CHM/COS/MOL/PHY 231/2

Fall 2007

An integrated, quantitative introduction to the natural sciences

Problem Set 2 **Due Monday, 1 October 2007**

Problem 1: Consider the motion of a particle subject to a drag force, as in the experiments you are doing in the lab. In the absence of any other forces (including, for the moment, gravity), Newton's equation $F = ma$ can be written as

$$M\frac{dv}{dt} = -\gamma v, \tag{1}$$

where M is the mass of the particle and γ is the drag coefficient; we assume that the velocities are small, so the drag force is proportional to the velocity. For a spherical particle of radius r in a fluid of viscosity η, we have the Stokes' formula, $\gamma = 6\pi\eta r$. Assume that the particle also has a mass density of ρ. As discussed in the lecture, the solution to Eq (1) is an exponential decay: $v(t) = v(0)\exp(-t/\tau)$, where τ is time constant determined by all the other parameters in the problem. Be sure that you understand this before doing the rest of this problem!

a. Write the time constant τ in terms of M and γ. How does τ scale with the radius of the particle?

b. Suppose that the density ρ is close to that of water, and that the relevant viscosity is also that of water. What value (in seconds) do you predict for the time constant τ when the particle has a radius $r \sim 1\,\mathrm{cm}$? What about $r \sim 1\,\mathrm{mm}$ or $r \sim 10\,\mu\mathrm{m}$? Be careful about units!

c. A bacterium like *E coli* is approximately a sphere with radius $r = 1\,\mu\mathrm{m}$. Will you ever see the bacterium moving in a straight line because of its inertia?

d. What is the relationship between the position $x(t)$ and the velocity $v(t)$? Given that $v(t) = v(0)e^{-t/\tau}$, find a formula for $x(t)$ and sketch the result. Label clearly the major features of your sketch. What happens at long times, $t \gg \tau$?

e. *E coli* can swim at a speed of $\sim 20\,\mu\mathrm{m/s}$. Imagine that the motors which drive the swimming suddenly stop at time $t = 0$. Now there are no forces other than drag, but the bacterium is still moving at velocity $v(0) = 20\,\mu\mathrm{m/s}$. How far will the bacterium move before it finally comes to rest?

Problem 2: Just to be sure that you understand first order kinetics ... If the half life of a substance that decays via first order kinetics is τ, how long do you have to wait until 95% of the initial material has decayed? Explain why this question wouldn't make sense in the case of second order kinetics.

Problem 3: A simple model of shooting a basketball is that the ball moves through the air influenced only by gravity, so we neglect air resistance. Let's also simplify and not worry about the rotation of the ball, so the dynamics is described just by its position as a function of time. Choose coordinates so the basket is at position $x = 0$ and at a height $y = h$ above the floor (in fact $h = 10\,\text{ft}$, but it's best in these problems not to plug in numbers until the end). When a player located at $x = L$ shoots the ball, it leaves his or her hand at a speed v and at an angle θ measured from the floor (i.e., $\theta = \pi/2$ would be shooting straight up, $\theta = 0$ would correspond to throwing the ball horizontally, parallel to the floor). Assume that the shooter is standing still, and the release of the ball happens at some initial height $y = h_0$ above the floor (in practice h_0 is somewhere between 5 and 7 ft, depending on who's playing).

a. Draw a diagram that represents everything you know about the problem, labeling things with all the right symbols. Notice that we are treating this as a problem in two dimensions, whereas of course the real problem is three dimensional.

b. What is the equation for the trajectory of the ball with as a function of time after the player releases it? Write your answer as $x(t)$ and $y(t)$, with $t = 0$ the moment of release.

c. A perfect shot must arrive at the point $x = 0$, $y = h$ at some time. Presumably the ball also has to traveling downward at this time. Express these conditions as equations that constrain the trajetcory $\{x(t),\, y(t)\}$, and solve to find allowed values of the speed v and angle θ.

d. Saying that the ball must be traveling downward might not be enough. In fact the ball has radius $r = 4.5''$ and the basket has radius $R = 9''$. Continuing with the assumption that we want the ball to pass perfectly through the center of the basket (that is, $x = 0$, $y = h$), what is the real condition on the trajectory?

e. The fact that the basket is bigger than the ball means that you don't have to have x exactly equal to zero when $y = h$. To keep things simple let's assume that the shot still will go so long as we get within some critical distance $|x| < x_c$ at the moment when $y = h$. Given what you know so far, what is a plausible value of x_c? Turn this condition on the end of the trajectory into a range of allowed values for v and θ. With typical values for L (think about what these are, or go out to a basketball court and measure!), how accurately does someone need to control v and θ in order to make the shot?

f. (optional) What we have done here is oversimplified. You are invited to see how far you can go in making a more realistic calculation.[1] Some things to think about are the third dimension (e.g., how accurately does the trajectory need to be "pointed" toward the basket?), and a more careful treatment of the ball going through the hoop so that you can state more precisely the condition for making the shot. If you were really ambitious you could think about shots that bounce off the backboard, but that's probably too much for now!

Problem 4: The radioactive isotope ^{14}C has a half–life of $t_{1/2} = 5730\,\text{years}$. You find two human skeletons which you suspect are about $10,000\,\text{years}$ old. The setting in which you find these skeletons suggests that they died in two events separated by roughly $20\,\text{years}$. How accurately do you need to measure the abundance of ^{14}C in the skeleton in order to test this prediction? State as clearly as possible any assumptions that are made in interpreting such measurements.

[1] You might reasonably ask why we care. The fact that people (well, some people, at least) can make these shots with high probability from many different distances is telling us something about ability of the brain to deliver precise motor commands to our muscles, since it is the action of our muscles that determine the initial conditions of the ball leaving the hand of the shooter. Although the mechanisms are biological, the constraints are physical. Exploring the constraints makes precise what the system must do in order to achieve the observed level of performance.