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Convergence trading with wealth effects: an amplification mechanism in financial markets ☆

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Abstract

I study convergence traders with logarithmic utility in a continuous-time equilibrium model. In general, convergence traders reduce asset price volatility and provide liquidity by taking risky positions against noise trading. However, when an unfavorable shock causes them to suffer capital losses, thus eroding their risk-bearing capacity, they liquidate their positions, thereby amplifying the original shock. In extreme circumstances, this wealth effect causes convergence traders to be destabilizing in that they trade in exactly the same direction as noise traders. This situation is consistent with the near-collapse of Long-Term Capital Management in 1998.

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1. Introduction

Convergence trading strategies were made popular by the hedge fund Long-Term Capital Management (LTCM). A typical convergence trading strategy is to bet that the price difference between two assets with similar, but not identical, characteristics will narrow in the future. The near-collapse of LTCM in 1998 illustrates the effect of convergence traders' capital dynamics on financial market dynamics, and motivates my study of a continuous-time equilibrium model of convergence trading with wealth effects.[1] The wealth effect occurs when convergence traders suffer unfavorable shocks to their positions, their risk-bearing capacity decreases along with their wealth, and they are forced to unwind some of their positions. My model shows that the wealth effect can act as an amplification mechanism for financial market shocks.[2]

I model the equilibrium of an asset market involving three types of traders: noise traders, long-term investors, and convergence traders. Without loss of generality, the risky "asset" can be thought of as a synthetic spread position involving a long position in one underlying asset and a short position in another underlying asset. Examples of the risky asset are a spread position between two stocks, a spread position between a mortgage bond and a U.S. Treasury bond with similar maturity, or a spread position between off-the-run and on-the-run U.S. Treasury bonds. Consistent with this interpretation, the risky asset is assumed to have constant fundamental (cash flow) volatility. Noise traders create exogenous, stochastic supply shocks in the market and their trading is assumed to be mean reverting in the same way as in Campbell and Kyle (1993) and Wang (1993). The mean reversion of noise trading creates an opportunity for convergence trading. Long-term investors are modeled as a

[1] Fung and Hsieh (1999), Perold (1999), and Gatev et al. (1999) offer detailed descriptions of the market practices and risk profiles of convergence trading, and a report by the Bank for International Settlements (BIS, 1999) documents financial market conditions during the period of the near-collapse of LTCM. Campbell and Kyle (1993) offer an equilibrium model of convergence trading without wealth effects.

[2] Similar amplification mechanisms have been studied before under different contexts. Shleifer and Vishny (1992) study the effect of leverage on corporate asset sales. Stein (1995) studies the effect of homeowner's equity on housing prices. Kiyotaki and Moore (1997) and Krishnamurthy (1998) study amplification caused by the fluctuation of producers' collateral values (land prices).

group of prudent investors pursuing a robust, long-term, value-based investment strategy. This strategy requires long-term investors to demand the risky asset in proportion to the difference between its fundamental value and its price. This difference represents the net present value of profits to long-term investors in a worst-case scenario when they have to hold the asset forever and collect all the future cash flows, rather than trade out of the position early. The trading by long-term investors provides convergence traders with an exit strategy when they need to liquidate their positions as a result of capital losses.

This paper focuses on the behavior of a group of fully rational and perfectly competitive convergence traders who aggressively exploit the short-term opportunities created by noise traders. To capture two basic elements in convergence trading, i.e., Sharpe ratios and capital, I assume logarithmic utility for convergence traders. Sharpe ratios represent the trading opportunity, while capital represents the risk-bearing capacity of convergence traders. With continuous-time trading, logarithmic utility implies a trading strategy that dynamically exploits the Sharpe ratio in the market and at the same time prevents wealth from dropping down to zero. With this strategy, convergence traders always take risky positions proportional to their wealth. Their expected trading profits (in percentage terms) and percentage portfolio variance are both determined by the squared Sharpe ratio.

In equilibrium, there are two state variables. One is the level of noise trading, representing the total supply of risk in the market. The other is convergence traders' aggregate wealth, representing their total risk-bearing capacity. The equilibrium is derived as a fixed-point problem in a representative convergence trader's trading strategy. This fixed-point problem is equivalent to a nonlinear second-order partial differential equation. Numerical solution of the equilibrium (using a projection technique) makes it possible to discuss the amplification mechanism caused by the wealth effect of convergence traders. Simulation exercises make it possible to study the stationary distribution of the equilibrium.

The portfolio rebalancing of convergence traders associated with their wealth changes has interesting implications for the effects of fundamental shocks and noise trading shocks. The analysis of fundamental shocks is easy: an unfavorable fundamental shock always causes convergence traders to lose money, and the resulting wealth effect always causes convergence traders to amplify the shock. The analysis of noise trading shocks is more complicated. A shock that pushes noise trading further away from its mean makes convergence trading more profitable (higher Sharpe ratios), and this induces convergence traders to take larger positions relative to their wealth. This is a substitution effect, and it has been studied by Campbell and Kyle (1993). Such a noise trading shock also causes convergence traders to lose money on their current positions, and therefore leads them to reduce their positions. This is the wealth effect, and it operates in the opposite direction from the substitution effect. Most of the time, the substitution effect dominates the wealth effect, and convergence traders take larger positions in response to increased noise trading, in which case convergence trading reduces the effect of noise trading shocks on prices and improves market liquidity. Occasionally, however, when the wealth effect dominates the substitution effect, convergence traders liquidate their positions in response to increased noise trading. In this case, convergence traders become destabilizing in the sense that they trade in exactly the same direction as noise traders, which amplifies the price effect of noise trading shocks and reduces market liquidity. This situation is exactly illustrated by the LTCM crisis in 1998. When LTCM suffered large capital losses, they had to liquidate some of their positions, thereby causing liquidity to dry up and volatility to rise.

The interaction between the wealth effect and the substitution effect allows us to discuss an old economic question: Do speculators stabilize prices? A common-sense argument has been strongly expressed by Friedman (1953) that speculators always buy cheap and sell dear, and so always stabilize prices. My model suggests otherwise.[3] The wealth effect can cause speculators to unwind their positions by buying when prices are high and selling when prices are low after suffering large capital losses.

The amplification mechanism created by the convergence traders' wealth effect can cause asset price volatility to be excessive in the sense that it is too large to be explained by the volatility of asset fundamentals in a simple present value model with a constant discount rate. The amplification mechanism can also cause asset price volatility to be time-varying. The extreme liquidity risks caused by the convergence traders' wealth effect present a great challenge to the risk management of highly leveraged financial institutions. My model provides a way for risk managers to study market equilibrium dynamics and to forecast the extreme risks caused by the amplification mechanism. The wealth process of convergence traders is endogenously determined in a stationary equilibrium where convergence traders' trading profits can break even with their consumption in the long run. This allows a discussion of the determinants of convergence trading activity across different markets.

The wealth effect studied in this paper arises from the nature of convergence trading. Convergence traders usually specialize in a limited number of assets or

[3] Shleifer and Vishny (1997) have similarly observed that risk aversion can be a cause of destabilizing speculation, but they do not offer a formal model to characterize the mechanism. Several other explanations of destabilizing speculation have been offered in the literature. Hart and Kreps (1986) point out that speculators buy when chances of price appreciation are high, which may or may not be when prices are low. Stein (1987) suggests that an information externality can cause speculation to reduce social welfare. De Long et al. (1990) suggest that the incentive for rational speculators to take advantage of irrational positive feedback traders can also cause speculation to be price-destabilizing.

strategies due to the information costs of participating in many markets, as pointed out by Merton (1987) and Shleifer and Vishny (1997). Moreover, they often use high leverage to exploit short-term opportunities available to them. As a result, their portfolios can be both undiversified and highly levered, so shocks in these markets can cause large fluctuations in convergence traders' wealth.

Another important feature of market practices is that capital does not flow perfectly across different markets or strategies. When convergence traders suffer capital losses, it is especially difficult for them to raise new capital to maintain their positions, and it is equally difficult for them to find existing convergence traders to buy out their positions without deep discounts. As suggested by Shleifer and Vishny (1997) and Shleifer (2000), agency problems can cause this type of imperfect capital flow to professional traders (such as hedge funds). As these professional traders lose money, they also lose credibility among their investors, and can thus face difficulty raising new capital or even experience withdrawals from their investors, as occurred with LTCM during the financial market crisis of 1998. According to the New York Times article by Michael Lewis (Jan. 24, 1999, magazine section), LTCM had a very hard time raising new capital to maintain their positions after they suffered large capital losses.

My model captures this idea of imperfect capital flow by assuming that there is no capital flow into the asset market through entry of new convergence traders or through additional fundraising by existing convergence traders. In this way, the model extends the limits of arbitrage argument in Shleifer and Vishny (1997) to study market dynamics.

Kyle and Xiong (2001) study financial market contagion using a framework similar to this paper with two risky assets. They show that the wealth effect of convergence traders can also act as a mechanism for volatility to be transmitted from one market to another. When convergence traders suffer large capital losses, they need to unwind their positions across their whole portfolio, therefore causing the prices of fundamentally unrelated assets to move together. Aiyagari and Gertler (1998) and Gromb and Vayanos (2000) study equilibrium models with margin-constrained traders. Their results are similar in spirit in the sense that capital constraints can cause excess volatility in asset markets. The wealth effect is also studied in the context of portfolio insurance by Basak (1995) and Grossman and Zhou (1996). Omberg (1997) discusses a potential interaction between the wealth effect and the substitution effect with price jumps in a partial equilibrium model. In other related studies, Basak and Croitoru (2000) study riskless arbitrage trading with position limits, Loewenstein and Willard (2000) study the trading of hedge funds with credit constraints, and Liu and Longstaff (2000) study a portfolio choice problem of arbitrageurs when arbitrage opportunities follow an exogenous Brownian bridge process.

This paper proceeds as follows. Section 2 introduces the structure of the model. Section 3 derives the asset return process and the convergence traders' optimal policies, and then sets up the equilibrium as a fixed-point problem. Section 4 illustrates the equilibrium using a numerical example. Section 5 discusses some implications of the model. Section 6 concludes the paper.

2. The model

The model studies the equilibrium of an asset market (one sector of the aggregate financial markets) in a continuous-time framework with an infinite time horizon. There are three types of traders in this asset market. Noise traders create stochastic and mean-reverting supply shocks to the risky asset. Convergence traders are fully rational with logarithmic utility. They trade the risky asset to exploit the short-term opportunities created by noise traders. Long-term investors hold the risky asset based on the difference between the price and the fundamental value. Since the model is only concerned with one sector of the aggregate financial markets, it treats the interest rate as exogenous and assumes that all market participants can borrow and lend their capital at a constant risk-free rate r.

2.1. Asset fundamentals

The asset is risky and its cash flows D are assumed to follow an observable, mean-reverting stochastic process,

$$\mathrm{d}D = -\lambda_D(D - \bar{D})\mathrm{d}t + \sigma_D \mathrm{d}z_D \tag{1}$$

with constant volatility σ_D, constant rates of mean reversion λ_D, and known long-term mean $\bar{D}$. The term $\mathrm{d}z_D$ represents a fundamental shock, which follows a Wiener process. Thus, the dividend process has a normal distribution.

In the context of convergence trading, this risky asset can be regarded as a spread position between the two underlying assets involved. One typical example of such spread positions given by Perold (1999) is a long position in the stock of Shell and a short position in the stock of Royal Dutch. These two stocks have closely related fundamentals. Therefore, this spread position can reduce fundamental risks involved in the trade. The dividend from this spread position is thus the difference between the dividends from the two assets involved, and it can be either positive or negative consistent with a normal distribution.

The fundamental value F of the risky asset (not to be confused with the market price P described later) is defined as the expected payoff to a risk-neutral investor discounted at the risk-free rate:

$$F = E_t \int_0^{\infty} e^{-rs} D(t+s)\,ds \\ = \frac{\bar{D}}{r} + \frac{D(t) - \bar{D}}{r + \lambda_D}. \quad (2)$$

The second equation is just a variation of Gordon's growth formula.

The risk-neutral excess returns process dQ_F corresponding to the fundamental values (not to be confused with the actual returns process dQ discussed later) is given by the hypothetical mark-to-market profits of a fully levered one-share portfolio, which collects the dividend and pays the risk-free rate of interest:

$$dQ_F = dF + (D - rF)\,dt. \quad (3)$$

Using the cash flow process and fundamental price process above, it is straightforward to show that the risk neutral mark-to-market profit on the risky asset follows a Brownian motion with constant volatility, defined as σ_F:

$$dQ_F = \frac{\sigma_D}{r + \lambda_D}\,dz_D = \sigma_F\,dz_D. \quad (4)$$

Since this excess return process is associated with the risk-neutral price (fundamental value) process, there is no risk premium or drift term. The equilibrium discussed below depends on the fundamental cash flow process only through the parameter σ_F. In other words, the specific rate of mean reversion and the long-term mean of cash flows do not affect the equilibrium except through their effect on σ_F. Furthermore, the risky asset can be scaled arbitrarily (as in a stock split) to give any level of fundamental volatility, without changing the equilibrium.

2.2. Market-clearing condition

The equilibrium price for the risky asset (as opposed to the fundamental value discussed above) arises from trading by three different types of market participants: noise traders, long-term investors, and convergence traders. If noise traders supply θ shares of the risky asset while long-term investors demand X_L shares and convergence traders demand X shares, then the market-clearing condition (which holds at every point in time) can be written as

$$X_L + X = \theta. \quad (5)$$

Following Campbell and Kyle (1993) and Wang (1993), the supply of noise traders is assumed to follow an exogenous mean-reverting process,

$$d\theta = -\lambda_\theta(\theta - \bar{\theta})\,dt + \sigma_\theta\,dz_\theta \quad (6)$$

with long-term mean $\bar{\theta}$, mean-reversion parameter λ_θ, and standard deviation σ_θ. The notation dz_θ denotes a noise trading shock, which follows a Wiener process. Noise trading shocks are independent of the fundamental shocks.

2.3. Long-term investors

Long-term investors have the following aggregate demand curve for the risky asset:

$$X_{\mathrm{L}} = \frac{1}{k}(F - P), \tag{7}$$

where k, with $k > 0$, denotes the slope of the downward-sloping demand function. This demand curve is proportional to the spread between the fundamental value F and the actual price P. Graham (1973) calls this spread a safety margin, and it measures the net present value of profits to long-term investors in a worst-case scenario when they have to hold the asset forever and collect all the future cash flows, rather than trade out of the position early. If we assume that long-term investors have exponential utility and use this (suboptimal) strategy at the same time, the slope of this demand curve is determined by

$$k = \phi\sigma_F^2, \tag{8}$$

where ϕ is the long-term investors' aggregate absolute risk aversion and σ_F^2 is the variance of fundamental shocks. The exact number of long-term investors is not specified here, but it is incorporated in the aggregate risk aversion. When there are more long-term investors, the aggregate risk aversion will be lower, therefore the slope of the aggregate demand curve will be smaller.

According to this demand curve, long-term investors always provide liquidity in the market. When the price falls below the fundamental value, long-term investors will buy the asset. When the price falls further below the fundamental value, long-term investors will buy more. The slope of the demand curve k measures the liquidity provided by long-term investors. Larger k means a steeper demand curve, and thus represents less liquidity from long-term investors. Long-term investors are assumed to have deep pockets, i.e., they have no wealth constraints (consistent with exponential utility). As shown later, the liquidity from long-term investors provides an exit strategy for convergence traders during crises.

While this long-term strategy is profitable, it is not optimal. Because the inventory of noise traders θ changes randomly in a mean-reverting manner, a short-term strategy can improve the portfolio performance of the long-term investors. A short-term strategy implies trading more aggressively against noise trading than with the long-term strategy used by long-term investors. This creates an opportunity for convergence traders to prosper in the market by providing extra liquidity to noise traders.

The rationale behind the long-term strategy is its robustness. Graham (1973) observes that a short-term strategy that improves upon the long-term strategy for a given noise trading process can be subject to large model specification risks. Therefore, he advocates a long-term strategy to exploit long-term opportunities (measured by the safety margins) in the market. This view is consistent with recent studies on the aversion to model uncertainty by Epstein and Wang (1994) and Hansen et al. (1999). Since the focus of my model is on the effect of convergence traders, I assume a simplistic trading rule for long-term investors.

2.4. Convergence traders

Convergence traders behave optimally in response to a given noise trading process. Intuitively, this means that they can make profits not only by purchasing the risky asset when it is priced below fundamentals, but they can also make short-term profits by taking the other side of transitory noise trading. Due to the aggressive nature of convergence trading, convergence traders are subject to large wealth fluctuations with the leverage they are induced to use. This makes their wealth effect an important part of convergence trading. In order to capture the two sides of convergence trading, i.e., short-term opportunity and the wealth effect, convergence traders are assumed to be a continuum of competitors who maximize an additively separable logarithmic utility function with an infinite time horizon and a time-preference parameter ρ:

$$J(t) = \max E_t \int_0^\infty \mathrm{e}^{-\rho s} \ln(C_{t+s})\,\mathrm{d}s. \tag{9}$$

With logarithmic utility, convergence traders have decreasing absolute risk aversion. As their wealth approaches zero, convergence traders become infinitely risk averse. To prevent their wealth from becoming negative, convergence traders use the liquidity provided by long-term investors to liquidate their risky positions as their wealth decreases. Note that there can be no equilibrium with only convergence traders and noise traders (i.e., no long-term investors), because wealth cannot be guaranteed to stay positive for convergence traders when fundamentals have a normal distribution.

Since logarithmic utility prevents convergence traders' wealth from falling below zero in this model, there are no bankruptcy risks for convergence traders, and creditors are always willing to lend money to them at the risk-free rate r. The trading opportunity to convergence traders is the excess return process

$$\mathrm{d}Q = \mathrm{d}P + (D - rP)\,\mathrm{d}t \tag{10}$$

with P denoting the price of the risky asset (not the fundamental value F). The process $\mathrm{d}Q$ represents the cash flow to a fully levered portfolio long one share of the risky asset.

I assume that convergence traders specialize in trading only in this asset market. Their budget constraint is then

$$\mathrm{d}W = X\,\mathrm{d}Q + (rW - C)\,\mathrm{d}t, \tag{11}$$

where W denotes their wealth, C denotes their consumption, and X denotes their demand for the risky asset in shares. Consumption C can also be interpreted as a dividend paid to investors in the convergence traders' funds. The convergence traders' demand X and consumption C are derived from their utility optimization problem. The budget constraint in Eq. (11) incorporates the assumption that convergence traders will not receive any capital inflow at any time. This assumption is motivated from imperfect capital inflow to convergence trading discussed earlier.

3. Equilibrium

This paper studies a symmetric and perfectly competitive equilibrium. In this equilibrium, each individual convergence trader is a price taker. Given everyone else's trading strategy, an individual convergence trader will optimally choose the same strategy. This equilibrium condition implies that a representative convergence trader's trading strategy solves a fixed-point problem.

In this model, there are two sources of uncertainty, the fundamental shock ($\mathrm{d}z_D$) and the noise trading shock ($\mathrm{d}z_\theta$). Since there is only one risky asset, markets are incomplete. There are also two state variables, the level of noise trading θ and the aggregate wealth of convergence traders W. The variables θ and W represent, respectively, the total supply of risk and the risk-bearing capacity of convergence traders. Due to logarithmic utility, the total wealth of all convergence traders can be aggregated to represent their aggregate risk-bearing capacity. Unlike models with constant absolute risk aversion, the exact number of convergence traders is not important for the equilibrium.

The fundamental variable D is not a state variable. Due to the normal distribution assumption for the cash flow process, the fundamental risk is constant and the variable D only measures the level of fundamental value. Since long-term investors trade on long-term opportunities measured by the difference between the price and the fundamental value, while convergence traders trade on short-term opportunities measured by the Sharpe ratio (as shown later by the model), the level of fundamental value has no effect on the trading strategies of either long-term investors or convergence traders. Therefore, the variable D has no effect on the equilibrium.

The only function to be solved in the equilibrium is the convergence traders' trading strategy or demand function for the risky asset $X(\theta, W)$. This equilibrium trading strategy should solve the convergence traders' utility optimization problem, while simultaneously satisfying the market-clearing condition. With logarithmic utility, the convergence traders' consumption function is trivial, because they always consume their wealth at a constant rate equal to their time-preference parameter. Given convergence traders' trading strategy $X(\theta, W)$, the price function of the risky asset can be derived by plugging the long-term investors' demand function into the market-clearing condition:

$$P(\theta, W, F) = F - k(\theta - X(\theta, W)). \tag{12}$$

This equation reveals the key feature of the model, which is that convergence traders' wealth dynamics influence the asset price dynamics. Actually, the wealth dynamics and the price dynamics need to be determined simultaneously in the equilibrium.

The equilibrium can be set up in three steps. The first step is to derive the asset return process given the convergence traders' trading strategy. The second step is to derive a representative convergence trader's optimal investment and consumption policies given the asset return process. Finally, the equilibrium involves solving a fixed-point problem which is a nonlinear second-order partial differential equation. This equation can be solved numerically.

3.1. Asset return process

The asset return process $\mathrm{d}Q$ in Eq. (10) can be expressed in terms of a risk-premium term and two volatility terms associated with the two sources of risk, $\mathrm{d}z_D$ and $\mathrm{d}z_\theta$:

$$\mathrm{d}Q = \mu^Q(\theta, W)\,\mathrm{d}t + \sigma_D^Q(\theta, W)\,\mathrm{d}z_D + \sigma_\theta^Q(\theta, W)\,\mathrm{d}z_\theta, \tag{13}$$

where μ^Q denotes the risk premium and σ_D^Q and σ_θ^Q denote the two volatility components. The risk premium and the volatility components are functions of the two state variables W and θ.

The wealth effect shows up through the simultaneous relation between convergence traders' wealth W and the return process $\mathrm{d}Q$. On the one hand, any shocks to $\mathrm{d}Q$ (either fundamental shocks, $\mathrm{d}z_D$, or noise trading shocks, $\mathrm{d}z_\theta$) can change convergence traders' wealth W through their budget constraints as in Eq. (11) when they are taking some risky positions ($X \neq 0$). On the other hand, fluctuations of the convergence traders' wealth induce fluctuations of their local risk aversion due to logarithmic utility. If the risk premium and volatility of convergence traders' risky positions were to remain unchanged, the convergence traders would need to rebalance their risky positions.[4] The rebalancing can further move the asset prices through the market-clearing condition as in Eq. (12). Therefore, any shock to dQ can be fed back to itself, and the mechanism of this feedback effect is through the convergence traders' risk aversion. This feedback effect is exactly the wealth effect studied in this paper. As shown later, this wealth effect always amplifies original shocks.

The wealth effect appears as a common factor in the expressions of both the risk premium, μ^Q, and the two volatility components, σ_D^Q and σ_θ^Q of the excess return process dQ. This factor $A(\theta, W)$ measures the magnitude of the wealth effect, and is defined as

$$A(\theta, W) = \frac{1}{1 - kX(\theta, W)X_W(\theta, W)}. \tag{14}$$

The subscripts θ and W denote partial derivatives of a function with respect to θ or W, i.e., X_W is the derivative of X with respect to W, and X_{WW} is the second derivative of X with respect to W.

The excess return process dQ is derived in Appendix A with the drift and volatility terms given by

$$\begin{aligned}\mu^Q(\theta, W) = \{&k\lambda_\theta(\theta - \bar{\theta})(1 - X_\theta(\theta, W)) + kX_W(\theta, W)[rW - C(\theta, W)] \\ &+ rk(\theta - X(\theta, W)) + \frac{k\sigma_\theta^2}{2}X_{\theta\theta}(\theta, W) \\ &+ \frac{k[\sigma^W(\theta, W)]^2}{2}X_{WW}(\theta, W) \\ &+ k\sigma_\theta\sigma_\theta^W(\theta, W)X_{\theta W}(\theta, W)\}A(\theta, W),\end{aligned} \tag{15}$$

$$\sigma_D^D(\theta, W) = \sigma_F A(\theta, W), \tag{16}$$

$$\sigma_\theta^Q(\theta, W) = -k\sigma_\theta(1 - X_\theta(\theta, W))A(\theta, W). \tag{17}$$

Appendix A also gives expressions for the convergence traders' aggregate wealth process.

Eq. (16) shows that the factor A measures the amplification of fundamental shocks due to the wealth effect. Therefore, this amplification factor has the same shape as the fundamental component of the return volatility. Eq. (17) gives the noise trading component of the return volatility. It has three factors. The first factor, $-k\sigma_\theta$, represents the effect of noise trading shocks on return volatility when there are no convergence traders in the market and noise trading shocks are buffered only by long-term investors. This factor can be used as a benchmark to evaluate the effect of convergence traders on noise

[4] The risk premium and volatility do change with convergence traders' wealth in equilibrium as discussed in Section 4. I ignore them here for the sake of argument.

trading shocks. The second factor, $1 - X_\theta$, represents the tendency for convergence traders to reduce noise trading shocks if they are not wealth constrained. This is a substitution effect. Due to the mean reversion of noise trading, convergence traders tend to increase (reduce) their demand X when noise trading supply θ goes up (down). For profits to increase, the changes in their demand must be less than the changes in θ. Therefore, $1 - X_\theta$ is always between zero and one. The third factor of Eq. (17), $A(\theta, W)$, represents the amplification of noise trading shocks by the wealth effect. The wealth effect forces convergence traders out of their positions in response to unfavorable noise trading shocks. It therefore operates in the opposite direction from the substitution effect. The net effect of convergence traders on noise trading shocks is determined by the product of the second factor and the third factor. If this product is below (above) one, convergence traders reduce (amplify) noise trading shocks.

Market liquidity can be measured as $\partial P/\partial\theta$, the magnitude of price changes caused by the innovations in asset supply shocks. It is easy to derive that

$$\frac{\partial P}{\partial \theta} = -k(1 - X_\theta)A(\theta, W) = \frac{\sigma_\theta^Q}{\sigma_\theta}. \tag{18}$$

Since σ_θ is a constant, the noise trading component of return volatility σ_θ^Q measures the amount of liquidity in the market. When noise trading shocks can cause large asset price fluctuations (large σ_θ^Q), there is little market liquidity. When noise trading shocks can only cause small asset price fluctuations (smaller σ_θ^Q), there is more market liquidity.

3.2. Optimal strategy of convergence traders

Given the return process $\mathrm{d}Q$ to an individual convergence trader, the value function J is a function of wealth W^i and the two state variables W and θ:

$$J(W^i, \theta, W) = \max_{\{X^i, C^i\}} E_t \int_0^\infty \mathrm{e}^{-\rho s} \ln(C^i_{t+s})\, \mathrm{d}s. \tag{19}$$

Note that W^i measures the individual convergence trader's wealth, while W represents the aggregate wealth of all convergence traders. The optimal consumption and portfolio strategies can be solved using a Bellman equation (see Appendix B):

$$X^i = \frac{\mu^Q}{(\sigma^Q)^2} W^i, \tag{20}$$

$$C^i = \rho W^i. \tag{21}$$

Consumption is a constant fraction ρ of wealth, where ρ is the time-preference parameter. The consumption strategy can be interpreted as a constant dividend rate. The trading strategy is also proportional to the convergence trader's wealth, because logarithmic utility implies that the convergence trader's risk-bearing capacity is proportional to wealth. This trading strategy can prevent wealth from falling to zero through dynamic portfolio rebalancing. Whenever wealth drops, the convergence trader needs to liquidate some risky positions if the risk premium μ^Q and the variance $(\sigma^Q)^2$ are unchanged. As wealth approaches zero, the convergence trader becomes infinitely risk averse and takes almost no positions. In equilibrium, the existence of long-term investors in the market is crucial to the implementation of this strategy, because the liquidity from long-term investors provides a means of exit for convergence traders.

The optimal trading strategy is short-term in the sense that it only depends upon the instantaneous risk premium and the variance of the return process. This contrasts with the long-term strategy used by long-term investors. This trading strategy is also myopic, i.e., there is no hedging demand (against changes in the future investment opportunity set), as discussed in Merton (1971) and Breeden (1979). This is a well-known property of logarithmic utility, and it makes the model more tractable. I will briefly discuss the effect of hedging motives at the end of the paper.

The instantaneous mean and variance of the convergence trader's wealth growth rate are

$$E_t\left[\frac{\mathrm{d}W^i}{W^i}\right] = \left[\left(\frac{\mu^Q}{\sigma^Q}\right)^2 + r - \rho\right] \mathrm{d}t, \tag{22}$$

$$\mathrm{Var}_t\left[\frac{\mathrm{d}W^i}{W^i}\right] = \left[\left(\frac{\mu^Q}{\sigma^Q}\right)^2\right] \mathrm{d}t. \tag{23}$$

From Eq. (22), the expected trading profit in percentage terms is determined by the squared Sharpe ratio, while the expected wealth growth rate equals the expected trading profits plus return from the risk-free asset minus the consumption rate. From Eq. (23), the Sharpe ratio determines the volatility of the convergence traders' portfolio, which measures the leverage used by the trader. These two equations highlight the importance of the Sharpe ratio to convergence traders. Also, we see the great benefit of using logarithmic utility. Logarithmic utility implies an intuitive trading strategy in terms of Sharpe ratios, similar to the way in which Sharpe ratios are actually used in markets.

3.3. Fixed-point problem

In equilibrium, the trading and consumption rules, $X(\theta, W)$ and $C(\theta, W)$, should solve the log-utility optimization problem and satisfy the market-clearing condition at the same time. Since an individual convergence trader's optimal consumption and trading rules are proportional to his or her wealth, the consumption and trading rules of all convergence traders can be aggregated by replacing the individual wealth variable W^i with aggregate wealth W. Denote the aggregate optimal trading rule by $X^*(\theta, W)$ and the aggregate optimal consumption rule by $C^*(\theta, W)$. Notice that X^* and C^* are functions of the conjectured rules X and C as derived explicitly in Appendix B. It is evident that the equilibrium is equivalent to a fixed-point problem:

$$X^*(\theta, W) = X(\theta, W), \tag{24}$$

$$C^*(\theta, W) = C(\theta, W). \tag{25}$$

These fixed-point conditions represent that given convergence traders' trading and consumption rules, the optimal trading and consumption rules of a representative convergence trader should be the same. Thus, if a transversality condition holds, the calculation of the equilibrium boils down to solving a fixed-point problem.

To make the equilibrium interesting, the model assumes that the convergence traders' time-preference parameter (which is also their consumption rate) is higher than the risk-free rate ($\rho > r$). Otherwise, convergence traders could gradually accumulate wealth to infinity by investing in the risk-free asset. In the limit as wealth approaches infinity, the risky asset will be priced in a risk-neutral manner ($P = F$). This is not an interesting case for us to study. The assumption of $\rho > r$ insures that there is only limited wealth for convergence traders in a stationary equilibrium. Thus, interesting implications can be derived about the dynamics of the convergence traders' wealth process and its effect on the asset price dynamics.

No existence or uniqueness theorems are available at this point. It is conjectured that the existence of an equilibrium with a stationary distribution of wealth is guaranteed by the assumption that long-term investors have a fixed linear downward-sloping demand curve for the risky asset. Without long-term investors, it is clear that convergence traders might not be able to liquidate their positions in crises, resulting in no equilibrium. This paper uses a numerical method to find a conjectured equilibrium, and discusses the implications for convergence traders' behavior and asset price dynamics.

In general, there will be no closed-form solution to the fixed-point problem. From Eqs. (15)–(17), the solution for X^* and C^* in terms of X and C involves derivatives of X up to second order. Thus, to solve the fixed-point problem, it is necessary to solve a nonlinear second-order partial differential equation of X with two state variables (W and θ). This partial differential equation is presented in Eq. (C.4) of Appendix C. Due to its non-linearity, this partial differential equation is much more tedious than the equations in most linear equilibrium models, where all the first derivatives become constant and the second derivatives become zero. Thus, a solution to the partial differential equation is obtained numerically.

While a numerical solution of the partial differential equation is necessary, the partial differential equation does satisfy obvious boundary conditions for $W = 0$ and ∞. When their wealth is zero, convergence traders do not trade, so the boundary condition at $W = 0$ is

$$X(\theta, 0) = 0. \tag{26}$$

On this bound, the price is given by

$$P = \frac{\bar{D}}{r} + \frac{D - \bar{D}}{r + \lambda_D} - k\theta. \tag{27}$$

The innovation on the per-share return for the risky asset is $\sigma_F \, \mathrm{d}z_D - k\sigma_\theta \, \mathrm{d}z_\theta$, and the volatility of the per-share return on the risky asset is $\sqrt{\sigma_F^2 + (k\sigma_\theta)^2}$. This is the price volatility without convergence traders, and it will be used later as a benchmark to evaluate the impact of convergence traders on price volatility.

When wealth approaches infinity, the risk premium is driven toward zero, i.e., the risky asset is priced in a risk-neutral manner. This drives long-term investors out of the market, and convergence traders absorb all of the noise trading. The boundary condition for $W = \infty$ is

$$X(\theta, \infty) = \theta. \tag{28}$$

On this bound, the price is equal to the fundamental value $P = F$, where F is given in Eq. (2). The innovation on the per-share return for the risky asset is $\sigma_F \mathrm{d}z_D$, and the volatility of the per-share returns on the risky asset is σ_F.

4. A numerical illustration of the equilibrium

I solve the equilibrium numerically using a projection method. The basic idea is to approximate the equilibrium trading strategy of convergence traders by rational functions using Chebyshev polynomials. The details of this numerical method are discussed in Appendix D. For different parameter sets, the calculated equilibria have similar qualitative features. Table 1 shows a summary of some of these equilibria. To illustrate the equilibrium, I choose the following values for the seven parameters needed to describe the model: $\sigma_F = 0.3, \bar{\theta} = 0, \lambda_\theta = 0.5, \sigma_\theta = 0.25, k = 1.0, r = 0.5\%, \rho = 2.5\%$. The time unit for these values is per year. These seven parameters describe seven features of the model. The first four of these features describe the equilibrium when there are no convergence traders (or their wealth is zero):

1. The mean of the Sharpe ratio in the risky asset is zero.

Table 1
Summary of equilibria with different parameters

The following seven parameters are needed to specify an equilibrium: σ_F is the volatility of fundamental shocks of one share of the risky asset. $\bar{\theta}$ is the mean of noise trading. λ_θ is the mean reverting speed of noise trading. σ_θ is the volatility of noise trading shocks. k is the slope of long-term investors' demand curve for the risky asset. r is the risk-free interest rate. ρ is the time-preference parameter of convergence traders.

The following variables are shown in this table: n_{u} and n_{d} are the orders of Chebyshev polynomials in the numerator and denominator of the rational function used to approximate each equilibrium demand function. The variable "Error" is the numerical error involved in the numerical solution of each equilibrium as discussed in Appendix D. $\mathrm{E}[\sigma^Q]$ is the long-run average of the total asset price volatility. $\mathrm{E}[\sigma^Q_D]$ is the long-run average of the fundamental component of the asset price volatility. $\mathrm{E}[\sigma^Q_\theta]$ is the long-run average of the noise trading component of the asset price volatility. $\mathrm{E}[W]$ is the long-run average of convergence traders' wealth. $\mathrm{E}[(\mu^Q/\sigma^Q)^2]$ is the long-run average of the squared Sharpe ratio in equilibrium. All these variables' long-run averages are estimated through Monte Carlo simulation. $2(\rho - r)$ is roughly the long-run mean of the squared Sharpe ratio as discussed in Section 4.

Panel A: Equilibrium dependence on fundamental volatility σ_F[a]

σ_F	n_{u}	n_{d}	Error	$\mathrm{E}[\sigma^Q]$	$\mathrm{E}[\sigma^Q_D]$	$\mathrm{E}[\sigma^Q_\theta]$	$\mathrm{E}[W]$	$\mathrm{E}[(\mu^Q/\sigma^Q)^2]$	$2(\rho-r)$
0.25	13	13	1.1(−3)	0.283	0.256	−0.113	0.199	3.95(−2)	4.0(−2)
0.30	12	12	7.1(−4)	0.335	0.305	−0.132	0.179	4.02(−2)	4.0(−2)
0.35	10	10	1.5(−3)	0.387	0.354	−0.151	0.144	4.03(−2)	4.0(−2)

Panel B: Equilibrium dependence on mean-reverting speed λ_θ[b]

λ_θ	n_{u}	n_{d}	Error	$\mathrm{E}[\sigma^Q]$	$\mathrm{E}[\sigma^Q_D]$	$\mathrm{E}[\sigma^Q_\theta]$	$\mathrm{E}[W]$	$\mathrm{E}[(\mu^Q/\sigma^Q)^2]$	$2(\rho-r)$
0.4	10	10	4.4(−3)	0.341	0.305	−0.148	0.158	3.84(−2)	4.0(−2)
0.5	12	12	7.1(−4)	0.335	0.305	−0.132	0.179	4.02(−2)	4.0(−2)
0.6	10	10	2.5(−3)	0.330	0.306	−0.116	0.202	3.89(−2)	4.0(−2)

Panel C: Equilibrium dependence on noise trading volatility σ_θ[c]

σ_θ	n_{u}	n_{d}	Error	$\mathrm{E}[\sigma^Q]$	$\mathrm{E}[\sigma^Q_D]$	$\mathrm{E}[\sigma^Q_\theta]$	$\mathrm{E}[W]$	$\mathrm{E}[(\mu^Q/\sigma^Q)^2]$	$2(\rho-r)$
0.15	10	10	1.0(−3)	0.331	0.302	−0.134	0.038	4.30(−2)	4.0(−2)
0.20	12	12	7.1(−4)	0.335	0.305	−0.132	0.179	4.02(−2)	4.0(−2)
0.25	13	13	3.0(−3)	0.342	0.308	−0.139	0.306	3.96(−2)	4.0(−2)
0.30	13	13	6.7(−3)	0.349	0.310	−0.149	0.434	3.96(−2)	4.0(−2)

Panel D: Equilibrium dependence on liquidity parameter k[d]

k	n_{u}	n_{d}	Error	$\mathrm{E}[\sigma^Q]$	$\mathrm{E}[\sigma^Q_D]$	$\mathrm{E}[\sigma^Q_\theta]$	$\mathrm{E}[W]$	$\mathrm{E}[(\mu^Q/\sigma^Q)^2]$	$2(\rho-r)$
0.9	10	10	1.52(−3)	0.333	0.304	−0.130	0.142	4.03(−2)	4.0(−2)
1.0	12	12	7.1(−4)	0.335	0.305	−0.132	0.179	4.02(−2)	4.0(−2)
1.1	12	12	2.99(−4)	0.338	0.307	−0.134	0.210	3.99(−2)	4.0(−2)

Table 1 (*continued*)

Panel E: Equilibrium dependence on time preference ρ[e]

ρ	n_u	n_d	Error	$E[\sigma^Q]$	$E[\sigma^Q_D]$	$E[\sigma^Q_\theta]$	$E[W]$	$E[(\mu^Q/\sigma^Q)^2]$	$2(\rho-r)$
2.0%	10	10	3.0(−3)	0.328	0.306	−0.111	0.267	3.00(−2)	3.0(−2)
2.5%	12	12	7.1(−4)	0.335	0.305	−0.132	0.179	4.02(−2)	4.0(−2)
3.0%	10	10	2.9(−3)	0.342	0.305	−0.151	0.116	5.04(−2)	5.0(−2)

Panel F: Equilibrium dependence on average noise supply $\bar{\theta}$[f]

$\bar{\theta}$	n_u	n_d	Error	$E[\sigma^Q]$	$E[\sigma^Q_D]$	$E[\sigma^Q_\theta]$	$E[W]$	$E[(\mu^Q/\sigma^Q)^2]$	$2(\rho-r)$
0	12	12	2.4(−3)	0.343	0.311	−0.139	0.250	4.04(−2)	4.0(−2)
0.5	12	12	2.3(−3)	0.333	0.315	−0.103	0.444	4.02(−2)	4.0(−2)
1.0	12	12	1.8(−3)	0.345	0.329	−0.094	0.702	4.05(−2)	4.0(−2)

Panel G: Equilibrium dependence on interest rate r[g]

r	n_u	n_d	Error	$E[\sigma^Q]$	$E[\sigma^Q_D]$	$E[\sigma^Q_\theta]$	$E[W]$	$E[(\mu^Q/\sigma^Q)^2]$	$2(\rho-r)$
5%	14	14	2.2(−3)	0.345	0.313	−0.139	0.271	5.98(−2)	6.0(−2)
6%	12	12	2.3(−3)	0.333	0.315	−0.103	0.444	4.02(−2)	4.0(−2)
7%	12	12	1.1(−3)	0.323	0.315	−0.064	0.808	2.00(−2)	2.0(−2)

[a] Every equilibrium in this panel shares the following parameters: $\bar{\theta}=0$, $\lambda_\theta=0.5$, $\sigma_\theta=0.2$, $k=1.0$, $r=0.5\%$, $\rho=2.5\%$.
[b] Every equilibrium in this panel shares the following parameters: $\sigma_F=0.3$, $\bar{\theta}=0$, $\sigma_\theta=0.2$, $k=1.0$, $r=0.5\%$, $\rho=2.5\%$.
[c] Every equilibrium in this panel shares the following parameters: $\sigma_F=0.3$, $\bar{\theta}=0$, $\lambda_\theta=0.5$, $k=1.0$, $r=0.5\%$, $\rho=2.5\%$.
[d] Every equilibrium in this panel shares the following parameters: $\sigma_F=0.3$, $\bar{\theta}=0$, $\lambda_\theta=0.5$, $\sigma_\theta=0.2$, $r=0.5\%$, $\rho=2.5\%$.
[e] Every equilibrium in this panel shares the following parameters: $\sigma_F=0.3$, $\bar{\theta}=0$, $\lambda_\theta=0.5$, $\sigma_\theta=0.2$, $k=1.0$, $r=0.5\%$.
[f] Every equilibrium in this panel shares the following parameters: $\sigma_F=0.3$, $\lambda_\theta=0.5$, $\sigma_\theta=0.25$, $k=1.0$, $r=6\%$, $\rho=8\%$.
[g] Every equilibrium in this panel shares the following parameters: $\sigma_F=0.3$, $\lambda_\theta=0.5$, $\sigma_\theta=0.25$, $k=1.0$, $\rho=8\%$, $\bar{\theta}=0.5$.

2. The standard deviation of the Sharpe ratio is 0.323 ($(r+\lambda_\theta)k\sigma_\theta/\sqrt{2\lambda_\theta[\sigma_F^2+(k\sigma_\theta)^2]}$, from Appendix A).

3. Noise traders cause the price volatility in the risky asset to be 0.391 ($\sqrt{\sigma_F^2+(k^A\sigma_\theta)^2}$), which is 30.2% higher than what it would be if noise trading volatility were zero.

4. The half-life of noise trading is 1.39 years ($\ln(2)/\lambda_\theta$).

The remaining three features show the scales of units in the equilibrium:

5. The convergence traders' wealth decreases at a rate of 2% ($\rho - r$) per year if they do not make any trading profits at all.

6. The value $\sigma_F = 0.3$ gives the units in which shares of the risky asset are measured.

7. The value $r = 0.5\%$ gives the rate at which the present value is calculated.

The equilibrium is described with graphs depicting various functions of the two state variables, wealth W and noise trading θ. Notice that both state variables have been transformed into the region of $[-1, 1]$ (see Appendix D). The domain of all graphs is a square in the transformed W, θ plane centered at the origin. Each graph fits into a rectangular box with this square as its base, and the graph is rotated so that the intersection of the graph with the vertical faces of the box indicate the behavior of the variable at extreme values of the state variables as follows:

Southeast face: Convergence traders have zero wealth.

Northwest face: Convergence traders have infinite wealth.

Northeast face: Noise traders have a four-standard-deviation short position.

Southwest face: Noise traders have a four-standard-deviation long position.

4.1. Convergence traders' demand function

Fig. 1 shows the demand function of convergence traders for the risky asset. The intersection of the graph and the southeast face is a horizontal line at zero, reflecting the boundary condition that convergence traders have a zero aggregate position when they have no wealth. The northwest face contains a 45-degree line, indicating the boundary condition that convergence traders absorb all the noise supply when they have infinite wealth. The northeast face indicates that when noise traders are big sellers, convergence traders' demand goes monotonically (but not linearly) from zero to 100% of the noise trading as their wealth goes from zero to infinity. The southwest face indicates that when noise traders are big buyers, convergence traders' supply goes monotonically from zero to 100% of the noise trading as their wealth goes from zero to infinity. On both the northeast and southwest faces, the asset offers large (positive or negative) expected returns which convergence traders exploit as their wealth permits.

4.2. The Sharpe ratio and wealth dynamics

Fig. 2 shows the squared Sharpe ratio. As discussed earlier, this variable represents convergence traders' expected trading profits measured as a percentage of their wealth. It is also the instantaneous variance of the convergence traders' wealth growth rate, therefore representing the risk of their portfolio. From the graph, the squared Sharpe ratio is zero when convergence

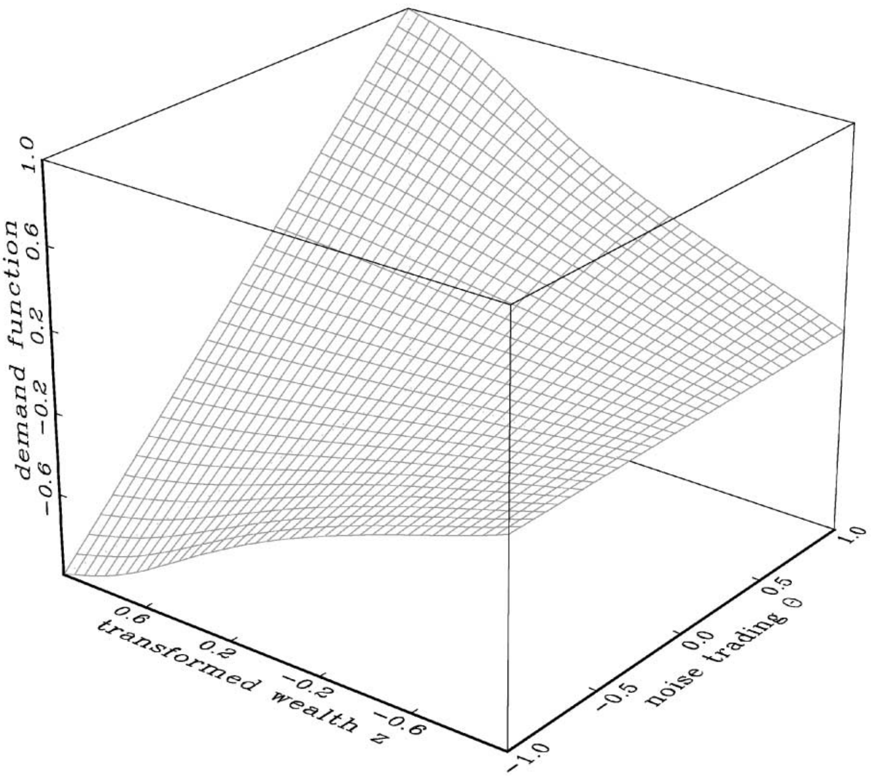

Fig. 1. Equilibrium demand function X of convergence traders for the risky asset. The two independent variables are convergence traders' aggregate wealth and noise trading. Aggregate wealth has been transformed monotonically using $z = (W-1)/(W+1)$ from $(0, \infty)$ into $(-1, 1)$. As the transformed wealth z ranges from -1 to 1, the aggregate wealth W ranges from zero to infinity. Noise trading θ ranges from -1 (four standard deviations below its mean of zero) to 1 (four standard deviations above its mean of zero).

traders have infinite wealth, indicating zero expected trading profits and also zero risk for their portfolio. When convergence traders have zero wealth, the squared Sharpe ratio can be very large as the noise trading moves away from its long-term mean of zero. This indicates very profitable trading opportunities for convergence traders. At the same time, convergence traders face large risks in their portfolio when they exploit these opportunities by taking the other side of noise trading.

For a given level of noise trading, the squared Sharpe ratio gradually decreases as convergence traders' wealth goes from zero to infinity. This is due to the increase in risk-bearing capacities among convergence traders, which cause decreased risk premia. This property of the Sharpe ratio results in mean-

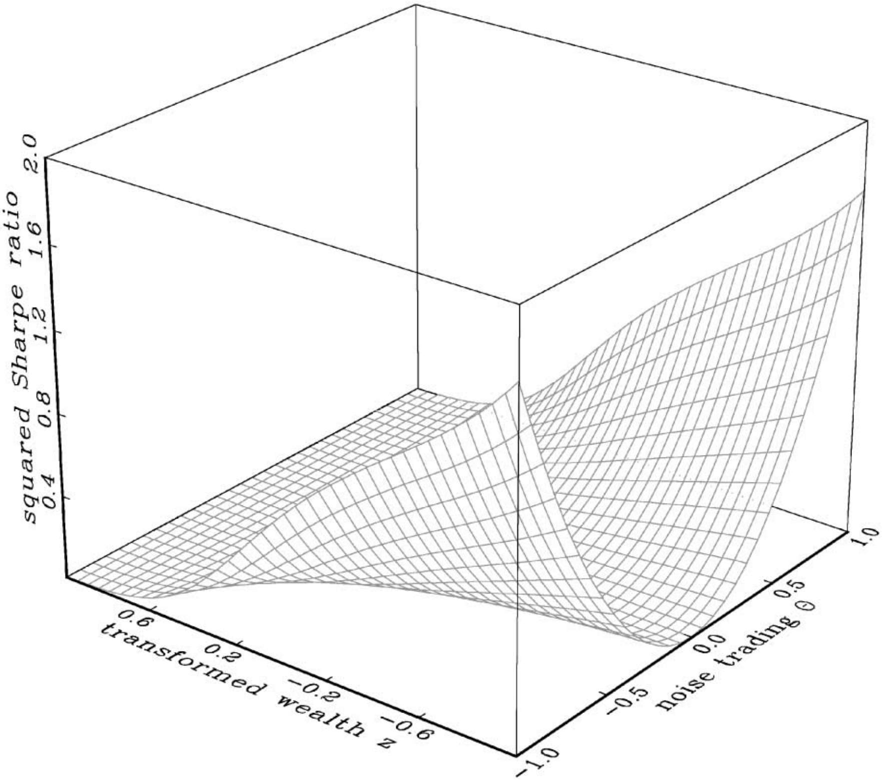

Fig. 2. Squared Sharpe ratio $(\mu^Q/\sigma^Q)^2$ of the risky asset. The two independent variables are convergence traders' aggregate wealth and noise trading. Aggregate wealth has been transformed monotonically using $z = (W-1)/(W+1)$ from $(0, \infty)$ into $(-1, 1)$. As the transformed wealth z ranges from -1 to 1, the aggregate wealth W ranges from zero to infinity. Noise trading θ ranges from -1 (four standard deviations below its mean of zero) to 1 (four standard deviations above its mean of zero).

reverting dynamics for the convergence traders' wealth process. The drift rate of the logarithm of convergence traders' wealth can be derived from Eqs. (22) and (23) by using Ito's lemma:

$$E_t[\mathrm{d}\log(W)] = \left[\frac{1}{2}\left(\frac{\mu^Q}{\sigma^Q}\right)^2 - (\rho - r)\right]\mathrm{d}t, \tag{29}$$

where the coefficient of $\frac{1}{2}$ appears due to the second-order term in Ito's lemma. This formula can help us discuss the dynamics of the wealth process and the Sharpe ratio. When convergence traders' wealth is low, a large risk premium is needed to induce them to bear risk, resulting in a large squared Sharpe ratio in the market. If the squared Sharpe ratio is larger than $2(\rho - r)$, trading is so profitable that convergence traders' wealth is expected to go up. As wealth becomes large, the increased risk-bearing capacity of convergence traders will drive down the risk premium (or the squared Sharpe ratio). If the squared Sharpe ratio is less than $2(\rho - r)$, convergence traders cannot make enough money from trading to make up for their consumption, so their wealth is expected to go down. As a result, the wealth process follows mean-reverting dynamics.

Eq. (29) also implies that the long-run mean of the squared Sharpe ratio is roughly $2(\rho - r)$. This result is motivated from the fact that convergence traders' long-run average trading profits should be equal to their average consumption in order for their wealth process to be in balance. This result can be confirmed by simulations of equilibria with different parameter sets. As shown in Table 1, the long-run mean of the squared Sharpe ratio is always about $2(\rho - r)$ across a wide range of parameter sets. According to this result, the trading opportunities left in the market are primarily determined by the convergence traders' time-preference parameter, and it depends very little on market conditions such as the fundamental value process and the noise trading process. The time preference (or the consumption rate given by the logarithmic utility function) can be interpreted as the convergence traders' cost of capital. In this sense, this result implies that the long-run trading opportunities in the market are determined by the convergence traders' cost of capital, similar in spirit to the model of endogenous participation of liquidity provision by Grossman and Miller (1988).

4.3. Stationary probability density

Since both of the two state variables, noise trading and convergence traders' wealth, follow mean-reverting processes, the equilibrium is stationary. The stationary distribution of the equilibrium is obtained through a simulation of 1,000 years of weekly data (using an Euler approximation) and shown in Fig. 3. This figure shows that noise trading concentrates within two standard deviations of its unconditional distribution, and convergence traders' wealth is mostly between zero and an intermediate level.

4.4. The amplification mechanism and effect on fundamental shocks

Fig. 4 shows the fundamental component of the asset return volatility σ_D^Q. From Eq. (16), this volatility component has the same shape as the wealth effect amplification factor $A(\theta, W)$. From the graph, it equals the volatility of fundamental shocks ($\sigma_F = 0.3$) when the wealth is either zero or infinity. In between, it is always above σ_F, indicating that the wealth effect is always amplifying. To illustrate the intuition, consider a situation when noise trading

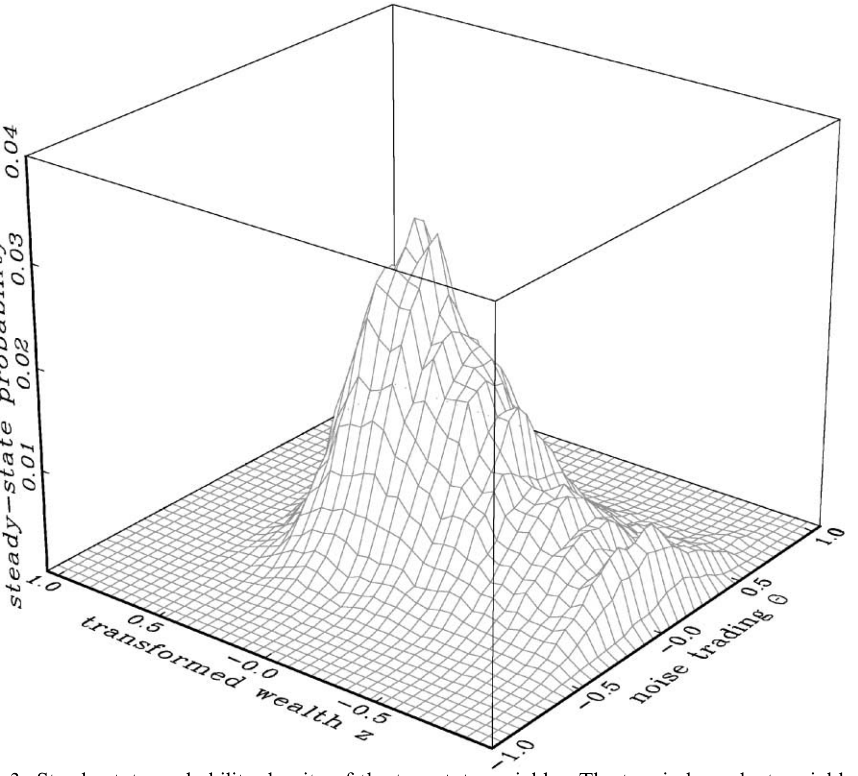

Fig. 3. Steady-state probability density of the two state variables. The two independent variables are convergence traders' aggregate wealth and noise trading. Aggregate wealth has been transformed monotonically using $z=(W-1)/(W+1)$ from $(0,\infty)$ into $(-1,1)$. As the transformed wealth z ranges from -1 to 1, the aggregate wealth W ranges from zero to infinity. Noise trading θ ranges from -1 (four standard deviations below its mean of zero) to 1 (four standard deviations above its mean of zero). The steady-state probability density of the two state variables is estimated by simulating 1,000 years of equilibrium trading.

is above zero. In this situation, convergence traders take long positions in the risky asset (from Fig. 1). If a negative fundamental shock hits the market and there is no change in noise trading, convergence traders lose money on their positions, and their risk aversion increases. This induces convergence traders to reduce their risky positions. The reduction of convergence traders' long positions further pushes down the asset price, and the fundamental shock is amplified. Similar intuition applies to other situations when positive fundamental shocks hit the market or noise trading is below zero. Therefore, the wealth effect of convergence traders provides an amplification mechanism for financial market shocks.

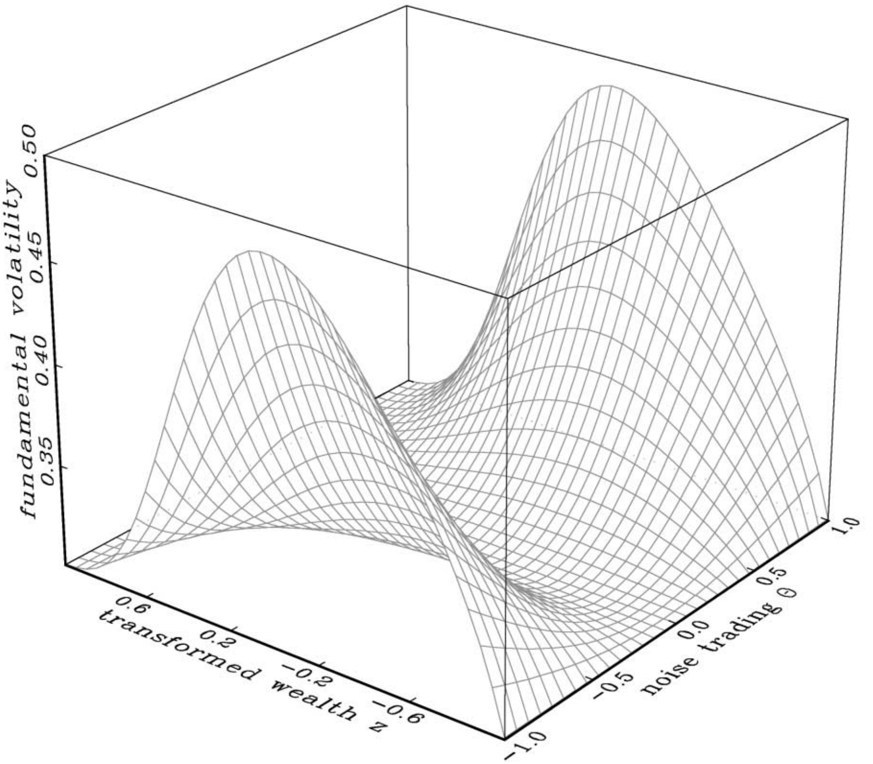

Fig. 4. Fundamental component of asset return volatility σ_D^Q. The two independent variables are convergence traders' aggregate wealth and noise trading. Aggregate wealth has been transformed monotonically using $z = (W-1)/(W+1)$ from $(0, \infty)$ into $(-1, 1)$. As the transformed wealth z ranges from -1 to 1, the aggregate wealth W ranges from zero to infinity. Noise trading θ ranges from -1 (four standard deviations below its mean of zero) to 1 (four standard deviations above its mean of zero).

The magnitude of the volatility amplification from the wealth effect changes with the two state variables, and it is most significant when noise trading is far from its mean of zero and convergence traders' wealth is at some intermediate level. From Eq. (14), there are two conditions necessary for the amplification effect to be large. First, the trading opportunity should be great, so that convergence traders will be induced to take large levered positions and therefore make their portfolio highly sensitive to shocks in the market. Second, the positions of convergence traders should be large so that the position rebalancing caused by exogenous shocks can generate a large price impact. Combining these two conditions, the amplification effect is large when noise trading is large and convergence traders' wealth is at some intermediate level. In this numerical example, the mean of noise trading is zero, so the amplification effect is symmetric with θ. If the mean of θ is nonzero, the amplification effect becomes asymmetric, but it is still most significant in the regions when noise trading is far from its mean and when convergence traders' wealth is in some intermediate level.

4.5. Destabilizing speculation and effect on noise trading shocks

Fig. 5 shows the noise trading component of the return volatility σ_θ^Q. From the discussion above, this component also measures market liquidity. It is zero when the wealth is infinite, reflecting a perfectly efficient market, i.e., noise trading has no effect on prices and the market is infinitely liquid. When wealth is zero, this volatility component $k\sigma_\theta$ equals 0.25. This level represents the effect of noise trading shocks on return volatility when there are no convergence traders and the noise trading shocks are purely buffered by long-term investors. This level measures the liquidity provided by long-term investors and can be used as a benchmark to evaluate the effect of convergence traders on noise trading shocks and market liquidity.

The shape of σ_θ^Q when wealth is between zero and infinity reveals the interaction between the wealth effect and the substitution effect discussed earlier. In the middle of the graph, there is a valley where the value of noise trading is near its mean ($\theta = 0$). Along this valley, the magnitude of σ_θ^Q declines monotonically from the benchmark level to zero as convergence traders' wealth increases from zero to infinity. This suggests that the substitution effect dominates the wealth effect in this valley. Convergence traders will be induced to take larger positions in response to increased noise trading in the market, because increased noise trading pushes the asset price further out of line and makes the Sharpe ratio higher. As a result, the convergence traders' trading reduces the effect of noise trading shocks and provides liquidity into the market. Furthermore, convergence traders become more effective in reducing the effect of noise trading shocks as their risk-bearing capacities increase with their wealth.

For regions outside the middle valley, where noise trading θ is far from its mean of zero and wealth is below some intermediate level, the magnitude of σ_θ^Q can be even larger than the benchmark level of 0.25, indicating that the effect of noise trading shocks has been amplified. This is exactly the region where the wealth effect dominates the substitution effect. The mechanism works as follows. When noise trading increases, the price moves further out of line. This causes convergence traders to suffer large capital losses on their current positions, and their risk-bearing capacities decrease so much that they need to unwind some of their positions, although the Sharpe ratio becomes even higher than before.

The situation in which the wealth effect dominates the substitution effect is interesting, because it indicates that speculation can be destabilizing in the sense that speculators (convergence traders in this paper) can be trading in

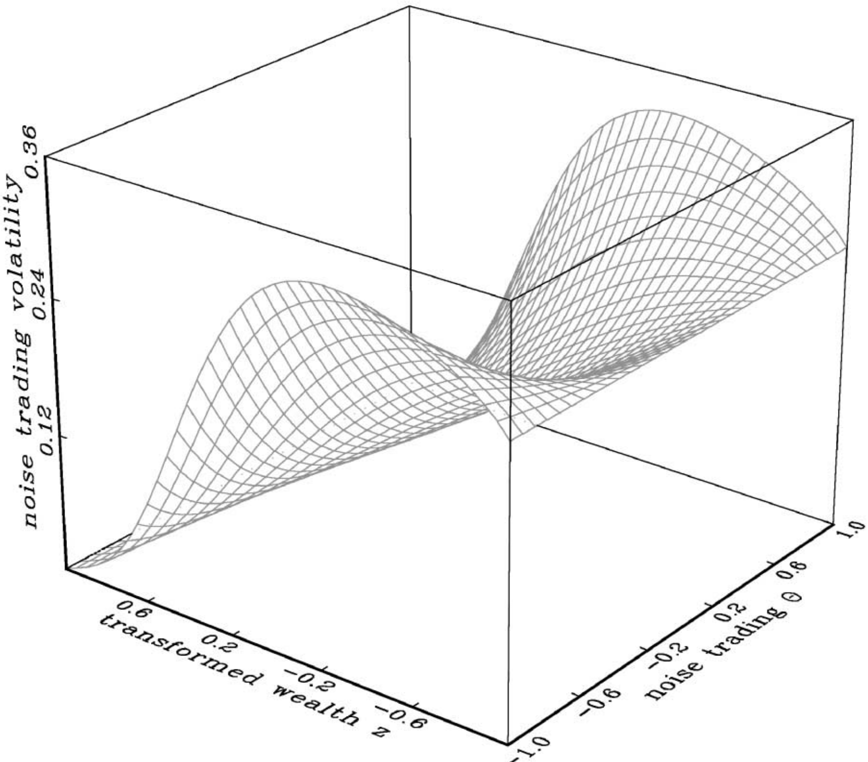

Fig. 5. Noise trading component of the asset return volatility σ_{θ}^{Q}. The two independent variables are convergence traders' aggregate wealth and noise trading. Aggregate wealth has been transformed monotonically using $z = (W-1)/(W+1)$ from $(0, \infty)$ into $(-1, 1)$. As the transformed wealth z ranges from -1 to 1, the aggregate wealth W ranges from zero to infinity. Noise trading θ ranges from -1 (four standard deviations below its mean of zero) to 1 (four standard deviations above its mean of zero).

exactly the same direction as noise traders, e.g., $\mathrm{d}X/\mathrm{d}\theta < 0$. It is shown that $\mathrm{d}X/\mathrm{d}\theta < 0$ is equivalent[5] to

$$(1 - X_{\theta})A(\theta, W) > 1. \tag{30}$$

As discussed before, the factor $1 - X_{\theta}$ represents the substitution effect, while the factor A represents the wealth effect. Using this definition of destabilizing speculation, the bound between destabilizing speculation and stabilizing speculation is shown in Fig. 6. This figure indicates that convergence trading

[5] By definition, $\mathrm{d}X(\theta, W)/\mathrm{d}\theta = X_{\theta} + X_W \mathrm{d}W/\mathrm{d}\theta$. From Eq. (11), $\mathrm{d}W/\mathrm{d}\theta = X\mathrm{d}Q/\mathrm{d}\theta$. From Appendix A, $\mathrm{d}W/\mathrm{d}\theta = [-k(1 - X_{\theta})]/(1 - kXX_W)$. Therefore, $\mathrm{d}X/\mathrm{d}\theta = 1 - [(1 - X_{\theta})]/(1 - kXX_W) = 1 - (1 - X_{\theta})A(\theta, W)$.

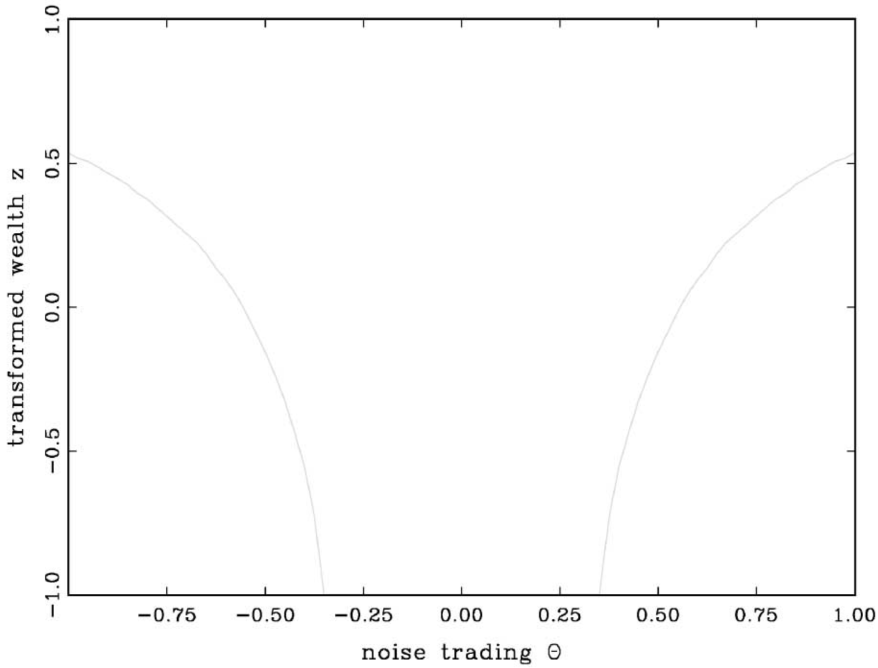

Fig. 6. Bound of destabilizing speculation. The bound is plotted between convergence traders' aggregate wealth and noise trading. Aggregate wealth has been transformed monotonically using $z = (W - 1)/(W + 1)$ from $(0, \infty)$ into $(-1, 1)$. As the transformed wealth z ranges from -1 to 1, the aggregate wealth W ranges from zero to infinity. Noise trading θ ranges from -1 (four standard deviations below its mean of zero) to 1 (four standard deviations above its mean of zero).

can be destabilizing over a large region where noise trading is far from its long-term mean and convergence traders' wealth is in a low or intermediate range. From Fig. 3 we know that the two state variables are highly concentrated in the middle. Therefore, convergence traders are stabilizing most of the time, while only in extreme circumstances do convergence traders become destabilizing.

This result contrasts with the common-sense observation that speculators always buy cheap and sell dear, and so always stabilize prices, as strongly expressed by the famous argument of Friedman (1953) that "to say that speculation is destabilizing is equivalent to saying that speculators lose money on average". The model in this paper is consistent with Friedman in the sense that, on average, convergence traders do make money and move prices towards their fundamentals. But in contrast to Friedman's intuition, convergence traders do not always make money. When they lose money, their increased risk aversion can induce them to sell when prices are cheap and to buy when prices are high, resulting in destabilizing speculation.

4.6. Total volatility

Fig. 7 shows the total return volatility in the risky asset. When wealth is infinite, volatility is a constant equal to the volatility of fundamental shocks ($\sigma_F = 0.3$) because convergence trading fully offsets noise trading. When wealth is zero, volatility is constant at the level of 0.391, which is higher than the fundamental volatility because of additional noise trading that is not offset. The latter level is a benchmark level used to evaluate the effect of convergence traders on total volatility. When noise trading is near its mean of zero, volatility declines monotonically as wealth increases from zero to infinity along the valley in the middle of the graph. This shows that convergence traders reduce total price volatility because the substitution effect causes them to reduce noise trading shocks more than the wealth effect causes them to amplify

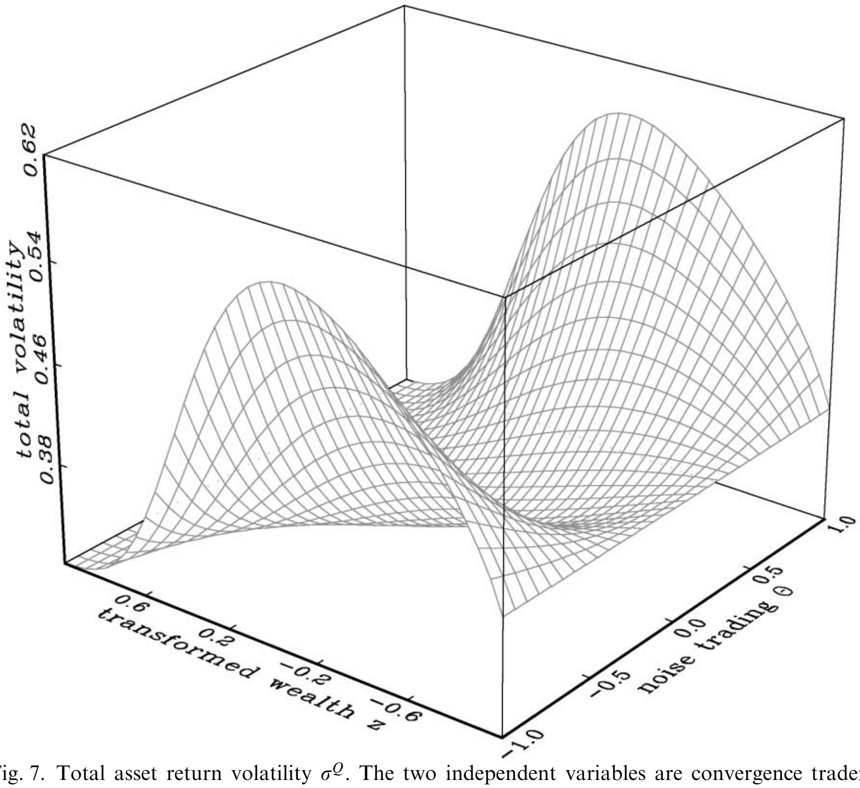

Fig. 7. Total asset return volatility σ^Q. The two independent variables are convergence traders' aggregate wealth and noise trading. Aggregate wealth has been transformed monotonically using $z = (W-1)/(W+1)$ from $(0, \infty)$ into $(-1, 1)$. As the transformed wealth z ranges from -1 to 1, the aggregate wealth W ranges from zero to infinity. Noise trading θ ranges from -1 (four standard deviations below its mean of zero) to 1 (four standard deviations above its mean of zero).

fundamental shocks. When noise trading is far from its mean and wealth is between zero and some intermediate level, the wealth effect dominates the substitution effect, and it causes volatility to be larger than the benchmark level. Fig. 4 shows that the two state variables stay near the middle valley most of the time. Therefore, convergence traders reduce volatility on average, but they can also increase volatility in extreme circumstances.

The shape of total volatility is consistent with two important aspects of asset price volatility: excess volatility and stochastic volatility. Campbell et al. (1998) provide a complete literature review on asset price volatility. Volatility can be excessive in the sense that it is too large to be explained by asset fundamentals from the simple present value model with a constant discount rate. There are two sources of extra volatility in my model in addition to fundamental volatility. One is noise trading shocks, and the other is the volatility amplification of the convergence traders' wealth effect. The first source has been modeled by Campbell and Kyle (1993). When speculators (or smart traders) are risk averse, they cannot eliminate all the effects of noise trading, with the result that noise trading shocks are part of total volatility. The volatility amplification from the convergence traders' wealth effect is the contribution of this paper to the literature. More specifically, the wealth effect causes fundamental shocks to be amplified.[6] Volatility also varies smoothly over time with the evolution of the two state variables (convergence traders' wealth and noise trading), because the magnitude of the amplification depends nonlinearly on the two state variables.

The amplification effect studied in this model does not imply that margin buying always destabilizes prices and increases volatility. This type of negative effect from (margin) leverage only occurs in extreme circumstances. On average, convergence trading reduces price volatility and improves market depth. These results are consistent with the empirical literature on the effect of margin buying on stock price volatility. There has been a long debate on this subject, as in Moore (1966) and Officer (1973). More recent contributions include Schwert (1989a,b), Hsieh and Miller (1990), Seguin (1990), and Hardouvelis (1990). In his review of the literature, Kupiec (1997) says there is no consistent empirical evidence supporting either the hypothesis that margin buying causes

[6] In the model, convergence traders will always increase asset price volatility if there is no noise trading in markets. Without noise trading, the only sources of asset return volatility are fundamental shocks, and only long-term investors and convergence traders trade the risky asset. Intuitively, we can think of convergence traders as investors using a dynamic risk management strategy to prevent their wealth from falling below zero. This situation is analogous to models on portfolio insurance by Grossman (1988) and Grossman and Zhou (1996). These models demonstrate that when some investors follow portfolio insurance strategies, the market volatility is increased. This is very similar to my model's prediction that, without noise trading in markets, the presence of convergence traders using dynamic risk management strategies increases price volatility.

larger volatility or the hypothesis that margin buying reduces volatility. My model confirms the subtlety of this issue, even from a theoretical point of view. For other theoretical work on this topic, see Chowdhry and Nanda (1998).

5. Discussions of the model

In this section, I discuss some implications of the model. First, I discuss the implications of liquidity risk for risk management by linking the model to the near-collapse of Long-Term Capital Management (LTCM) in 1998. Second, I discuss long-run implications for capital devoted to convergence trading.

5.1. LTCM and risk management of liquidity risks

My model is consistent with some of the observations about the critical situation faced by LTCM in the late summer of 1998. The market conditions during this period are described by the Bank for International Settlements (BIS, 1999, p. 10): "Following Russia's currency devaluation and default, yield spreads on corporate bonds widened sharply worldwide, particularly for instruments with lower credit standing. Day-to-day changes in financial prices were unusually volatile. Measures of implied volatility, inferred from options prices, rose sharply, peaking in October for most industrial country markets. Quoted bid–ask spreads rose in a number of markets, reflecting reduced liquidity. The yield premium for off-the-run government bonds in major industrial countries also widened." The report also provides detailed data on these market variables.

The severe market conditions were partly related to the trading of a group of specialized hedge funds represented by LTCM. According to the same report (BIS, 1999, p. 7), "LTCM sought high rates of return primarily by identifying small discrepancies in the prices of various instruments relative to historical norms and then taking highly leveraged positions in the instruments in the expectations that market prices would revert to such norms over time." The essence of this strategy is exactly the convergence trading studied in my model. The model captures one of the key ingredients of this event: as LTCM and other hedge funds following similar strategies scaled back their activities voluntarily to preserve their capital after initial losses, there was a dramatic widening in previously narrow swap spreads, credit spreads, etc., and the initial shocks that triggered the scaling back of these hedge funds were amplified.

By aggressively taking positions against noise trading, convergence traders effectively provide liquidity in markets. The episode of LTCM illustrates the disturbing possibility that liquidity providers can run into liquidity problems themselves due to capital constraints. In practice, liquidity provision is a low-margin business pursued by leveraged financial institutions. The use of leverage increases the possibility that these financial institutions might be forced out of their positions after capital losses, resulting in a one-way market. This liquidity risk creates a major challenge to the risk management system of highly leveraged financial institutions. As illustrated by the numerical example in the previous section, asset price volatility in certain extreme circumstances can be very different from historical average volatility or from volatility in normal periods. This type of extreme volatility can be forecast by the aggregate positions and capital of convergence traders using my model based on the trading strategy of convergence traders and liquidity provided by long-term investors. This type of liquidity risk only becomes significant in extreme circumstances. Therefore, it is very difficult for currently popular Value-at-Risk types of risk management systems to handle. A typical Value-at-Risk type of risk management system analyzes risks based on historical data, and can therefore be ineffective in extreme situations. After the LTCM episode in 1998, more and more practitioners and regulators started to realize the importance of managing liquidity risks and the ineffectiveness of Value-at-Risk types of risk management methods.

A more recent report issued by the Bank for International Settlements (BIS, 2000) discusses a new risk management method called "dynamic macro stress testing". By interviewing more than 20 large international financial institutions, the BIS collected information on these financial institutions' risk exposures to certain exceptional but plausible financial market scenarios. The BIS proposes to use this information on the aggregate risk exposure of financial institutions to manage liquidity risks in certain markets. This proposal is consistent with the results of my model in that it is important to take into account the amplification mechanism caused by the convergence traders' wealth effect. However, my model goes beyond the stress testing method discussed by the BIS. One of the weaknesses of stress testing is that it only reflects the potential losses corresponding to a specific stress scenario, but not the probability of the scenario. By studying the dynamics of market equilibrium, my model allows probabilities to be calculated for the endogenous liquidity risks.

5.2. Which markets attract convergence traders?

Specialization has been an important feature of convergence trading. As pointed out by Merton (1987) and Shleifer and Vishny (1997), both normal investors and professional traders can only trade in a limited number of assets due to the information costs of participating in more markets. Some empirical evidence suggests that convergence traders do specialize and only limited amounts of capital are allocated in certain specific markets. Mitchell and Pulvino (1999) and Baker and Savasoglu (2000) study the expected risk premium in merger arbitrage trades. They find that returns in merger arbitrage increase with ex ante completion risk and target size, and decrease with the general supply of arbitrage capital. Shleifer (1986) and Wurgler and

Zhuravskaya (2000) study the profits from convergence trading positions consisting of short-selling a stock newly added to the S&P 500 index and buying a substitute stock. They find a positive risk premium from this type of trade, and an especially large risk premium for stocks without close substitutes. Froot and O'Connell (1997) find evidence that risk premiums in the insurance industry rise when insurers' capital is low. Based on these studies, convergence traders demand risk premiums for bearing risks in specific idiosyncratic markets, and the risk premium demanded decreases with their capital. Furthermore, capital does not flow efficiently into markets where convergence traders are undercapitalized.

When convergence traders specialize, an interesting question raised by Shleifer and Vishny (1997) is this: Which markets attract convergence traders? They argue that since price volatility makes arbitrage (convergence trading) more difficult, high volatility deters arbitrage activity. My model allows us to look at this question more closely. The long-run average wealth accumulated in a market by convergence traders is a measure of convergence trading activity. By numerically computing the average wealth across different equilibria with different exogenous parameter values, I can discuss the effect of each parameter.

There are four variables that are relevant to this discussion: the volatility of fundamental shocks, the volatility of noise trading shocks, the mean-reverting speed of noise trading, and the slope of the demand curve of long-term investors. Simulation results in Panel A of Table 1 show that with all other parameters fixed, long-run average wealth decreases with the volatility of fundamental shocks. In this sense, high fundamental volatility discourages convergence trading activity because it makes convergence trading riskier. On the other hand, long-run average wealth increases with the volatility of noise trading shocks as shown in Panel C of Table 1. This suggests that a high volatility of noise trading shocks encourages convergence trading activity because it generates more trading opportunities in the market. Panel B of Table 1 shows that long-run average wealth increases with the mean-reverting speed of noise because convergence traders can expect their profits earlier. Panel D of Table 1 shows that long-run average wealth increases with the slope of the long-term investors' demand curve. This suggests that more convergence trading activity is expected if long-term investors provide less liquidity. With less liquidity from long-term investors, more trading opportunities are available for convergence traders, so they will accumulate more capital in the long run.

I do not claim that the amplification mechanism studied in this model applies to aggregate stock market volatility.[7] But it should be important for specific sectors where there is a group of specialized convergence traders with

[7] The crisis of LTCM eventually became a crisis of the aggregate financial markets due to the involvement of many banks and security firms as the creditors and counterparties of LTCM, but these issues are beyond my model. Edwards (1999) provides some discussion of these issues.

undiversified portfolios. Shocks in such markets can generate large fluctuations in convergence traders' capital, resulting in significant wealth effects. Even when convergence traders trade in more than one sector but are not fully diversified, Kyle and Xiong (2001) show that the wealth effect can still be generated and cause assets in their portfolio to move more closely together, resulting in reduced benefits from diversification.

Bond markets attract convergence trading on the spread positions between different bonds with larger than usual yield spreads, because the fundamental risks involved in this type of trade can be very limited. The effect of the amplification mechanism in bond markets was vividly illustrated by LTCM in 1998. Even one year after the event, the yield spread between corporate bonds and U.S. Treasury bonds stayed at a very high level compared with its historical level. This could be partly due to the fact that convergence traders in bond markets had lost most of their capital during the LTCM crisis, and were not fully recapitalized after a year. Merger arbitrage trades are also very popular among convergence traders. Since there are usually specific time limits for a merger deal to either succeed or fail, convergence speed is high and convergence traders can expect to realize profits quickly. With a group of specialized merger arbitrageurs in this market, the risk premium would be negatively related to their capital. After a series of failed deals, convergence traders can lose a significant percentage of their capital, and the risk premium as well as the volatility of the stocks involved are also likely to rise. Another type of trade widely used is "pairs trading" of stocks, which involves betting that the price differential between two stocks will converge. We would expect similar phenomena with pairs trading.

6. Conclusions

This paper develops an equilibrium model of a market with a group of specialized convergence traders. The assumption of a logarithmic utility function for convergence traders causes their risk-bearing capacity to change proportionally with their wealth. In equilibrium, the wealth effect occurs through the endogenous and simultaneously determined relation between convergence traders' wealth dynamics and asset price dynamics. When convergence traders suffer capital losses due to unfavorable shocks, they need to liquidate some of their positions, thereby causing the original shocks to be amplified. In this way, the wealth effect provides an amplification mechanism for financial market shocks, and this amplification mechanism can explain excess volatility and stochastic volatility. The model also studies the interaction between two effects in convergence trading that operate in opposite directions: the substitution effect and the wealth effect. Most of time, the substitution effect dominates the wealth effect, and convergence traders are induced to take larger positions in response to increased noise trading in the market. As a result, their trading provides liquidity and reduces asset price volatility. In certain extreme circumstances, however, the wealth effect can dominate the substitution effect. When this happens, convergence traders need to unwind some of their positions in response to increased noise trading. As a result, their trading becomes destabilizing in the sense that they are trading in exactly the same direction as noise traders, resulting in amplified price volatility and reduced market liquidity. This type of endogenous liquidity risk in extreme circumstances creates a challenge for the risk management systems of leveraged financial institutions. My model offers risk managers a tool to study market equilibrium dynamics and to forecast this type of extreme risk using information on market participants' aggregate positions and capital.

The wealth effect studied in this paper is driven by voluntary liquidation of convergence traders after their capital losses. Another possible mechanism to generate the wealth effect is through the involuntary liquidation of convergence traders caused by binding credit constraints imposed by their creditors. As convergence traders suffer large capital losses, their creditors can choose to call back their loans to avoid further losses. Both voluntary liquidation and involuntary liquidation of convergence traders have been recognized by Shleifer and Vishny (1997) and the Bank for International Settlements report (BIS, 1999) as possible mechanisms associated with stressed market conditions. The mechanism of involuntary liquidation should generate an amplification effect on price dynamics that is qualitatively similar to that of the mechanism of voluntary liquidation, only with an even larger magnitude. Loosely speaking, credit constraints impose a constant upper limit on the leverage of convergence traders. Conversely, the internal constraints on the leverage ratio generated by logarithmic utility can expand in response to better trading opportunities in the markets, because convergence traders with logarithmic utility take higher leverage when the Sharpe ratio is larger. Because of this, the involuntary liquidation caused by binding credit constraints can be larger than the voluntary liquidation caused by the logarithmic utility studied in my model.[8]

In many cases, convergence traders are agents managing other people's money, such as hedge funds or proprietary traders of publicly listed securities firms. This creates a potential agency problem between convergence traders and their investors. As highlighted by Shleifer and Vishny (1997), the agency problem can cause capital to flow out from convergence traders when they suffer capital losses, if their investors start to doubt their strategy or

[8] Liu and Longstaff (2000) study a portfolio choice problem of an arbitrageur facing margin constraints and an exogenous arbitrage opportunity represented by a Brownian bridge process. They show that an expost realized extreme opportunity can hurt arbitrageurs by forcing them to liquidate positions in response to binding margin constraints. Their exercise demonstrates the large effect of margin constraints.

ability. My model relies on this observation to assume that there is no capital inflow to convergence traders after their capital losses. Due to the complexity of specifically modeling the agency problem and the subsequent capital outflow from convergence traders, I do not incorporate these features into the model. But this feature will certainly generate even stronger amplification effects.

With the assumption of logarithmic utility, the model also ignores the potential hedging motives of convergence traders. Without a careful study of convergence traders' hedging demand, it is not clear whether the assumption of logarithmic utility overstates or understates the amplification mechanism in equilibrium. From Merton's (1971) dynamic portfolio theory, investors who are more risk averse than as implied by logarithmic utility have hedging motives, while investors who are less risk averse than as implied by logarithmic utility have speculative motives (negative hedging motives). These results are studied in detail by Kim and Omberg (1996). In my model, the trading opportunity (squared Sharpe ratio) is negatively related to convergence traders' wealth. This makes the current trading position a natural hedge for future opportunities, in the sense that when convergence traders suffer losses on their current positions, future opportunities will become better because of the decreased wealth of all convergence traders in the market. As a result, hedging motives will induce convergence traders to take larger positions for a given level of opportunity compared with their demand without hedging motives. Therefore, under the structure of my model, convergence traders who are more risk averse than as implied by logarithmic utility have a hedging motive to take larger positions. At the same time, because they are more risk averse, they are inclined to take smaller positions. Thus, it is not clear how asset demands change as convergence traders become more risk averse relative to that implied by logarithmic utility. Following the same intuition, if convergence traders are less risk averse than as implied by logarithmic utility, their speculative motives induce them to take smaller positions to store capital for better opportunities in the future, but at the same time they are less risk averse and tend to take larger positions. Thus, the net change on their demands is also not clear. The answer to this problem is left for future research.

Appendix A. Derivation of the asset return process

Given the aggregate trading strategy $X(\theta, W)$ for convergence traders, the asset return process can be derived by applying Ito's lemma. The market-clearing condition gives the price function for the risky asset:

$$P = F - k(\theta - X). \tag{A.1}$$

The excess return process for investing in one share of the risky asset is given by

$$\begin{aligned}\mathrm{d}Q &= \mathrm{d}P + (D - rP)\,\mathrm{d}t \\ &= \sigma_F\,\mathrm{d}z_D - k\,\mathrm{d}\theta + k\,\mathrm{d}X + rk(\theta - X)\,\mathrm{d}t.\end{aligned} \tag{A.2}$$

It is directly from Ito's lemma that

$$\begin{aligned}\mathrm{d}X = {} & X_\theta\,\mathrm{d}\theta + 1/2X_{\theta\theta}\mathrm{E}(\mathrm{d}\theta)^2 + X_W\,\mathrm{d}W \\ & + 1/2X_{WW}\mathrm{E}(\mathrm{d}W)^2 + X_{\theta W}\mathrm{E}(\mathrm{d}\theta\,\mathrm{d}W).\end{aligned} \tag{A.3}$$

Eqs. (A.2) and (A.3) show the dependence of the return process $\mathrm{d}Q$ on the convergence traders' aggregate wealth W. On the other hand, convergence traders' wealth depends on the return process from their budget constraint:

$$\mathrm{d}W = X\,\mathrm{d}Q + (rW - C)\,\mathrm{d}t. \tag{A.4}$$

Therefore, the asset return process $\mathrm{d}Q$ and convergence traders' wealth process W are both endogenously and simultaneously determined in equilibrium. This simultaneous relation can cause any shock to $\mathrm{d}Q$ to feed back to itself through W. This feedback effect is exactly the wealth effect. To deal with this simultaneous relation, Eq. (A.4) is substituted into Eq. (A.3), then Eq. (A.3) is substituted into Eq. (A.2). Finally, the return process is derived as

$$\mathrm{d}Q = \mu^Q\,\mathrm{d}t + \sigma_D^Q\,\mathrm{d}z_D + \sigma_\theta^Q\,\mathrm{d}z_\theta, \tag{A.5}$$

$$\begin{aligned}\mu^Q = \Bigg\{ & k\lambda_\theta(\theta - \bar{\theta})(1 - X_\theta) + kX_W(rW - C) + rk(\theta - X) \\ & + \frac{k\sigma_\theta^2}{2}X_{\theta\theta} + \frac{k(\sigma^W)^2}{2}X_{WW} + k\sigma_\theta\sigma_\theta^W X_{\theta W}\Bigg\} A(\theta, W),\end{aligned} \tag{A.6}$$

$$\sigma_D^Q = \sigma_F A(\theta, W), \tag{A.7}$$

$$\sigma_\theta^Q = -k\sigma_\theta(1 - X_\theta)A(\theta, W), \tag{A.8}$$

where $A(\theta, W) = 1/(1 - kXX_W)$ represents the amplification factor of the convergence traders' wealth effect. The total volatility of the asset's return is

$$\sigma^Q = \sqrt{(\sigma_D^Q)^2 + (\sigma_\theta^Q)^2}. \tag{A.9}$$

From the budget constraints, the process for convergence traders' aggregate capital can be derived as

$$\mathrm{d}W = \mu^W\,\mathrm{d}t + \sigma_D^W\,\mathrm{d}z_D + \sigma_\theta^W\,\mathrm{d}z_\theta, \tag{A.10}$$

$$\mu^W = X\mu^Q + rW - C, \tag{A.11}$$

$$\sigma_D^W = X\sigma_D^Q, \tag{A.12}$$

$$\sigma_\theta^W = X\sigma_\theta^Q. \tag{A.13}$$

The total volatility of the convergence traders' wealth is

$$\sigma^W = \sqrt{(\sigma_D^W)^2 + (\sigma_\theta^W)^2}. \tag{A.14}$$

It is interesting to show the return process when convergence traders have little wealth ($W \to 0$), because this return process represents the original trading opportunities when there are no convergence traders at all. Under this situation, the demand of convergence traders is small ($X \to 0$) and the excess return process is

$$\mathrm{d}Q = \sigma_F\,\mathrm{d}z_D - k\,\mathrm{d}\theta + rk\theta\,\mathrm{d}t. \tag{A.15}$$

The Sharpe ratio of the risky asset is

$$\frac{\mu^Q}{\sigma^Q} = \frac{rk\theta + k\lambda_\theta(\theta - \bar{\theta})}{\sqrt{\sigma_F^2 + (k\sigma_\theta)^2}}. \tag{A.16}$$

The Sharpe ratio fluctuates with noise trading θ. If $\bar{\theta} = 0$, the variance of the Sharpe ratio is

$$E\left(\frac{\mu^Q}{\sigma^Q}\right)^2 = \frac{(r+\lambda_\theta)^2 k^2 \sigma_\theta^2}{2\lambda_\theta[\sigma_F^2 + (k\sigma_\theta)^2]}. \tag{A.17}$$

Appendix B. Derivation of a convergence trader's optimal strategies

The trading opportunities of an individual convergence trader are

$$\mathrm{d}Q = \mu^Q(\theta, W)\,\mathrm{d}t + \sigma_D^Q(\theta, W)\,\mathrm{d}z_D + \sigma_\theta^Q(\theta, W)\,\mathrm{d}z_\theta. \tag{B.1}$$

The two state variables are θ and W. The variable θ denotes the noise trading in the risky asset, and

$$\mathrm{d}\theta = -\lambda_\theta(\theta - \bar{\theta})\,\mathrm{d}t + \sigma_\theta\,\mathrm{d}z_\theta.$$

The variable W is the aggregate capital of convergence traders, and

$$\mathrm{d}W = \mu^W(\theta, W)\,\mathrm{d}t + \sigma_D^W(\theta, W)\,\mathrm{d}z_D + \sigma_\theta^W\,\mathrm{d}z_\theta. \tag{B.2}$$

Denote an individual convergence trader's wealth, trading, and consumption policies as W^i, X^i, and C^i. The budget constraint is

$$\mathrm{d}W^i = X^i\mathrm{d}Q + (rW^i - C^i)\,\mathrm{d}t. \tag{B.3}$$

The convergence trader maximizes lifetime utility:

$$J(W^i, \theta, W) = \max_{X^i, C^i} E_t \int_0^\infty \mathrm{e}^{-\rho s} \ln(C_{t+s}^i)\,\mathrm{d}s. \tag{B.4}$$

The optimal trading and consumption policies are solved through a Bellman equation. The Bellman equation can be derived as

$$\begin{aligned}\rho J(W^i,\theta,w) &= \max_{X^i,C^i}\,[\ln(C^i)+\mathscr{L}^0 J]\\ &= \max_{X^i,C^i}\,[\ln(C^i)+J_{W^i}(X^i\mu^Q+rW^i-C^i)\\ &\quad +1/2J_{W^iW^i}(X^i)^2(\sigma^Q)^2\\ &\quad +\ \lambda_\theta(\bar\theta-\theta)J_\theta+\mu^W J_W+1/2\sigma_\theta^2 J_{\theta\theta}+1/2\sigma_W^2 J_{WW}\\ &\quad +\ J_{W^i\theta}\mathrm{E}(\mathrm{d}W^i\mathrm{d}\theta)/\mathrm{d}t+J_{W^iW}(\mathrm{d}W^i\mathrm{d}W)/\mathrm{d}t\\ &\quad +J_{\theta w}\mathrm{E}(\mathrm{d}\theta\mathrm{d}W)/\mathrm{d}t],\end{aligned} \tag{B.5}$$

where $\mathscr{L}^0$ is the drift operator. The value function of a log-utility optimizer can be specified as

$$J(W^i,\theta,w)=\frac{1}{\rho}\ln(W^i)+j(\theta,W). \tag{B.6}$$

The first-order condition of the Bellman equation gives the optimal trading and consumption policies:

$$X^i=\frac{\mu^Q}{(\sigma^Q)^2}W^i, \tag{B.7}$$

$$C^i=\rho W^i. \tag{B.8}$$

After substituting the optimal policies into the Bellman equation, W^i disappears from both sides of the equation, and the Bellman equation collapses into a partial differential equation in θ and W only:

$$\begin{aligned}\rho j(\theta,W)&=\ln(\rho)+\rho(r-\rho)+\rho\frac{(\mu^A)^2}{(\sigma^A)^2}+\lambda_\theta(\bar\theta-\theta)j_\theta\\ &\quad +\ \mu^W j_W+1/2\sigma_\theta^2 j_{\theta\theta}+1/2\sigma_W^2 j_{WW}+\sigma_\theta\sigma_\theta^W j_{\theta W}.\end{aligned} \tag{B.9}$$

Therefore, the convergence trader's policy functions and value function become separated. The solution to the PDE of the value function exists under certain technical conditions. This paper will focus on the policy functions and discuss the equilibrium of the asset market.

The logarithmic utility has interesting policy functions. Both trading and consumption policies are proportional to the convergence trader's wealth. The optimal trading strategy is myopic or short-term in the sense that it depends only on the instantaneous mean and variance of the return process. Thus, there is no need for hedging against changes in the future opportunity set with logarithmic utility even though there is for other utility functions with constant relative risk aversion. The assumption of logarithmic utility greatly simplifies the problem without losing the key feature of this model, the wealth effect.

From Appendix A, the drift and volatility terms μ^Q, μ^W, σ_D^Q, σ_θ^Q, σ_D^W, and σ_θ^W in $\mathrm{d}Q$ and $\mathrm{d}W$ are all determined by the convergence traders' aggregate demand function $X(\theta, W)$ and consumption function $C(\theta, W)$. Therefore, Eqs. (B.7) and (B.8) show that an individual convergence trader's optimal strategies X^i and C^i are explicit functions of the conjectured aggregate demand function $X(\theta, W)$ and consumption function $C(\theta, W)$. Since explicit expressions of these functions are extremely tedious, they are omitted here to save space.

Appendix C. The partial differential equation

Appendix C presents the partial differential equation from the fixed-point problem of the equilibrium. Given convergence traders' aggregate trading and consumption rules $X(\theta, W)$ and $C(\theta, W)$, the optimal aggregate trading and consumption rules can be easily derived from Eqs. (B.7) and (B.8) by replacing W^i with W:

$$X^* = \frac{\mu^Q}{(\sigma^Q)^2} W, \tag{C.1}$$

$$C^* = \rho W. \tag{C.2}$$

From the fixed-point problem, the equilibrium consumption rule is trivial ($C = \rho W$), and the equilibrium trading rule is determined by

$$X = \frac{\mu^Q}{(\sigma^Q)^2} W \tag{C.3}$$

with μ^Q and σ^Q given by Eqs. (A.6)–(A.8). By substituting all the necessary terms into the last equation, the following partial differential equation is obtained:

$$\begin{aligned} &k\lambda_\theta(\theta - \bar{\theta})(1 - X_\theta) + k(r-\rho)WX_W + rk(\theta - X) + \frac{k\sigma_\theta^2}{2} X_{\theta\theta} \\ &+ \frac{k}{2} X^2 X_{WW} \frac{\sigma_F^2 + k^2\sigma_\theta^2(1 - X_\theta)^2}{(1 - kXX_W)^2} - k^2\sigma_\theta^2 \frac{XX_{\theta W}(1 - X_\theta)}{1 - kXX_W} \\ &- \frac{X}{W} \frac{\sigma_F^2 + k^2\sigma_\theta^2(1 - X_\theta)^2}{1 - kXX_W} = 0. \end{aligned} \tag{C.4}$$

This is a nonlinear second-order partial differential equation in the two state variables θ and W. In addition to X itself, the equation involves first derivatives X_θ and X_W and second derivatives $X_{\theta\theta}$, $X_{\theta W}$, and X_{WW}.

Appendix D. Numerical method to the fixed-point problem

To study the equilibrium, a numerical method is needed to solve the fixed-point problem, since the partial differential equation in (C.4) is highly nonlinear in such a way that it is hopeless to solve it analytically. To calculate an approximate equilibrium numerically, a projection method is used. The trading strategy X is approximated with a rational function, where both the numerator and denominator are polynomials of the two state variables. The algorithm chooses coefficients of the polynomials so that the boundary conditions hold and the partial differential equation describing the equilibrium is approximately solved for a range of test points. Instead of ordinary polynomials, Chebyshev polynomials are used for reasons of numerical stability: with Chebyshev polynomials, the calculation of the values of polynomials is more stable, and there is less "collinearity" among estimated coefficients. Also, the use of Chebyshev polynomials makes it easier to impose boundary conditions as discussed in Appendix E. For a detailed introduction to projection methods and Chebyshev polynomials, see Judd (1998) and Press et al. (1992).

To use Chebyshev polynomials, whose natural range is $[-1,+1]$, it is first necessary to transform the state variables W and θ to fit this range. To transform W, whose range is $(0, \infty)$, a new variable z is introduced and defined (with an exogenously specified scale parameter γ) by

$$z = \frac{W-\gamma}{W+\gamma}, \quad z \in (-1, 1). \tag{D.1}$$

To transform θ, whose natural range is $(-\infty, +\infty)$, it is truncated at four standard deviations of its unconditional distribution and linearly transformed into a new state variable y:

$$y = \frac{\theta - \bar{\theta}}{4\sigma_\theta/\sqrt{2\lambda_\theta}}, \quad y \in [-1, 1]. \tag{D.2}$$

Both of these transformations are obviously monotonic and smooth. The reverse transformations are

$$\theta = \bar{\theta} + \frac{4\sigma_\theta}{\sqrt{2\lambda_\theta}} y, \tag{D.3}$$

$$W = \gamma \frac{1+z}{1-z}. \tag{D.4}$$

The derivatives of the two state variables θ and W can be transformed as

$$\frac{\partial}{\partial \theta} = \frac{4\sigma_\theta}{\sqrt{2\lambda_\theta}} \frac{\partial}{\partial y}, \tag{D.5}$$

$$\frac{\partial^2}{\partial\theta^2} = \frac{8\sigma_\theta^2}{\lambda_\theta}\frac{\partial^2}{\partial y^2}, \tag{D.6}$$

$$\frac{\partial}{\partial W} = \frac{(1-z)^2}{2\gamma}\frac{\partial}{\partial z}, \tag{D.7}$$

$$\frac{\partial^2}{\partial W^2} = \frac{(1-z)^4}{4\gamma^2}\frac{\partial^2}{\partial z^2} - \frac{(1-z)^3}{2\gamma^2}\frac{\partial}{\partial z}, \tag{D.8}$$

$$\frac{\partial^2}{\partial\theta\partial W} = \frac{\sqrt{2}\sigma_\theta(1-z)^2}{\sqrt{\lambda_\theta}}\frac{\partial^2}{\partial y\partial z}. \tag{D.9}$$

These formulas can transform the original partial differential equation of $X(\theta, W)$ in Eq. (C.4) into a partial differential equation of $X(y, z)$.

The equilibrium demand function $X(y, z)$ is approximated by

$$X(y,z) = \frac{\sum_{i+j\leqslant n_\mathrm{u}} a_\mathrm{u}(i,j)T_i(y)T_j(z)}{\sum_{i+j\leqslant n_\mathrm{d}} a_\mathrm{d}(i,j)T_i(y)T_j(z)}, \tag{D.10}$$

where $T_i()$ is the ith order Chebyshev polynomial, and n_u and n_d are the total orders of polynomials in the numerator and denominator of X. Let $\{a_\mathrm{u} \times (i,j)\}_{i+j\leqslant n_\mathrm{u}}$ and $\{a_\mathrm{d}(i,j)\}_{i+j\leqslant n_\mathrm{d}}$ denote the expansion coefficients. The total number of coefficients is $[(n_\mathrm{u}+1)(n_\mathrm{u}+2)]/2 + [(n_\mathrm{d}+1)(n_\mathrm{d}+2)]/2$.

In terms of these transformed state variables, the boundary conditions now hold for $z = -1$ (zero wealth) and $+1$ (infinite wealth). Furthermore, the boundary conditions are actually linear in terms of the transformed state variables. For the purpose of estimating the coefficient parameters, the boundary conditions can be implemented as a series of linear constraints on the coefficients in the Chebyshev polynomials. Appendix E explains in detail how the boundary conditions are implemented.

To capture the nonlinearities in the demand functions and the interactions between the two state variables, it is necessary to use high-order polynomials. Let n_u and n_u denote the total orders (maximum sum of powers of the two state variables) of the polynomials in the numerator and denominator of the estimated equilibrium demand function $X(\theta, W)$. The total number of coefficient parameters needed to specify the demand functions is $[(n_\mathrm{u}+1)(n_\mathrm{u}+2)]/2 + [(n_\mathrm{d}+1)(n_\mathrm{d}+2)]/2$. The boundary conditions, implemented as a series of linear constraints on the coefficients, reduce the number of coefficient parameters by $2n_\mathrm{u} + 2n_\mathrm{d} + 2$, resulting in $[n_\mathrm{u}(n_\mathrm{u}-1)]/2 + [n_\mathrm{d}(n_\mathrm{d}-1)]/2$ free parameters. In the numerical illustration of Section 4, the degree of both the numerator and the denominator in X is 13, which results in 210 coefficients. The constraints implied by the boundary conditions reduce this number by 54. Thus, 156 coefficient parameters in total need to be estimated.

To estimate the coefficients involves minimization of the sum of squared errors in the partial differential equations over a fixed set of test points. It appears that the demand function has more curvature near the boundaries $z = +1$ and -1, so instead of using a uniformly spaced grid of test points in the transformed state variables, more points near the boundaries are chosen. The grid size is 21 (for variable y) by 64 (for variable z), so the partial differential equations are evaluated at 1,344 points. Since there are 156 parameters to be estimated, the system is overdetermined by a factor of roughly nine.

Two types of error functions have been used at the same time. One is defined as

$$\text{Error1} = \frac{X - X^*}{\sigma_\theta}, \tag{D.11}$$

the difference between the given strategy X and the optimal strategy X^* normalized by the volatility of noise trading. Since the magnitude of X or X^* is very small when convergence traders' wealth is small, this method of calculating error underestimates errors to the convergence traders' portfolio over the region where wealth is small. The other error function is defined as

$$\text{Error2} = \frac{\sigma_F(X - X^*)}{W}, \tag{D.12}$$

the difference between the percentage wealth volatility caused by the fundamental shocks using X and X^*. Since this error function is defined by the percentage of wealth, it can correctly estimate errors over the region where convergence traders' wealth is small. But it may underestimate the errors to the market-clearing condition over the region where wealth is large, because values of X and X^* can be small relative to wealth W. To give precise estimates of the numerical errors over all regions, a combination of these two types of errors is used:

$$\text{Error} = \sqrt{\text{Error1}^2 + \text{Error2}^2}. \tag{D.13}$$

For the example described below, the maximum error is about 10^{-3}. This indicates that both types of numerical errors in the fixed-point problem are small, and an equilibrium has probably been found.

To solve the minimization problem, a Levenberg-Marquart algorithm is used. Despite the use of Chebyshev polynomials, the Hessian in this problem is not well behaved because of the linear constraints from the boundary conditions. Therefore, a gradient method has the potential to work better than Newton's method. The Levenberg-Marquart algorithm is designed to adjust smoothly between these two methods and thus deals with this problem.

Appendix E. Boundary constraints

The boundary conditions are linear in the after-transformation state variables y:

$$X(y,1) = \bar{\theta} + \frac{4\sigma_\theta}{\sqrt{2\lambda_\theta}} T_1(y), \tag{E.1}$$

$$X(y,-1) = 0. \tag{E.2}$$

Due to the properties of Chebyshev polynomials, $T_j(1) = 1$ and $T_j(-1) = (-1)^j$. The function X in Eq. (D.10) becomes an expansion in y when $z = 1$ or -1. To match the coefficients of y on the two bounds with the boundary conditions (E.1) and (E.2), the following constraints on the expansion coefficients are obtained:

$$\sum_{j=0}^{n_\mathrm{d}} a_\mathrm{d}(0,j) = 1, \tag{E.3}$$

$$\sum_{j=0}^{n_\mathrm{d}} (-1)^j a_\mathrm{d}(0,j) = 1, \tag{E.4}$$

$$\sum_{j=0}^{n_\mathrm{d}-i} a_\mathrm{d}(i,j) = 0 \quad \forall i \neq 0, \tag{E.5}$$

$$\sum_{j=0}^{n_\mathrm{d}-i} (-1)^j a_\mathrm{d}(i,j) = 0 \quad \forall i \neq 0, \tag{E.6}$$

$$\sum_{j=0}^{n_\mathrm{u}} a_\mathrm{u}(0,j) = \bar{\theta}, \tag{E.7}$$

$$\sum_{j=0}^{n_\mathrm{u}-1} a_\mathrm{u}(1,j) = \frac{4\sigma_\theta}{\sqrt{2\lambda_\theta}}, \tag{E.8}$$

$$\sum_{j=0}^{n_\mathrm{d}-i} a_\mathrm{u}(i,j) = 0 \quad \forall i > 1, \tag{E.9}$$

$$\sum_{j=0}^{n_\mathrm{d}-i} (-1)^j a_\mathrm{d}(i,j) = 0 \quad \forall i. \tag{E.10}$$

These linear constraints can be implemented by determining the first two columns of the expansion coefficients from the rest of the columns of the expansion coefficients:

$$a_{\mathrm{d}}(0,0) = 1 - \frac{1}{2}\sum_{j=2}^{n_{\mathrm{d}}} [1 + (-1)^j] a_{\mathrm{d}}(0, j), \tag{E.11}$$

$$a_{\mathrm{d}}(0,1) = -\frac{1}{2}\sum_{j=2}^{n_{\mathrm{d}}} [1 - (-1)^j] a_{\mathrm{d}}(0, j), \tag{E.12}$$

$$a_{\mathrm{d}}(i,0) = -\frac{1}{2}\sum_{j=2}^{n_{\mathrm{d}}-i} [1 + (-1)^j] a_{\mathrm{d}}(i,j) \quad \forall i \neq 0, \tag{E.13}$$

$$a_{\mathrm{d}}(i,1) = -\frac{1}{2}\sum_{j=2}^{n_{\mathrm{d}}-i} [1 - (-1)^j] a_{\mathrm{d}}(i,j) \quad \forall i \neq 0, \tag{E.14}$$

$$a_{\mathrm{u}}(0,0) = \frac{\bar{\theta}}{2} - \frac{1}{2}\sum_{j=2}^{n_{\mathrm{u}}} [1 + (-1)^j] a_{\mathrm{u}}(0,j), \tag{E.15}$$

$$a_{\mathrm{u}}(0,1) = \frac{\bar{\theta}}{2} - \frac{1}{2}\sum_{j=2}^{n_{\mathrm{u}}} [1 - (-1)^j] a_{\mathrm{u}}(0,j), \tag{E.16}$$

$$a_{\mathrm{u}}(1,0) = \frac{\sqrt{2}\sigma_\theta}{\sqrt{\lambda_\theta}} - \frac{1}{2}\sum_{j=2}^{n_{\mathrm{u}}} [1 + (-1)^j] a_{\mathrm{u}}(1,j), \tag{E.17}$$

$$a_{\mathrm{u}}(1,1) = \frac{\sqrt{2}\sigma_\theta}{\sqrt{\lambda_\theta}} - \frac{1}{2}\sum_{j=2}^{n_{\mathrm{u}}} [1 - (-1)^j] a_{\mathrm{u}}(1,j), \tag{E.18}$$

$$a_{\mathrm{u}}(i,0) = -\frac{1}{2}\sum_{j=2}^{n_{\mathrm{u}}-i} [1 + (-1)^j] a_{\mathrm{u}}(i,j) \quad \forall i > 1, \tag{E.19}$$

$$a_{\mathrm{u}}(i,1) = -\frac{1}{2}\sum_{j=2}^{n_{\mathrm{u}}-i} [1 - (-1)^j]\, a_{\mathrm{u}}(i,j) \quad \forall i > 1. \tag{E.20}$$

In this way, the total number of parameters is reduced by $2n_{\mathrm{u}} + 2n_{\mathrm{d}} + 2$ (first two columns of these two coefficient matrices). Therefore, the total number of parameters needed to specify the demand functions X is $[n_{\mathrm{u}}(n_{\mathrm{u}} - 1)]/2 + [n_{\mathrm{d}} (n_{\mathrm{d}} - 1)]/2$.

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