CHM/COS/MOL/PHY 231/2

Fall 2007

An integrated, quantitative introduction to the natural sciences

Problem Set 1 **Due Monday, 24 September 2007**

It might seem strange to give you a problem set before we have taught you anything. Hopefully you'll find that these problems draw on things you know from high school. We want to encourage you to think about these things in slightly different ways, and to have a little fun playing with ideas and with numbers. We expect you to collaborate in working through the problems, so don't be shocked if you don't immediately see all the answers; you'll have a chance to get help.

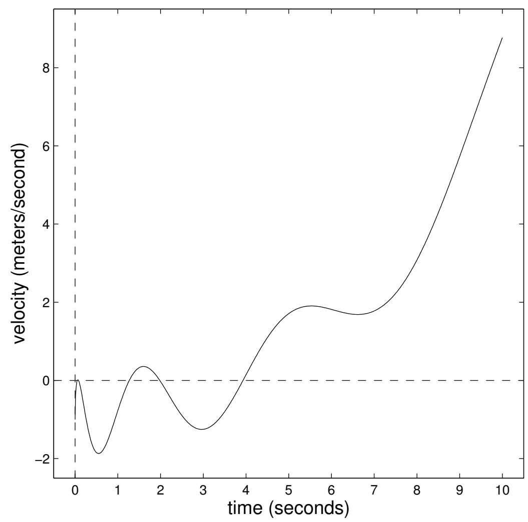

Figure 1: Velocity vs time for some hypothetical particle.

Problem 1: In Fig 1 we plot the velocity vs time $v(t)$ for an object moving in one dimension. Sketch the corresponding plots of position $x(t)$ and acceleration $a(t)$ vs time. If you need additional assumptions, please state them clearly. Be careful about units.

Problem 2: In fact the funny looking plot in Fig 1 corresponds to

$$v(t) = \sin(2\pi\sqrt{t}) + \left(\frac{t}{5}\right)^3 - \exp(-t/4). \tag{1}$$

a. Find analytic expressions for the position and acceleration as functions of time. You may refer to a table of integrals (or to its electronic equivalent), but you must give references in your written solutions.
b. Use MATLAB to plot your results in [a].[1] To get you started, here's a small bit of MATLAB code that should produce something like Fig 1:

```
t = [0:0.01:10];
v = sin(2*pi*sqrt(t)) + (t/5).^3 - exp(-t/4);
figure(1)
plot(t,v); hold on
plot([-1 11],[0 0],'k--',[0 0],[-3 10],'k--'); hold off
axis([-0.5 10.5 -2.5 9.5])
```

[1] We are going to use MATLAB repeatedly in the course. If you have your own computer, you can go to http://www.princeton.edu/licenses/software/matlab.xml to find out about how to get started; we'll also make sure that you get access to local computers that have MATLAB running on them. Hopefully, this problem is a good introduction. Note that you can type `help command` to get MATLAB to tell you how things work; for example, `help plot` will tell you something about those mysterious symbols `'k--'`.

There are just two lines of math, and the rest is to make the graph and have it look nice. How do these plots compare with your sketches in Problem 1 above?

Problem 3: It is important that you think about orders of magnitude. It might be hard to explain why something comes out to be exactly 347 (in some units), but you should be able to understand why it is ~ 300 and not ~ 30 or ~ 3000.

a. What is the typical distance between molecules in liquid water? You should start with the density of water, $\rho = 1\,\text{gram}/\text{cm}^3$.
b. Many bacteria are roughly spherical, with a diameter of $d \sim 1\,\mu\text{m}$. If you divide up the weight of the bacterium, you find that it is 50% water, 30% protein, and 20% other molecules (e.g., RNA, DNA, lipid). A typical protein has a molecular weight of 30,000 atomic mass units (or Daltons).[2] Roughly how many protein molecules make up a bacterium? A typical bacterium has genes that code for about 5,000 different proteins. On average, how many copies of each protein molecule is present in the cell?
c. In [b] you computed an average number of copies for all proteins, but different proteins are present at very different abundances inside the cell. Indeed, there are important proteins (such as the transcription factors that help to turn genes on and off) that function at concentrations of $\sim 1 - 10\,\text{nM}$. How many molecules of these proteins are present in the cell?
d. To encourage this kind of thinking, Enrico Fermi famously asked "How many piano tuners are there in America?" during a PhD exam in Physics. Similar questions include: How many students enter high school in the United States each year? How many college students each year need to become teachers in order to educate all these people? How many houses does the tooth fairy visit each night?[3] Answer these questions, and formulate one of your own.

Problem 4: Imagine that we have two flat parallel plates, each of area A, separated by a distance L, and that this space is filled with fluid. If we slide the plates relative to each other slowly, at velocity v (parallel to plates), then we will find that there is a drag force $F_{\text{drag}} = -\gamma v$ which acts to resist the motion. Intuitively, the bigger the plates (larger A) and the closer they are together (smaller L) the larger the drag, and in fact over a range of interesting scales one finds experimentally that $\gamma = \eta A/L$, where the proportionality constant η is called the viscosity of the fluid.

a. What are the units of viscosity? Instead of expressing your answer in terms of force, length and time, try to express the viscosity as a combination of energy, length and time.
b. Viscosity is something we can measure (and "feel") on a macroscopic scale. But the properties of a fluid depend on the properties of the molecules out of which it is made. So if we want to understand why the viscosity of water is $\eta = 0.01$ in the cgs (centimeter–gram–second) system of units, we need to think about the scales of energy, length and time that are relevant for the water molecules. Plausibly relevant energy scales are the energies of the hydrogen bonds between the water molecules (which you can look up), and the thermal energy $k_B T \sim 4 \times 10^{-21}$ J at room temperature (more about this later in the semester). The characteristic length is the size of an individual water molecule, or the distance between molecules. What is the range of time scales that combines with these energies and volume to give the observed viscosity? What do you think this time scale means—i. e., what event actually happens on this time scale?

[2]Recall that one atomic mass unit is a mass of one gram per mole.
[3]Admittedly, this is a bit more hypothetical than Fermi's problem.