



# THE DISTANCE BETWEEN TWO STOCHASTIC PROCESSES

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One of the principal aims of probability theory is to explain the statistical regularity associated with a macroscopic view of uncertainty. This is accomplished by the law of large numbers and the central limit theorem. This is also accomplished by stochastic process limits, i.e. limits in which a sequence of stochastic processes converges to a limiting stochastic process. The familiar example is a sequence of appropriately scaled random walks converging to Brownian motion.

*But what is a stochastic-process limit?* To specify what we mean by convergence for a sequence of stochastic processes, as well as what we mean by rates of convergence, it suffices to define a *metric* on the space of stochastic processes, which gives the distance between any two stochastic processes that we might encounter.

Perhaps the key idea is to regard a stochastic process as a random function. Then we can define a metric on the space of stochastic processes in two steps: First, we define a metric on the space of all probability measures on an abstract metric space. Second, we define a metric on the function space containing the possible sample paths of the stochastic process. These two metrics together determine a metric on the space of stochastic processes. We review some of the specific metrics that have proved to be useful.