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Computing the Unitary Dual of a Reductive Lie Group: An Introduction

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A Lie group is a continuous group of symmetries. Mathematicians study them because they preserve certain structures of interest in physics, chemistry and mathematics. Symmetry can provide critical insights into a problem and lead to interesting scientific discoveries. The largest of the elementary Lie groups is called E_8. Recently, a set of parameters that describe all its representations was calculated with a super computer.

E8 is not easy to describe, so here is a more familiar example of a Lie group. If I take a sphere and insert a long needle at any point so that it goes through the sphere's center, I get an axis of rotation. Then by rotating the sphere by any small amount around that axis, I leave the sphere looking the same. I also can choose any other point on the sphere in which to insert my needle. The end result is a three dimensional (infinite) set of rotations. The set of rotations of the sphere is a three dimensional reductive Lie group.

What do we want to know about such groups? The group of rotations of the sphere also acts on other spaces in different ways. Mathematicians study all possible, so to speak, 'symmetries' of a Lie group. It is like trying to get to know something better by studying the different ways in which it interacts with its environment. We call all these possibilities the representations of the group. The representations of the rotation group are described by positive integers (discrete data). More complicated representations of other groups can be described by continuous parameters. The data that describes the representations of a group can be arranged in a matrix called the character table of the group. This is a way to classify the representations of the group.

The Atlas of Lie Groups is an international project in which a team of mathematicians is trying to understand and describe explicitly all the representations of an arbitrary reductive Lie group. Among these representations is a special set of representations that have not been completely determined - the unitary representations. This is one of the most important unsolved problems in mathematics. I will discuss how we are trying to understand that problem in order to get an explicit description of all the unitary representations for each reductive Lie group. In particular, I will describe cases when we can separate the unitary representations from the non-unitary ones. This work is supported by the American Institute of Mathematics (AIM) and by several National Science Foundation (NSF) grants.