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COMBINATORIAL INTERPRETATIONS OF HANKEL DETERMINANTS

Naiomi T. Cameron
Department of Mathematics
Harvey Mudd College
cameron@math.hmc.edu

Given a sequence of real numbers $A = \{a_0, a_1, a_2, \ldots\}$, the *Hankel determinant* of order n of A is the determinant of the upper $n \times n$ submatrix of:

$$\begin{bmatrix} a_0 & a_1 & a_2 & a_3 & \ldots \\ a_1 & a_2 & a_3 & a_4 & \ldots \\ a_2 & a_3 & a_4 & a_5 & \ldots \\ a_3 & a_4 & a_5 & a_6 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

If A is a sequence of some combinatorial significance, it seems natural to expect that the Hankel determinants of that sequence will have a related combinatorial meaning. In fact, many authors have found this to be the case for a variety of sequences, including my favorite sequence, the Catalan numbers. In this talk, I will explain by example some of the techniques used by these authors to compute and interpret Hankel determinants, starting with the Catalan number sequence as a base case. As we will see, depending on the given sequence, these computations will yield interesting connections to other combinatorial objects, such as Riordan matrices, continued fractions, orthogonal polynomials, nonintersecting lattice paths and plane partitions. Finally, I will discuss a particular Hankel determinant sequence related to a certain symmetry class of alternating sign matrices and the questions raised by this relationship.