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We study  $N$ -player repeated Cournot competitions that model the determination of price in an oligopoly where firms choose quantities. These are nonzero-sum (ordinary and stochastic) differential games, whose value functions may be characterized by systems of nonlinear Hamilton-Jacobi-Bellman partial differential equations. When the quantity being produced is in finite supply, such as oil, exhaustibility enters as boundary conditions for the PDE's. We analyze the problem when there is an alternative, but expensive, resource (for example solar technology for energy production), and give an asymptotic approximation in the limit of small exhaustibility. We illustrate the two-player problem by numerical solutions, and discuss the impact of limited oil reserves on production and oil prices in the duopoly case.

This is joint work with Chris Harris (Cambridge University) and Sam Howison (Oxford University).