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Ultrafilters on the Collection of Finite Subsets of an Infinite Set

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Let J be an infinite set and let $I = \mathcal{P}_f(J)$, i.e., I is the collection of all non empty finite subsets of J. Let βI denote the collection of all ultrafilters on the set I. In this presentation, we consider $(\beta I, \uplus)$, the compact (Hausdorff) right topological semigroup that is the Stone-Čech compactification of the semigroup $(I, \uplus)$ equipped with the discrete topology. It will be shown that there is an injective map $A \to \beta_A(I)$ of $\mathcal{P}(J)$ into $\mathcal{P}(\beta I)$ such that each $\beta_A(I)$ is a closed subsemigroup of $(\beta I, \uplus)$, the set $\beta_J(I)$ is a closed ideal of $(\beta I, \uplus)$ and the collection $\{\beta_A(I) \mid A \in \mathcal{P}(J)\}$ is a partition of βI. The algebraic structure of βI will be displayed. In particular, the following results will be presented: (1) $\beta_J(I) = K(\beta I)$, i.e., $\beta_J(I)$ is the smallest ideal of βI and (2) for each non empty $A \subseteq J$, the set $\mathcal{V}_A = \cup\{\beta_B(I) \mid B \subseteq A\}$ is a closed subsemigroup of $(\beta I, \uplus)$, $\beta_A(I)$ is a proper ideal of $\mathcal{V}_A$ and $\beta_A(I) = K(\mathcal{V}_A)$ (the smallest ideal of $\mathcal{V}_A$) if and only if $J \setminus A$ is finite.