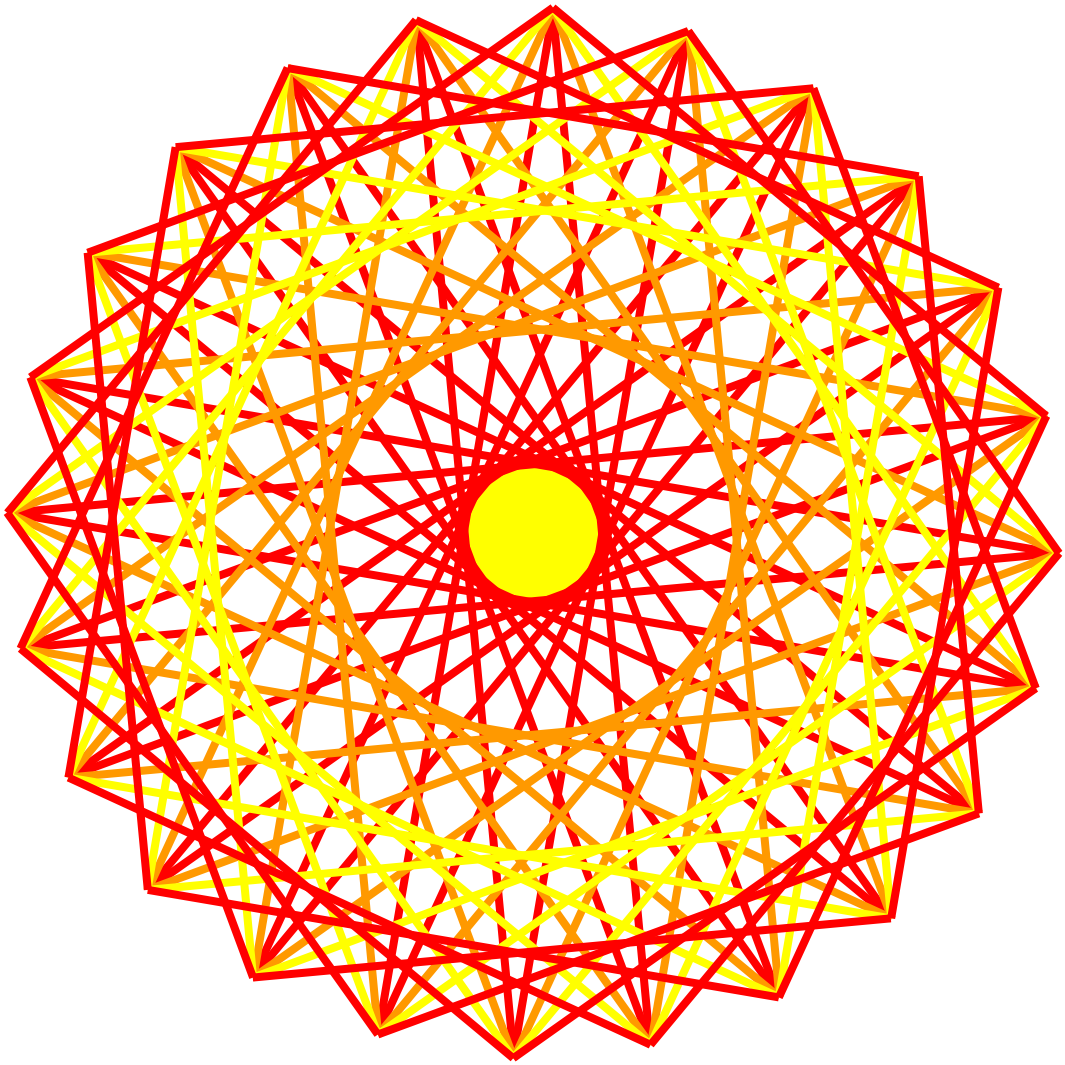

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Poster Presentation Abstracts

Chase Adams III – Howard University

"Largeness of the Set of Finite Sums of Sequences in $\mathbb{N}$*"*

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In 1974, Neil Hindman established that whenever $\mathbb{N}$ is partitioned into finitely many cells, one of these must contain $FS(\langle x_n\rangle_{n\in\mathbb{N}})=\{\sum_{t\in F} x_t : F \in \mathcal{P}_f(\mathbb{N})\}$, where $\mathcal{P}_f(\mathbb{N})$ is the set of finite subsets of $\mathbb{N}$. Subsequently, a proof of the Finite Sums Theorem involving the algebraic structure of $\beta\mathbb{N}$, the Stone-Čech compactification of $\mathbb{N}$, was provided by Fred Galvin and Steven Glazer. It established an intimate relationship between finite sums in $\mathbb{N}$ and idempotents in $\beta\mathbb{N}$.

In $(\mathbb{N},+)$, a set *A* is *syndetic* if and only if it has bounded gaps and a set is *piecewise syndetic* if and only if there exist a fixed bound *b* and arbitrarily long intervals in which the gaps of *A* are bounded by *b*. We say a set $A \subseteq \mathbb{N}$ is *central* if and only if there is some idempotent in the smallest ideal of $\beta\mathbb{N}$ that contains *A*.

Under the direction of Neil Hindman, I used combinatorial and algebraic methods to investigate when $FS(\langle x_n\rangle_{n\in\mathbb{N}})$ is piecewise syndetic, syndetic, or central.

Onobu Akogwu – Princeton University

"Instabilities and Pattern Formation in Crystal Growth"

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Several common modes of crystal growth provide particularly simple and elegant examples of spontaneous pattern formation in nature. Phenomena of interest are those in which an advancing solidification front suffers instability and subsequently reorganizes itself into a more complex mode of behavior. The systems examined here are those in which solidification is controlled entirely by single diffusion process either by flow of latent heat away from a moving interface or the analogous redistribution of chemical constituents. Convective effects and crystalline anisotropy are ignored. The Mullins-Sekerka linear stability analysis is used to analyze simple planar, spherical interfaces and special case of directional solidification. These techniques can be extended to freely growing dendrites, which leads to an understanding of side-branching and tip-splitting instabilities, which occurs naturally in snowflakes.

Caleb Ashley – Howard University

"A Classical Case of Knots in $\mathbb{R}^3$: A Theorem Attributed to Gauss"

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In a Calculus 3 course, many integral theorems are given which are related to physical phenomena, for example Gauss' Divergence Theorem. After one year in Algebraic Topology, I have encountered a variety of new characters which are tools for generalizing calculus to higher dimensions.

This poster presentation highlights the results of the classical integral equation of Gauss with a corresponding theorem which is stated and proved using characters and techniques of algebraic topology. Ultimately, the linking number of two manifolds is expressed by Gauss' Integral.

Yakira Braden – University of Wisconsin-Madison

"The Threat of Predatory Generation Control to the U.S. Electric Power Infrastructure"

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As structural changes for electric power in the U.S. and world-wide occur, market competition heightens for the provision of generation services. To increase market share, an unethical energy company owning generation might be motivated to exercise "predatory control," to undermine perceived reliability of existing competitors. Perhaps more seriously, a malicious intruder, "hacking" controller software, could induce wide-spread blackouts via predatory control.

Synchronous generators connected in an AC network form a single dynamical system with strong coupling across considerable geographic distances. This natural coupling between synchronous machines offers an opportunity for predatory control to take place. Predatory control occurs when a group of generators destabilizes competing machines in the network while minimizing the impact of the unstable mode within the group exercising control and maintaining nearly satisfactory performance.

Kareem Carr – Illinois State University

"Resonance in a Driven, Relativistic Harmonic Oscillator"

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In many undergraduate texts, the phenomenon of resonance is explored for a harmonic oscillator in the non-relativistic case. The physically impossible conclusion of the non-relativistic analysis is that as the frequency of the driving force approaches the so-called natural frequency of a harmonic oscillator, the amplitude of its oscillations approach infinity. To our knowledge, the relativistic case has not been studied. For this case, we have shown, using numerical integration, that:

1. Resonance occurs below the natural frequency of the system.
2. There is a region of bistability that does not occur in the non-relativistic case.
3. The maximum amplitude does not approach infinity at resonance.

By applying the method of multiple scales for nonlinear oscillators, and other techniques, we approximate the resonance function for this phenomenon and explore the underpinnings of the novel resonance behaviors introduced by the relativistic nonlinearity.

Abdoul Kane – University of Toronto

"Propagating Bursts in a Model of the Subthalamo-Pallidal Loop"

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We use a biophysically constrained model to study the propagation of waves in the subthalamo-pallidal network, a neuronal pacemaker involved in the planning and control of movement. Within that network the membrane potential of subthalamic and pallidal cells display activity over many time scales. This makes it difficult to carry a straightforward computational or analytical investigation. By combining perturbative and asymptotic methods we describe a general technique for reducing such complex network equations to simpler yet biophysically relevant systems. We then use the reduced model to formulate conditions for structured self sustained propagation and compute the functional dependence of the propagation speed on network parameters.

Jon Middleton–State University of New York at Buffalo

"On Rational Embeddings of Countable Ordinals"

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It is possible to embed countable ordinals into the set of rational numbers with the usual order. We prove the existence of such embeddings and exhibit various methods of explicitly representing ordinal numbers via an embedding.

Irene Moshesh – Howard University

"Image Partition Regularity of Affine Transformations"

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In his famous 1933 paper, Richard Rado studied partition regularity of systems of linear equations. That is, given a system of equations

$$a_{1,1}x_1 + a_{1,2}x_2 + \cdots + a_{1,v}x_v = b_1$$
$$a_{2,1}x_1 + a_{2,2}x_2 + \cdots + a_{2,v}x_v = b_2$$
$$\cdots$$
$$a_{u,1}x_1 + a_{u,2}x_2 + \cdots + a_{u,v}x_v = b_u$$

and given a finite partition of the set N of positive integers, could one guarantee a solution set $\{x_1, x_2, \ldots, x_v\}$ contained in one cell of the partition? In coloring terminology, one is asking whether, whenever N is finitely colored, must there be a monochromatic solution set.

In the case where the system of equations is homogeneous, in other words when $\mathbf{b} = \mathbf{0}$, Richard Rado gave the following definition:

Definition: Let $u, v \in \mathbb{N}$ and let $\mathbf{A}$ be a $u \times v$ matrix with entries from $\mathbb{Q}$. Let S be one of $\mathbb{N}$, $\mathbb{Z}$, or $\mathbb{Q}$. Then $\mathbf{A}$ is kernel partition regular over S (KPR/S) if and only if, whenever $S \setminus \{0\}$ is finitely colored, there must exist monochromatic $\mathbf{x} \in S^v$ such that $\mathbf{Ax} = 0$.

Rado was able to establish precisely when a u×v matrix $\mathbf{A}$ with entries in $\mathbb{Q}$ was kernel partition regular. This characterization was of importance since one could use kernel partition regularity to prove Schur's Theorem published in 1916, which stated that whenever $\mathbb{N}$ is finitely colored there must exist monochromatic x, y, and $x + y$ for example. However, kernel partition regularity fails to establish van der Waerden's Theorem which states: Whenever $\mathbb{N}$ is finitely colored and $\ell \in \mathbb{N}$ is given, there exist a and d such that $a, a + d, a + 2d, \ldots, a + \ell \cdot d$ are all the same color. In order to prove van der Waerden's Theorem using kernel partition regularity, one must require that the increment d also be the same color.

Neil Hindman and Imre Leader used image partition regularity, a notion introduced by Deuber, to establish van der Waerden's Theorem without requiring the increment d to be the same color. Image partition regularity is a nice alternative to kernel partition regularity because it guarantees nonconstant solutions, is computable and offers a canonical matrix representation. In 1993, Neil Hindman and Imre Leader characterized when a $u \times v$ matrix $\mathbf{A}$ with entries from $\mathbb{Q}$ is image partition regular over $\mathbb{N}$.

Just as Rado considered the kernel partition regularity of the affine transformation $\mathbf{x} \rightarrow \mathbf{A} \cdot \mathbf{x} + \mathbf{b}$, one can ask about the image partition regularity of the same transformation. This matter will be the focus of this talk.

Wilfred Ndifon – Princeton University

"Putative Self-Organized Criticality of RNA Molecules and RNA Viral Genomes"

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The distribution of the number of structural rearrangements of an RNA molecule is shown to be log-normal over short time scales, and to converge asymptotically to a power-law, with a scaling exponent $0 < \gamma < 2$, which is believed to be characteristic of self-organized critical systems. This suggests that RNA molecules and RNA viral genomes may exist in a self-organized critical state and predicts, among other things, a scale-invariant distribution of fluctuations in viral fitness during evolutionary stasis. Interestingly, this prediction is borne out by the results of virus evolution experiments. (This presentation is based on joint work with Dr. Santiago Elena of the Spanish National Research Council and Dr. Simon Levin of Princeton University).

Etienne Ogoubi – Université de Montréal

"Near-Complete Decomposability Concept for On-Chip Multiprocessor Architecture"

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Partitioning applications using *Domain Decomposition* (DD) for a multiprocessor architecture is seen as a powerful technique. Many areas of science have extensively and successfully used this technique to solve problems of great size and complexity. A part of our research tackles the *Near - Complete Decomposability* (NCD) concept, with its associated technique of aggregation of variables, to model and then solve problems based on a parallel on-chip multiprocessor architecture with reconfigurable devices. This approach is based on conjectures made from analogies between aggregates in nearly completely decomposable systems and computational sub-applications.

Olawale O. Oladehin – Princeton University

"Data Driven Strategic Gaming"

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What attributes are required to play a game well? Of course all sports require talent, but exactly how do strategies and different approaches affect or benefit a player? For this project we separate the aspects of games into three different categories—facts, tactics, and strategy. Each player must know the facts and rules of the game, they must then apply those facts to create different tactics for a specific moment in the game, and then strategy becomes the culmination of facts and tactics that a player gains over time. This strategy is the attribute that the player will use to view the game in its entirety.

Through the use of game theory, data mining, and simulation, we will approach a very popular card game — Texas Hold 'em. The approach for this project will be to create a database of basic poker knowledge — pot odds, starting hand values, etc. This basic knowledge will then be linked to several components in order to create an artificial "short-term" and "long-term" memory for the poker player. The goal is to be able to train a computer program to learn and improve at the game of poker through learning and accumulating "knowledge."

Derrick Raphael – Princeton University

"A Collective Analysis of African American Researchers in the Mathematical Sciences"

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For this project, I am conducting brief surveys of Black mathematicians attending CAARMS12 at UNC-Chapel Hill. I also plan to conduct extended interviews with various participants as well to get a more in-depth view of their path to become mathematicians. In addition to the gleaning of information from participants, I will incorporate various past studies on this issue and relevant studies to get a more informed view of these unique individuals. Overall this study seeks to find out what are the building blocks that help to produce Black mathematicians.

Charles Rogers – North Carolina State University

"The Effect of Alcohol on Neuron Firing"

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Neurons are responsible for transmitting messages throughout the body via long distance electrical signals known as action potentials. These depend on the active transport of sodium and potassium ions across the cell membrane. The effect of various drugs on the process of neuron firing is a current research interest. The Hodgkin-Huxley equations, a system of four nonlinear ordinary differential equations, mathematically model the influx and efflux of these ions across the cell membrane. In the presence of alcohol, the release of potassium ions is accelerated. We propose a modified version of these equations, which incorporates the effect of alcohol, and examine its implications through mathematical analysis in dynamical systems. We investigate the qualitative behavior and interpret the results of the steady-state solutions in the fast and fast-slow phase planes.

This is joint work with Jeannine T. Abiva, Edna S. Joseph, and Arpy K. Mikaelian, with faculty mentors Erika T. Camacho and Stephen A. Wirkus.

Stephanie Somersille – University of California at Berkeley

"Adjoints of Composition Operators on Hilbert Spaces"

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Many seemingly basic questions regarding composition operators on Hilbert spaces remain open. One such problem is finding explicit formulas for their adjoints. We will consider a new class of functions, an extension of composition, weighted composition and Toeplitz operators to include operators derived from multiple valued functions. We will discuss their properties such as boundedness and compactness. We will give a formula for the adjoints of composition operators on Hardy Hilbert spaces whose symbols are rational functions as composits of such (weighted) composition operators and backward shift operators.

Luke J. Stewart – Duke University

"Solving Numerically the Lens Equation in Stochastic Gravitational Lensing"

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Solving analytically the lens equation in gravitational lensing tends to be impossible, especially as the parameters of the lensing system are allowed to become more complicated. Therefore, numerical methods are used to solve the lens equation for set of lensed images that are produced for a given light source and lensing parameters. The goal of my project is to determine the mean and variance of the number of lensed images produced when the parameters are varied uniformly, as in the case of stochastic gravitational lensing.

Alberto Mokak Teguia – Duke University

"A Sierpiński Graph and Some of its Properties"

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The *Sierpiński fractal* or *Sierpiński gasket* Σ is a familiar object studied by specialists in dynamical systems and probability. In this paper, we consider a graph S_n derived from the first *n* iterations of the process that leads to Σ and study some of its properties, including the cycle structure, domination number and pebbling number. Various open questions are posed. This is joint work with Anant P. Godbole.

Conrad Tucker – University of Illinois, Urbana-Champaign

"Analytical Design of Product Family Cell Phone Architecture"

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Product platform design enables companies to minimize manufacturing and design costs by allowing a family of products to be developed around shared, efficient product architecture. Under this product family design paradigm, this paper introduces a method to analytically determine the optimal product platform configuration in the multi-product hierarchy that will evolve in upcoming generations of product launch. By sharing the most common and labor intensive components, the cost of a product together with its predecessors can be drastically reduced, hereby appealing more to market demand.

Multilevel, multidisciplinary optimization has become an effective alternative to solve complex, large-scale system design problems that are conventionally solved by all-in-one (AIO) approach. To link product design and product planning effectively, however, traditional single stage static formulation should be expanded to multistage formulation to model the changing product specifications and market demand. In this article, a product family design model that optimizes the overall enterprise objective spanning multiple stages of product launch is presented. At each stage, the hierarchical product family design and planning problem are modeled using analytical target cascading (ATC) and are further expanded to accommodate changing design variables and parameters for different stages. Multistage modeling allows for simultaneous and independent optimization of each individual stage system design while still maintaining the overall enterprise objective.

The motivation of this research is to explain how the ATC methodology can be extended in a multistage setting to include the optimization of engineering designs for a product family in an extremely volatile and competitive market space. To illustrate the methodology, an automotive vehicle brake system design and a cellular phone family design will be presented.

Brian Williams – University of Mississippi

"Large Circuit Pairs in Matroids"

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Scott Smith conjectured in 1979 that two distinct longest cycles of a k connected graph meet in at least k vertices when $k \geq 2$. This conjecture is known to be true for $k \leq 10$. Only the case $k \leq 6$ appears in the literature, however. Reid and Wu generalized Smith's conjecture to k connected matroids by considering largest circuits. The case $k = 2$ of the matroid conjecture follows from a result of Seymour. In addition, McMurray, Reid, Sheppardson, Wei, and Wu established an extension of the matroid conjecture for $k = 2$ and proved it for co-graphic matroids when $k \leq 6$. In his Ph.D. dissertation, McMurray established the matroid conjecture for matroids of circumference four. I establish Reid and Wu's conjecture for several classes of matroids which include those that have connectivity three, circumference five, and spanning circuits. For this talk I will focus on Reid and Wu's conjecture for k = 3.

Adrian Wilson – University of Mississippi

"Graph Groupoids and Their Topology"

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I investigate graph groupoids and the path spaces associated with their unit spaces. It was shown by Paterson and Welch that for a general directed graph $\mathcal{E}$, the path space X which is of the form $Y \cup Z$, where Y is the set of finite paths and Z the set of infinite paths in $\mathcal{E}$, is a locally compact metric space. I solved three main questions. For the first, a natural question that was asked by A. Kumjian (University of Nevada at Reno) in the case of the Cuntz graph $\mathcal{E}^{\infty}$ was how that topology relates to an earlier topology investigated by J. Renault (Orléans). I show that the two topologies are homeomorphic and so can be identified. I then discuss the graph groupoid for $\mathcal{E}$ in the general case. For this investigation, it is important to be able to use the axiomatic approach to groupoids, by showing that this is equivalent to the usual definition of a groupoid as a "small category with inverses". The proof of this is given, and is the second main question answered by my dissertation. The third main question is to construct the graph groupoid $G_{\mathcal{E}}$ for $\mathcal{E}$ and prove that it is a second countable, locally compact Hausdorff groupoid. The construction and proof of this are given in the last section of my dissertation. (The proof of the corresponding result for row finite graphs had been sketched earlier in the work of, e.g., Kumjian, Raeburn and Renault.)

Michael Young – Carnegie Mellon University

"Results in Ramsey Theory"

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A typical result in Ramsey Theory states that if a given set of objects are partitioned into a finite number of partitions, then there exists some significant object in one of the partitions. The Pigeon Hole Principle is a trivial example. The Ramsey Number R(ℓ,m) is the smallest n such that the edges of the complete graph on n vertices (K_n) can be two colored (red and blue) and contain no blue K_ℓ or red K_m. R(k,k) is known for $k < 5$. I will be presenting a new method to find bounds on $K_3 + I_3$ and $K_4 + I_4$. Finding these bounds will prove to be useful in finding R(5,5).