



DIFFERENT EXPRESSIONS OF A MATHEMATICAL OBJECT

John A. Ewell
Department of Mathematics
Northern Illinois University

For given odd primes p, q , $p > q$, the first mathematical object is the number $N = N(p, q)$ of lattice points on the interior of the rectangle having vertices

$$(0, 0), (p/2, 0), (p/2, q/2), (0, q/2)$$

in the coordinate plane. In the proper setting two different expressions of N reveal an elegant proof of the celebrated Euler-Legendre-Gauss Law of Quadratic Reciprocity.

The second mathematical object is the infinite product

$$P(x) := \prod_{n=1}^{\infty} (1 + x^{2n-1}),$$

which converges for each complex number x such that $|x| < 1$. Two different expressions of $P(x)$ reveal a new recurrence for the partition function $p(\cdot)$. Recall that $p(\cdot)$ is defined (generated) by the infinite-product expansion

$$\prod_{n=1}^{\infty} \frac{1}{1 - x^n} = \sum_{n=0}^{\infty} p(n)x^n,$$

valid for each complex number x such that $|x| < 1$.