The other half of western civilization: Communicating a mathematical view of nature

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http://www.princeton.edu/~wbialek/wbialek.html

Boundary conditions

First, do no harm.

For the first year, we want to create an alternative to the combination of freshman physics and chemistry ...

While we want lots of connections to biology, we don't want the responsibility of communicating the factual content of intro bio.

All relevant departments need to agree that we have delivered the equivalent of freshman physics and chemistry (+ a little CS) at some level.

Thus, students from our course will have access to the full range of majors.

As in our physics courses for majors, we build on previous mathematical experience, but will teach some of what we need as we go along.

We simplify our problem by taking students who have had a calculus course at the level of AP Calculus BC.

You can't satisfy the boundary conditions without genuine collaboration among the departments.

Faculty:

W Bialek (physics)
D Botstein (molecular biology)
B Chazelle (computer science)
JT Groves (chemistry)
M Hecht (chemistry)
L Hodges (chemistry)
D Marlow (physics)
O Troyanskaya (computer science)
EF Wieschaus (molecular biology)

Lewis-Sigler fellows: M Dunham & W Ryu

<u>many</u> teaching assistants from all departments

Freshman physics topics

Newtonian mechanics
Electricity and magnetism (up to Maxwell)
Waves
Thermodynamics and a little statistical physics
(sometimes) “Modern physics”

Freshman chemistry topics

Thermodynamics and chemical equilibrium
Reaction kinetics
A tour of the periodic table
Chemical potential, electrochemistry, ...
Orbitals, bonds, ...

There are obvious commonalities, and some more subtle relationships through the common mathematical structures

Can we organize around these more general ideas?

What kinds of mathematical structures do we use in describing nature?

Functional relations

$V = IR$, $Q = CV$, $F = -kx$, $F = -\gamma v$, ...

Dynamical models (differential equations)

$F = ma$ (in all its manifestations), $\frac{dC_A}{dt} = -kC_A C_B$, ...

Probabilistic models

$P(\text{state } s) = \frac{1}{Z}\exp\left[-\frac{E_s}{k_B T}\right]$, but also genetics, ...

Fields

Electricity and magnetism, but also diffusion, ...

(example) Six weeks on probabilistic models

Genes, combinations and probability (with some inference)
Gas laws and the Boltzmann distribution
Brownian motion and the reality of molecules
Chemical equilibria and thermodynamics
Entropy, from Carnot to Shannon

$$\begin{aligned}
\langle E \rangle &= \frac{1}{Z}\sum_s E_s \exp(-\beta E_s) \\
&= \frac{1}{Z}\sum_s \left[-\frac{\partial}{\partial\beta}\exp(-\beta E_s)\right] \\
&= -\frac{1}{Z}\frac{\partial}{\partial\beta}\sum_s \exp(-\beta E_s) \\
&= -\frac{1}{Z}\frac{\partial}{\partial\beta}Z,
\end{aligned}$$

we aimed high: mathematical sophistication at the level of our honors freshman physics course

a. Suppose that we make one measurement of the velocity of a molecule and find the results $v_x = v_1$. What is the value of the temperature that is most likely to have generated this data point?

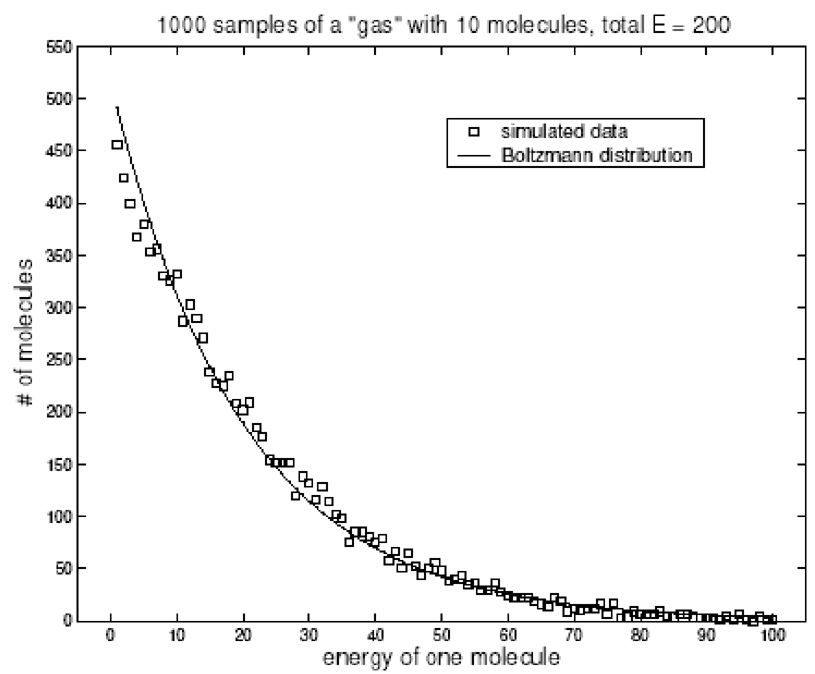
b. Let us make measurements on two molecules, with results v_1 and v_2. What is joint the probability distribution of v_1 *and* v_2? Generalize this to making n measurements.

c. Given n measurements $v_1, v_2, \cdots, v_n$, what is the value of the temperature that is most likely have generated this whole set of data?

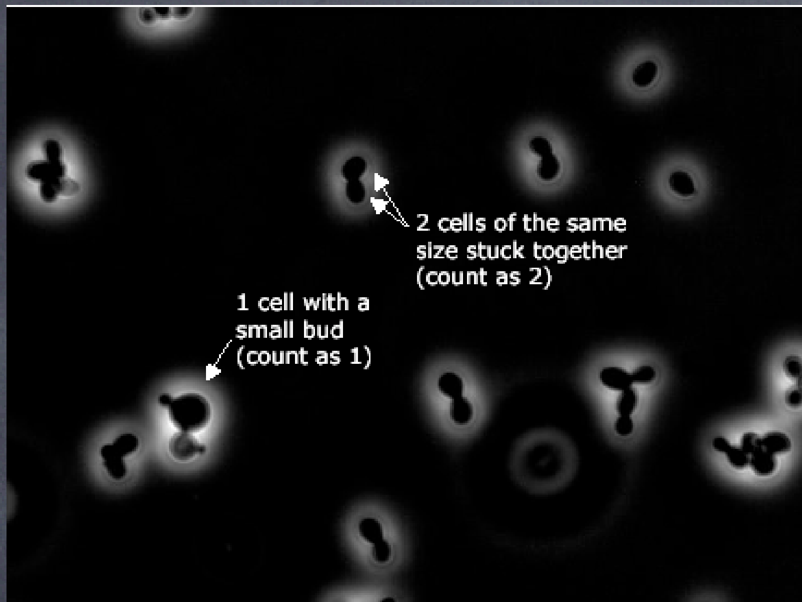
Getting some help from the computer: Exploring the combinatorics that leads to the Boltzmann distribution

In MATLAB, we can let `states` be 1000 samples of a list of 10 numbers, so it is a 10×1000 array. To pick the energies we use `rand`, with a little trick to turn the continuous distribution into random integers from 1 to 100. We have to try many times in order to be sure that we will find 1000 examples that meet our criterion for the total energy.

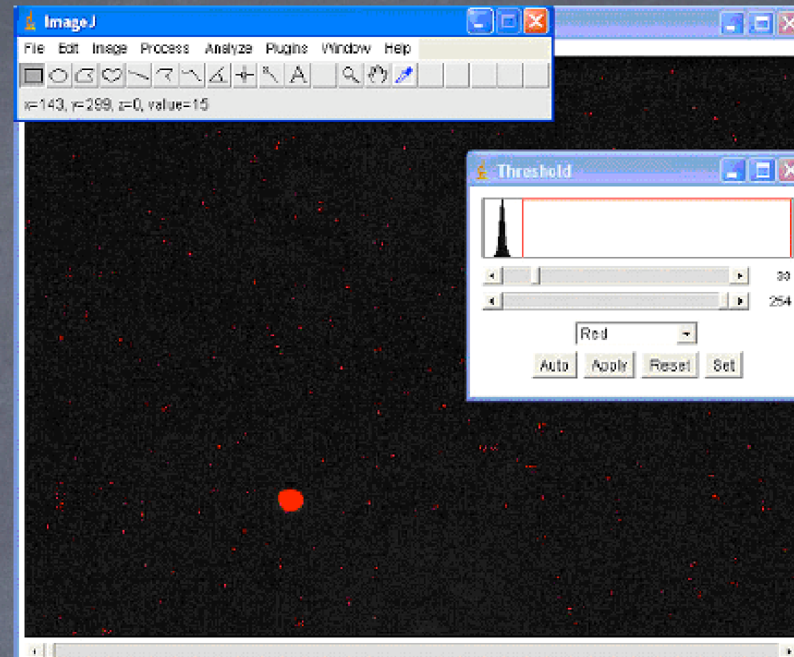
```
Nsamples = 1000;
states = zeros(10,Nsamples);
k =1;
for N = 1:1e8;
    if k <= Nsamples;
       s = ceil(100*rand(10,1));
       if sum(s)==200;
          states(:,k) = s;
          k = k+1;
       end;
    end;
end;
```



Meanwhile, in the lab ...



Delbrück-Luria experiment with yeast



Direct measurements of Brownian motion

(conveniently, the first "mechanics" lab was with viscosity)

keeping labs, lectures, problems all tied together ....