)\\
\underline{\mathbf{X}}_J &= \{X_{i_{J+1}}, \ldots, X_{i_K}\}, && (10)\\
\bar{\mathbf{A}}_J &= \mathsf{diag}\{\mathsf{A}_{i_1}, \ldots, \mathsf{A}_{i_J}\}, && (11)\\
\underline{\mathbf{A}}_J &= \mathsf{diag}\{\mathsf{A}_{i_{J+1}}, \ldots, \mathsf{A}_{i_K}\}. && (12)
\end{aligned}$$

Analogously, $\bar{\mathbf{S}}_J$ denotes the $N \times J$ matrix of signatures of the J strongest users and $\underline{\mathbf{S}}_J$ denotes the $N \times (K-J)$ matrix of signatures of the remaining users.

Let $F(x)$ designate the cumulative probability distribution function (cdf) of the fading power $|\mathsf{A}_k|^2$. Then, the distribution of ν_k is given by (e.g. [1, p.388])

$$F_{\nu_k}(x) = \int_0^x \frac{K!}{(K-k)!(k-1)!} \Big(1 - F(z)\Big)^{k-1} F(z)^{K-k}\, dF(z) \tag{13}$$

which for $K \to \infty$, yields [5]

$$\lim_{K\to\infty} F_{\nu_{k(K)}}(x) - \left\{ \begin{array}{ccc} 0 & , & 1 - F(x) > \frac{k(K)-1}{K-1} \\ 1 & , & 1 - F(x) \leq \frac{k(K)-1}{K-1} \end{array} \right\} = 0, \tag{14}$$

where we have explicitly denoted the fact that we are interested in a strongest-user index $k(K)$ which depends on the total number of users. The asymptotic expectation of the kth strongest power is

$$
\begin{aligned}
E(\nu_{k(K)}) &= \int_0^\infty \left(1 - F_{\nu_{k(K)}}(x)\right) dx && (15)\\
&= \int_0^\infty 1\left\{1 - F(x) > \frac{k(K)-1}{K-1}\right\} dx + o(1) && (16)\\
&= F^{-1}\left(1 - \frac{k(K)-1}{K-1}\right) + o(1)\ , && (17)
\end{aligned}
$$

where $1\{\cdot\}$ designates the indicator function. Regarding (17) note that for a continuous and differentiable function $g(x)$

$$
\begin{aligned}
\frac{1}{K}\sum_{k=w_L K}^{k=w_H K} g(\nu_k) \ \overset{K\to\infty}{\longrightarrow}\ & \int_{w_L}^{w_H} g\left(F^{-1}(1-x)\right) dx \\
=& \int_{F^{-1}(1-w_H)}^{F^{-1}(1-w_L)} g(y)\, dF(y)\ . && (18)
\end{aligned}
$$

4. NONASYMPTOTIC CAPACITY REGION

In principle, the fact that some users are decoded whereas the code structure of some users is not exploited could lead to analytical difficulties. However, it is possible to sidestep those difficulties crisply by using Kolmogorov's identity. The mutual information between the channel output and the information transmitted by the users to be demodulated can be decomposed as

$$
\begin{aligned}
I(\bar{\mathbf{X}}_J;\mathbf{Y}) &= I(\bar{\mathbf{X}}_J,\underline{\mathbf{X}}_J;\mathbf{Y}) - I(\underline{\mathbf{X}}_J;\mathbf{Y}|\bar{\mathbf{X}}_J) && (19)\\
&= I(\mathbf{X};\mathbf{Y}) - I(\underline{\mathbf{X}}_J;\mathbf{Y}|\bar{\mathbf{X}}_J) && (20)
\end{aligned}
$$

where the input and output random vectors in (1) are denoted by the upper case symbols $\mathbf{X}$ and $\mathbf{Y}$. Both mutual informations appearing in (20) find closed form expressions. It was shown in [9] that

$$
I(\mathbf{X};\mathbf{Y}) = \log\left(\mathbf{I} + \mathsf{SNR}\mathbf{A}^*\mathbf{S}^*\mathbf{S}\mathbf{A}\right). \qquad (21)
$$

Analogously, since the information transmitted by different users is independent

$$
I(\underline{\mathbf{X}}_J;\mathbf{Y}|\bar{\mathbf{X}}_J) = \log\left(\mathbf{I} + \ \mathsf{SNR}\ \underline{\mathbf{A}}_{\mathrm{J}}^*\underline{\mathbf{S}}_{\mathrm{J}}^*\underline{\mathbf{S}}_{\mathrm{J}}\underline{\mathbf{A}}_{\mathrm{J}}\right). \qquad (22)
$$

It should be emphasized that the fact that the signature sequences are known also for the weak $K - J$ users that are not decoded is fully exploited in (20)-(22). Unlike [5] where no spreading is considered, it would be suboptimal to have a front end of J matched filters which neglects the $K-J$ weak users.

Conditioned on the channel fades and signature waveforms, the capacity region determining the rates achievable by the J strongest users is given by

$$\mathbf{C} = \bigcap_{\mathcal{D}\subset\mathcal{D}_J} \left\{ \sum_{i\in\mathcal{D}} R_i \le I(\mathbf{X}_{\mathcal{D}_J};\mathbf{Y} \mid \mathbf{X}_{\mathcal{D}_J-\mathcal{D}}) \right\} \tag{23}$$

As we see in the following result, due to the particular structure of our problem, J of the rate constraint equations among the 2^J-1 equations in (23) are sufficient in order to describe the capacity region.

Theorem 1 *The capacity region achievable by the J strongest users is equal to (bits/sec)*

$$\mathbf{C} = \bigcap_{j=1}^{J} \left\{ \sum_{\ell=j}^{J} R_{i_\ell} \le I(\underline{\mathbf{X}}_j;\mathbf{Y} \mid \bar{\mathbf{X}}_j) - -I(\underline{\mathbf{X}}_J;\mathbf{Y} \mid \bar{\mathbf{X}}_J) \right\} \tag{24}$$

Proof.

First we need to show that J equations are sufficient:

$$\mathbf{C} = \bigcap_{j=1}^{J} \left\{ \sum_{\ell=j}^{J} R_{i_\ell} \le I(X_{i_j},\ldots,X_{i_J};\mathbf{Y} \mid \{X_{i_1},\ldots X_{i_{j-1}}\}). \right\} \tag{25}$$

To that end, proceeding as in (20), note that all the mutual informations arising in (23) take the form

$$\begin{aligned} I(\mathbf{X}_{\mathcal{D}_J};\mathbf{Y} \mid \mathbf{X}_{\mathcal{D}_J-\mathcal{D}}) &= I(\mathbf{X}_{\mathcal{D}},\underline{\mathbf{X}}_J;\mathbf{Y} \mid \mathbf{X}_{\mathcal{D}_J-\mathcal{D}}) \\ &- I(\underline{\mathbf{X}}_J;\mathbf{Y} \mid \bar{\mathbf{X}}_J) && (26) \\ &= \log\left(\mathbf{I} + \mathsf{SNR}\ \underline{\mathbf{A}}_{\mathcal{B}}^{*}\underline{\mathbf{S}}_{\mathcal{B}}^{*}\underline{\mathbf{S}}_{\mathcal{B}}\underline{\mathbf{A}}_{\mathcal{B}}\right) \\ &- I(\underline{\mathbf{X}}_J;\mathbf{Y} \mid \bar{\mathbf{X}}_J) && (27) \\ & && (28) \end{aligned}$$

where $\underline{\mathbf{A}}_{\mathcal{D}}$ and $\underline{\mathbf{S}}_{\mathcal{D}}$ denote the fade coefficients and signature matrices of the users in the subset

$$\mathcal{B} = \mathcal{D}\cup\mathcal{D}_J^C = (\mathcal{D}_J-\mathcal{D})^c.$$

The right side of (28) is always lower bounded by the choice of $\mathcal{D}$ corresponding to the l-weakest users among the J strongest users, i.e.

$J-l+1,\ldots,J-1,J$. Thus, the corresponding equation dominates. Note that the interfering part $I(\underline{\mathbf{X}}_J;\mathbf{Y}\mid\bar{\mathbf{X}}_J)$ is fixed for fixed J and thus is not affected by the specific subset of users.

Finally, generalizing the decomposition in (20) we can write every mutual information appearing in (25) as

$$\begin{aligned}
I(X_{i_k},\ldots,X_{i_J};\mathbf{Y}\mid\{X_{i_1},\ldots X_{i_{k-1}}\}) &= I(X_{i_k},\ldots,X_{i_J},\underline{\mathbf{X}}_J;\mathbf{Y}\mid\{X_{i_1},\ldots X_{i_{k-1}}\}) \\
&- I(\underline{\mathbf{X}}_J;\mathbf{Y}\mid\{X_{i_1},\ldots X_{i_{k-1}}\},\{X_{i_k},\ldots,X_{i_J}\}) \\
&= I(\underline{\mathbf{X}}_k;\mathbf{Y}\mid\bar{\mathbf{X}}_k)-I(\underline{\mathbf{X}}_J;\mathbf{Y}\mid\bar{\mathbf{X}}_J) \qquad (29)
\end{aligned}$$

for $k=1,\ldots J$.

Naturally, the explicit expression in (22) not only can be used to compute the second mutual information in the rate constraints of (24) but also the first, since they both can be viewed as coming from a channel where there are no users other than the (j or J) weakest ones.

5. ASYMPTOTIC CAPACITY

From (22) and (24), we need to focus on the behavior of

$$\log\left(\mathbf{I}+\mathsf{SNR}\;\underline{\mathbf{A}}_L^*\underline{\mathbf{S}}_L^*\underline{\mathbf{S}}_L\underline{\mathbf{A}}_L\right),$$

for values of L ranging from 1 to J. Using results on the asymptotic behavior of the singular values of random matrices, the main result in [3] can be used to show that as $K\to\infty$, $N\to\infty$, $K/N=\beta$, $L/K=\delta$ (where $0\le\delta\le1$ and $\beta>0$ are arbitrary parameters) we obtain the asymptotic behavior:

$$\begin{aligned}
\frac{1}{N}\log\left(\mathbf{I}+\mathsf{SNR}\;\underline{\mathbf{A}}_L^*\underline{\mathbf{S}}_L^*\underline{\mathbf{S}}_L\underline{\mathbf{A}}_L\right) &= -\log\eta_K^L+(\eta_K^L-1)\log e+o(1) \\
&\quad\frac{1}{N}\sum_{m=L}^{K}\log(1+\mathsf{SNR}\,\nu_m\,\eta_K^L), \qquad (30)
\end{aligned}$$

where η_K^L solves the equation

$$\frac{1}{\eta_K^L}=1+\frac{1}{N}\sum_{m=L}^{K}\frac{\nu_m}{\frac{1}{\mathsf{SNR}}+\eta_K^L\,\nu_m}\,. \qquad (31)$$

Using (18), we can see that

$$\lim_{K\to\infty}\eta_K^{\delta K}=\eta_\delta \qquad (32)$$

where η_δ is the solution to

$$1-\beta=\eta-\beta\Theta(\delta;\;\mathsf{SNR}\,\eta)\,, \qquad (33)$$

and

$$\Theta(w;a)=\int_{F^{-1}(0)}^{F^{-1}(1-w)}\frac{1}{1+a\phi}\,dF(\phi) \qquad (34)$$

Analogously, in order to deal with the summation in (30), we define

$$\Gamma(w;a) = \int_{F^{-1}(0)}^{F^{-1}(1-w)} \log(1+a\phi)\, dF(\phi) \tag{35}$$

and

$$\mathsf{C}(\delta) = \beta\, \Gamma(\delta; \mathsf{SNR}\eta_\delta) - \log \eta_\delta + (\eta_\delta - 1)\log e. \tag{36}$$

It follows from the above definitions that (30) converges to

$$\lim_{K\to\infty} \frac{1}{N} \log\left(\mathbf{I} + \ \mathsf{SNR}\ \underline{\mathbf{A}}_L^* \underline{\mathbf{S}}_L^* \underline{\mathbf{S}}_L \underline{\mathbf{A}}_L\right) = \mathsf{C}(\delta) \tag{37}$$

It is convenient to realize that the functions defined in (34)-(35) are related by

$$\Theta(w;a) = \frac{-a\partial\Gamma(w;a)}{\partial a} + \frac{1}{a}\left(F^{-1}(1-w) - F^{-1}(0)\right). \tag{38}$$

Now we proceed to plug the asymptotic expressions in the capacity region (24). Because of the equal-rate assumption, and recalling (5), the capacity region becomes

$$\mathbf{C}{=}\bigcap_{j=1}^{J}\left\{(J-j+1)R \le \frac{1}{N}I(\underline{\mathbf{X}}_j;\mathbf{Y}\mid\bar{\mathbf{X}}_j) - \frac{1}{N}I(\underline{\mathbf{X}}_J;\mathbf{Y}\mid\bar{\mathbf{X}}_J)\right\} \tag{39}$$

$$= \bigcap_{j=1}^{J}\left\{(J-j+1)R \le \mathsf{C}\left(\frac{j}{K}\right) - \mathsf{C}\left(\frac{J}{K}\right)\right\}. \tag{40}$$

We are now ready to state the main result. Letting $1-q$ and $1-x$ take the asymptotic role of J/K and j/J respectively, leads to the following result on the total achievable rate RJ in the notation of (40):

Theorem 2 *Let the tolerable outage probability be q, then the total achievable rate (bits per second per Hz) is*

$$R_T(q) = \inf_{0<x\le 1} \frac{\mathsf{C}\left((1-q)(1-x)\right) - \mathsf{C}(1-q)}{x} \tag{41}$$

where the function C is defined in (36).

Note that in the limit with ordered received power the fraction $\frac{J}{K}$ of reliably decoded user hardens and is not a random variable anymore (see [5]). It is only the identity of the J operating users, which remains absolutely random.

The results can also be cast in form of an outage of an arbitrary user signaling at either zero rate (if it is not strong enough) or at a rate r times the expected throughput [3], that is:

$$R = \frac{r}{K}\,\mathcal{C}^{\mathrm{opt}}\left(\beta,\ \mathsf{SNR}\right)\,, \tag{42}$$

where $\mathcal{C}^{\mathrm{opt}}\left(\beta,\ \mathsf{SNR}\right)$ is as in [3]. Thus $R_T(q) = RJ$ (41) should satisfy

$$R_T(q) = (1-q)r\,\mathcal{C}^{\mathrm{opt}}\left(\beta,\ \mathsf{SNR}\right)\,. \tag{43}$$

The outage as a function of the parameter r can be simply obtained by solving for q in (43).

6. RAYLEIGH FADING

For Rayleigh fading,

$$F(\nu) = 1 - e^{-\nu}\,,\ F^{-1}(x) = -\log(1-x)\,. \tag{44}$$

Using the exponential integral function

$$- - \mathcal{E}_i(-x) = \int_x^{\infty} \frac{e^{-t}}{t}\,dt\,, \qquad x \geq 0 \tag{45}$$

we can write the key functions defined in (34) and (35) explicitly:

$$\begin{aligned}
\Gamma(w;a) = \int_0^{-\log w} & \log(1+a\phi)\,e^{-\phi}\,d\phi \\
= \quad & -x\log(1-a\log x) + xe^{\left(\frac{1}{a}-\log x\right)}\,\mathcal{E}_i\left(-\left(\frac{1}{a}-\log x\right)\right) \\
& -e^{\left(\frac{1}{a}\right)}\,\mathcal{E}_i\left(-\frac{1}{a}\right) = -x\log\left(1-a\log x\right) \\
& +e^{\left(\frac{1}{a}\right)}\left[\mathcal{E}_i\left(-\Big(\frac{1}{a}-\log\,x\Big)\right) - \mathcal{E}_i\left(-\frac{1}{a}\right)\right]\,.
\end{aligned} \tag{46}$$

$$\begin{aligned}
\Theta(w;a) &= \int_0^{-\log w} \frac{1}{1+a\phi}\,e^{-\phi}\,d\theta \\
&= \frac{e^{\left(\frac{1}{a}\right)}}{a}\left\{\mathcal{E}_i\left(-\left(\frac{1}{a}-\log x\right)\right) - \mathcal{E}_i\left(-\tfrac{1}{a}\right)\right\}\,.
\end{aligned} \tag{47}$$

7. DISCUSSION

In this analysis, we approached the outage issue of a joint detector by first ranking the received powers, following the approach in [5]. At the limit of large systems, the randomness manifests itself only through the actual identity of the successfully operating users. The fraction of successfully decoded user actually converges to a deterministic constant. Hence, the detector does not have to seek what the surrounding of the set of the reliably decoded users is, as the proportion of successfully decoded users becomes deterministic. Yet, the variance can be an important factor as $J \to \infty$, so the receiver needs to carry on with the elimination process at the vicinity of $J = y \cdot K$. This view can also be adopted for all the linear receivers considered in [3]. See also [4]. Clearly, the results of [3] will follow but now with the interpretation of $y = J/K$ designating the availability probability. (See [5] for a matched filter single user receiver in case of no spreading.)

We have assumed throughout that the fading channel state information is ideally provided to the receiver. Clearly, in the case of no dynamics this can be relaxed as this information could in principle be estimated with arbitrary accuracy.

Following the conclusions in [5, 10] we expect a dramatic effect in terms of rate-outage performance of this joint detector, even when compared to say DFE-MMSE (cancellation based) detectors capable of achieving the same overall throughput in a variety of models [3], [5], [2].

It is questionable whether this detector is optimal in a more general setting where codes are designed to account for the fact that some of the users are not supposed to be decoded (a given outage design goal), which resembles a broadcast setting [7]. Regardless of whether random spreading is used, that more general problem can be cast within the class of problems addressing combined multiple access broadcast and interference channels–a class of channels whose solution remains elusive.

8. ACKNOWLEDGEMENTS

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Dedication after King Henry VIII, Act iv. Sc. 2. by William Shakespeare.

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