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Four-Covering Maps for Elliptic Curves

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There has been much interest in computing the Mordell-Weil groups of elliptic curves defined over the rational numbers. In 1963 and 1965, Bryan Birch and Peter Swinnerton-Dyer made extensive computations on the ranks of such elliptic curves, leading to surprising conjectures about the relationship with certain complex functions arising from Galois representations (namely the L-series) and certain groups arising from Galois cohomology (namely the Shafarevich-Tate group). In 1997, with the advent of stronger computing power and more efficient algorithms, John Cremona extended these computations.

The computational aspects of arithmetic algebraic geometry have expanded from simply computing special values of L-series to understanding how the ranks of elliptic curves are distributed. Many of these computations involve families of quadratic twists of elliptic curves. While this method helps to keep control over how fast the conductor grows, it does not keep track of how the torsion subgroup changes. Indeed, there is no extensive data at present on how the ranks of elliptic curves are distributed according to their torsion.

In this talk, we discuss ways to gain a better understanding of the ranks of those elliptic curves having as much torsion as possible. Many of the current methods in the literature used to compute the rank employ a descent via 2-isogeny. We outline new modifications by using 4-covering maps. Much of this work was done by undergraduates in the Summer Undergraduate Mathematical Sciences Research Institute (SUMSRI) at Miami University of Ohio.