Internet Appendix for Informational Frictions and Commodity Markets

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In this appendix, we present in detail an extended model with a futures market, in supplement to the summary of the model extension in Section IV of the main paper.

A. Model Setting

We introduce a new date $t = 0$ before the dates $t = 1$ and 2 in the baseline model, and a centralized futures market at $t = 0$ for delivery of the commodity at $t = 1$. All agents can take positions in the futures market at $t = 0$, and can choose to revise or unwind their positions before delivery at $t = 1$. The ability to unwind positions before delivery is an advantage that makes futures market trading appealing in practice.

We keep all of the agents in the baseline model: island households, goods producers, and commodity suppliers and add a group of financial traders. These traders invest in the commodity by taking a long position in the futures market at $t = 0$ and then unwinding this position at $t = 1$ without taking delivery.

To focus on information aggregation through trading in the futures market, we assume that there is no spot market trading at $t = 0$. At $t = 1$, a spot market naturally emerges through commodity delivery for the futures market. Commodity suppliers take a short position in the futures market at $t = 0$ and then make delivery at $t = 1$. Suppliers' marginal cost of supplying the commodity determines the spot price. When a trader chooses to unwind a futures position at $t = 1$, his gain/loss is determined by this spot price.

*Citation format: Sockin, Michael, and Wei Xiong, Internet Appendix for "Informational Frictions and Commodity Markets," Journal of Finance. Please note: Wiley-Blackwell is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the authors of the article.

Table IA.I

Timeline of the Extended Model

	t=0	t=1	t=2
	Futures Market	Spot Market	Goods Market
Households			Trade/Consume Goods
Producers	Observe Signals Long Futures	Take Delivery Produce Goods	
Suppliers	Short Futures	Observe Supply Shock Deliver Commodity	
Fin Traders	Long/Short Futures	Unwind Position	

Table IA.I specifies the timeline of the extended model. We keep the same specification for the island households, who trade and consume both home and away goods at $t = 2$ as described in Section I.A of the main paper. We modify some of the specifications for goods producers and commodity suppliers and describe our specifications for financial traders below.

A.1 Goods Producers

As in the main model, we allow goods producers to have the same production technology and receive their private signals at $t = 0$. Each producer takes a long position in the futures market at $t = 0$ and then commodity delivery at $t = 1$. The timing of the producer's information flow is key to our analysis. At $t = 0$, producer i's information set $\mathcal{I}_i^0 = \{s_i, F\}$ includes its private signal s_i and the traded futures price F. At $t = 1$, its information set $\mathcal{I}_i^1 = \{s_i, F, P_X\}$ includes the updated spot price P_X.

We allow the producer to use its updated information set at $t = 1$ to revise its futures position for commodity delivery. That is, its production decision is based on not only its private signal and the futures price but also the updated spot price. Thus, it is not obvious that noise in the futures market can affect the producer's production decision and commodity demand. We examine this key issue with our extended model.

At $t = 1$, the producer optimizes its production decision X_i (i.e., commodity demand) based on its updated information set $\mathcal{I}_i^1$:

$$\max_{X_i} \; E\left[P_i Y_i \middle| \, \mathcal{I}_i^1\right] - P_X X_i + (P_X - F)\tilde{X}_i.$$

The first two terms above represent the producer's expected profit from goods production and the last term is the gain/loss from its futures position. The producer's optimal production decision is then

$$X_i = \left\{ \phi E\left[A X_j^{\phi\eta} \middle| \mathcal{I}_i^1 \right] \Big/ P_X \right\}^{1/(1-\phi(1-\eta))}. \tag{IA.1}$$

When deciding its futures position at $t = 0$, the producer faces a nuanced issue in that, because it does not need to commit its later production decision to the initial futures position, it may engage in dynamic trading. In other words, it could choose a futures position to maximize its expected trading profit at $t = 0$. This trading motive is not essential for our focus on analyzing the aggregation of the producers' information but significantly complicates derivation of the futures market equilibrium. To avoid this complication, we make a simplifying assumption that the producers are myopic at $t = 0$. That is, at $t = 0$, each producer chooses a futures position as if it commits to taking full delivery and using the good for production, even though the producer can revise its production decision based on the updated information at $t = 1$. While this simplifying assumption affects each producer's trading profit, it is innocuous for our analysis of how the futures price feeds back to the producers' later production decisions because each producer still makes good use of its information and the futures price is informative by aggregating each producer's information.

Specifically, at $t = 0$ the producer chooses a futures position $\tilde{X}_i$ to maximize the following expected production profit based on its information set $\mathcal{I}_i^0$:

$$\max_{\tilde{X}_i} \; E\left[P_i Y_i \middle| \, \mathcal{I}_i^0\right] - F\tilde{X}_i,$$

where it treats $\tilde{X}_i$ as its production input at $t = 1$. Throughout the rest of this appendix, we use a tilde to denote variables and coefficients associated with the futures market at $t = 0$; we maintain the same notation without the tilde for variables related to the spot market at $t = 1$. The producer's futures position is then

$$\tilde{X}_i = \left\{ \phi E\left[A \tilde{X}_j^{\phi\eta} \middle| \mathcal{I}_i^0 \right] \Big/ F \right\}^{1/(1-\phi(1-\eta))}. \tag{IA.2}$$

A.2 Financial Traders

We introduce a group of financial traders, who trade in the futures market at $t = 0$ and unwind their position at $t = 1$ before delivery. For simplicity, we assume that the aggregate position of financial traders and goods producers is given by the aggregate position of producers multiplied by a factor $e^{\kappa \log A+\theta}$:

$$e^{\kappa \log A+\theta} \int_{-\infty}^{\infty} \tilde{X}_i\left(s_i, F\right) d\Phi\left(\varepsilon_i\right),$$

where the factor $e^{\kappa \log A+\theta}$ represents the contribution of financial traders. This multiplicative specification is useful for ensuring the tractable log-linear equilibrium of our model.[1]

We allow the contribution of financial traders $e^{\kappa \log A+\theta}$ to contain a component $\kappa \log A$, where $\kappa > 0$, to capture the possibility that the trading of financial traders is partially driven by their knowledge of the global fundamental $\log A$.

The trading of financial traders also contains a random component θ, which is unobservable by other market participants. This assumption is realistic in two respects. First, in practice, the trading of financial traders is often driven by portfolio diversification and risk-control purposes unrelated to fundamentals of commodity markets. Second, market participants cannot directly observe others' positions.[2] Specifically, we assume that θ has a normal distribution independent of other sources of uncertainty in the model,

$$\theta \backsim \mathcal{N}\left(\overline{\theta}, \tau_\theta^{-1}\right),$$

with mean $\overline{\theta}$ and variance τ_θ^{-1}.

The presence of financial traders introduces an additional source of uncertainty to the futures market, as both goods producers and commodity suppliers cannot observe θ at $t = 0$. At $t = 1$, financial traders unwind their positions, and commodity suppliers make delivery only to goods producers.

[1] From an economic perspective, this specification implies that the position of financial traders tends to expand and contract with producers' futures position, which is broadly consistent with the expansion and contraction of the aggregate commodity futures positions of portfolio investors and hedge funds in the recent commodity price boom-and-bust cycle (e.g., Cheng, Kirilenko, and Xiong (2012)). Also note that $e^{\kappa \log A+\theta}$ can be less than one. This implies that financial traders may take a net short position at some point, which is consistent with short positions taken by hedge funds in practice.

[2] Despite the fact that large traders need to report their futures positions to the Commodities Future Trading Commission (CFTC) on a daily basis, ambiguity in trader classification and netting of positions taken by traders who are involved in different lines of business nevertheless make the aggregate positions provided by the CFTC's weekly Commitment of Traders Report to the public imprecise. See Cheng, Kirilenko, and Xiong (2012) for a more detailed discussion of the trader classification and netting problems in the CFTC's Large Trader Reporting System and a summary of positions taken by commodity index traders and hedge funds.

A.3 Commodity Suppliers

Commodity suppliers take a short position of $\tilde{X}_S$ in the futures market at $t = 0$ and then make delivery of X_S units of the commodity at $t = 1$. We maintain the same convex cost function for the suppliers: $\frac{k}{1+k}e^{-\xi/k}(X_S)^{\frac{1+k}{k}}$, where the supply shock ξ has a Gaussian distribution $\mathcal{N}(\bar{\xi}, \tau_\xi^{-1})$.

We assume that the suppliers observe their supply shock ξ only at $t = 1$, which implies that the supply shock does not affect the futures price at $t = 0$ and instead hits the spot market at $t = 1$. Due to this timing, the supply shock provides a camouflage for the unwinding of financial traders' aggregate futures position at $t = 1$. That is, even after financial traders unwind their position, the commodity spot price does not reveal their position.[3]

In summary, the suppliers' information set at $t = 0$ is $\mathcal{I}_S^0 = \{F\}$, and at $t = 1$ is $\mathcal{I}_S^1 = \{F, P_X, \xi\}$. At $t = 1$, the suppliers face the following optimization problem:

$$\max_{X_S} P_X X_S - \frac{k}{1+k}e^{-\xi/k}X_S^{\frac{1+k}{k}} + (F - P_X)\tilde{X}_S,$$

where they choose X_S—the quantity of commodity delivery—to maximize the profit from delivery in the first two terms. The last term is the gain/loss from their initial futures position. The suppliers' optimal supply curve is then given by $X_S = e^\xi P_X^k$, which is identical to their supply curve in the baseline model.

At $t = 0$, like the goods producers, the suppliers also face a nuanced issue related to dynamic trading. As their initial futures position does not necessarily equal their later commodity delivery, they may also choose to maximize the trading profit from $t = 0$ to $t = 1$. To be consistent with our earlier assumption about the myopic behavior of goods producers, we assume that at $t = 0$ the suppliers believe that goods producers will take full delivery of their futures positions and that the suppliers choose their initial short position to myopically maximize the profit from making delivery of $e^{-(\kappa \log A + \theta)}\tilde{X}_S$ units of the commodity to goods producers:

$$\max_{\tilde{X}_S} E\left[Fe^{-(\kappa \log A+\theta)}\tilde{X}_S \middle| \mathcal{I}_S^0\right] - E\left[\frac{k}{1+k}e^{-\xi/k}\left(e^{-(\kappa \log A+\theta)}\tilde{X}_S\right)^{\frac{1+k}{k}} \middle| \mathcal{I}_S^0\right].$$

[3]This timing may appear special in our static setting with only one round of futures market trading followed by physical commodity delivery, as there is no particular reason to argue whether letting the suppliers observe the supply shock at $t = 0$ or $t = 1$ is more natural. However, if we view this setting as one module of a more realistic setting with many recurrent periods and a supply shock arriving in each period, then there is always a supply shock hitting the market when financial traders unwind their futures position.

Since ξ is independent of θ and $\log A$, it is easy to derive

$$\tilde{X}_S = e^{\bar{\xi}-\sigma_\xi^2/2k}\left\{E\left[e^{-(\kappa\log A+\theta)}\middle|\ \mathcal{I}_S^0\right]/E\left[e^{-\frac{1+k}{k}(\kappa\log A+\theta)}\middle|\ \mathcal{I}_S^0\right]\right\}^k F^k, \tag{IA.3}$$

which is a function of the futures price F.

A.4 Joint Equilibrium of Different Markets

We analyze the joint equilibrium of a number of markets: the goods markets between each pair of matched islands at $t=2$, the spot market for the commodity at $t=1$, and the futures market at $t=0$. Equilibrium requires clearing of each of these markets:

- At $t=2$, for each pair of randomly matched islands $\{i,j\}$, the households of these islands trade their produced goods and clear the market of each good:

$$\begin{aligned} C_i + C_j^* &= AX_i^\phi, \\ C_i^* + C_j &= AX_j^\phi. \end{aligned}$$

- At $t=1$, the commodity supply equals the goods producers' aggregate demand:

$$\int_{-\infty}^{\infty} X\left(s_i, F, P_X\right) d\Phi\left(\varepsilon_i\right) = X_S\left(P_X, \xi\right).$$

- At $t=0$, the futures market clears:

$$e^{\kappa\log A+\theta}\int_{-\infty}^{\infty} \tilde{X}_i\left(s_i, F\right) d\Phi\left(\varepsilon_i\right) = \tilde{X}_S\left(F\right).$$

B. The Equilibrium

The goods market equilibrium at $t=2$ remains identical to that derived in Proposition 1 for the main model. The futures market equilibrium at $t=0$ and the spot market equilibrium at $t=1$ also remain log-linear and can be derived following a similar procedure as the derivation of Proposition 2. The following proposition summarizes the key features of the equilibrium with explicit expressions for all coefficients given in Section D.

Proposition 1 *At $t=0$, the futures market has a unique log-linear equilibrium: the futures price is a log-linear function of $\log A$ and θ,*

$$\log F = \tilde{h}_A \log A + \tilde{h}_\theta \theta + \tilde{h}_0, \tag{IA.4}$$

with the coefficients $\tilde{h}_A > 0$ *and* $\tilde{h}_\theta > 0$*, while the long position taken by goods producer* i *is a log-linear function of its private signal* s_i *and* $\log F$*,*

$$\log \tilde{X}_i = \tilde{l}_s s_i + \tilde{l}_F \log F + \tilde{l}_0, \tag{IA.5}$$

with the coefficient $\tilde{l}_s > 0$.

At $t = 1$*, the spot market also has a unique log-linear equilibrium: the spot price of the commodity is a log-linear function of* $\log A$*,* $\log F$*, and* ξ*,*

$$\log P_X = h_A \log A + h_F \log F + h_\xi \xi + h_0, \tag{IA.6}$$

with the coefficients $h_A > 0$*,* $h_F > 0$*, and* $h_\xi < 0$*, while the commodity consumed by producer* i *is a log-linear function of* s_i*,* $\log F$*, and* $\log P_X$*,*

$$\log X_i = l_s s_i + l_F \log F + l_P \log P_X + l_0, \tag{IA.7}$$

with the coefficients $l_s > 0$ *and* $l_F > 0$*, and the sign of* l_P *undetermined.*

There are two rounds of information aggregation in the equilibrium. During the first round of trading in the futures market at $t = 0$, goods producers take long positions based on their private signals. The futures price $\log F$ aggregates producers' information, and reflects a linear combination of $\log A$ and θ, as given in (IA.4). The futures price does not fully reveal $\log A$ due to the θ noise originated from the trading of financial traders. The spot price that emerges from the commodity delivery at $t = 1$ represents another round of information aggregation by pooling together the goods producers' demand for delivery. As a result of the arrival of the supply shock ξ, the spot price $\log P_X$ does not fully reveal either $\log A$ or θ, and instead reflects a linear combination of $\log A$ and ξ, as derived in (IA.6).

Despite the updated information from the spot price at $t = 1$, the informational content of $\log F$ is not subsumed by the spot price, and still has an influence on goods producers' expectations of $\log A$. As a result of this informational role, equation (IA.7) confirms that each goods producer's commodity demand at $t = 1$ is increasing with $\log F$, as $l_F > 0$, and equation (IA.6) shows that the spot price is also increasing with $\log F$, as $h_F > 0$. This is the key feedback channel through which futures market trading affects commodity demand and the spot price despite the availability of information from the spot price.

The simplifying assumptions we make regarding the myopic trading of goods producers and commodity suppliers at $t = 0$ are innocuous to the informational role of the futures price at $t = 1$. As long as goods producers trade on their private signals, the futures price would aggregate the information, which in turn establishes the futures price as a useful price signal for the later round at $t = 1$. Our simplifying assumptions have quantitative consequences for goods producers' trading profits and the efficiency of the futures price signal, but should not critically affect the qualitative feedback channel of the futures price, which we characterize in the next subsection.[4]

Interestingly, Proposition 1 also reveals that l_P can be either positive or negative, due to the offsetting cost effect and informational effect of the spot price, similar to our characterization of the main model.

C. Implications

C.1 Feedback on Commodity Demand

As financial traders do not take or make any physical delivery, their trading in the futures market does not have direct effect on commodity supply or demand. However, their trading affects the futures price, through which it can further impact commodity demand and spot prices. By substituting equation (IA.4) into (IA.6), we express the spot price $\log P_X$ as a linear combination of primitive variables $\log A$, θ, and ξ:

$$\log P_X = \left(h_A + h_F \tilde{h}_A\right) \log A + h_F \tilde{h}_\theta \theta + h_\xi \xi + h_F \tilde{h}_0 + h_0. \tag{IA.8}$$

The θ term arises through the futures price. As $h_F > 0$ and $\tilde{h}_\theta > 0$, the noise from financial traders' trading in the futures market, θ, has a positive effect on the spot price.

Furthermore, by substituting the equation above and (IA.4) into (IA.7), we obtain an individual producer's commodity demand as

$$\begin{aligned} \log X_i \quad = \quad & l_s s_i + \left(l_F \tilde{h}_A + l_P \left(h_A + h_F \tilde{h}_A\right)\right) \log A + \left(l_F + l_P h_F\right) \tilde{h}_\theta \theta + l_P h_\xi \xi \\ & + \left(l_F + l_P h_F\right) \tilde{h}_0 + l_P h_0 + l_0, \end{aligned}$$

and the producers' aggregate demand as

$$\begin{aligned} \log\left[\int_{-\infty}^{\infty} X\left(s_i, F, P_X\right) d\Phi\left(\varepsilon_i\right)\right] \quad = \quad & \left[l_s + l_P h_A + l_F \tilde{h}_A + l_P h_F \tilde{h}_A\right] \log A + \left(l_F + l_P h_F\right) \tilde{h}_\theta \theta + l_P h_\xi \xi \\ & + \left(l_F + l_P h_F\right) \tilde{h}_0 + l_P h_0 + l_0 + \frac{1}{2} l_s^2 \tau_s^{-1}. \end{aligned} \tag{IA.9}$$

[4] Note that despite the different information content of the futures price and the spot price, there is no arbitrage between the two prices because the two prices are traded at different points in time and the spot price is exposed to the supply shock realized later.

By using equation (IA.28) in the proof of Proposition 1, the coefficient on θ in the aggregate commodity demand is

$$l_F + l_P h_F = k h_F > 0.$$

Thus, θ also has a positive effect on aggregate commodity demand.

The effects of θ on commodity demand and the spot price clarify the simple yet important conceptual point that traders in commodity futures markets, who never take or make physical commodity delivery, can nevertheless impact commodity markets through the informational feedback channel of commodity futures prices.

C.2 Market Transparency

Information frictions in the futures market, originating from the unobservability of the positions of different participants, are essential in order for the trading of financial traders to impact the demand for the commodity and spot prices. The following proposition confirms that as $\tau_\theta \to \infty$ (i.e., the position of financial traders becomes publicly observable), the spot market equilibrium converges to the perfect-information benchmark.

Proposition 2 *As $\tau_\theta \to \infty$, the spot price and aggregate demand converge to the perfect-information benchmark.*

Proposition 2 shows that by improving transparency of the futures market, one can achieve the perfect-information benchmark because by making the position of financial traders publicly observable, the θ noise no longer interferes with the information aggregation in the futures market. As a result, the futures price fully reveals the global fundamental, which allows goods producers to achieve the same efficiency allowed by the perfect-information benchmark. This nice convergence result relies on the assumption that the supply noise ξ does not affect the futures market trading at $t = 0$ and hits the spot market only at $t = 1$. Nevertheless, this result highlights the importance of improving market transparency.[5]

[5]While our analysis focuses on the noise effect of their trading, financial traders can also contribute to information aggregation. As κ increases, the futures position of financial traders builds more on the global economic fundamental $\log A$, in which case the futures price $\log F$ becomes more informative of $\log A$. This is because one can prove based on Proposition 1 that $\tilde{h}_A/\tilde{h}_\theta$, the ratio of the loadings of $\log F$ on $\log A$ and θ, increases with κ.

Imposing position limits on speculators in commodity futures markets has occupied much of the post-2008 policy debate, while improving market transparency has received much less attention. By highlighting the feedback effect originating from information frictions as a key channel for noise in futures market trading to affect commodity prices and demand, our model suggests that imposing position limits may not address the central information frictions that confront participants in commodity markets and thus may not be effective in reducing potential distortion caused by speculative trading. Instead, increasing the transparency of trading positions might be more effective.

D. Technical Proofs

D.1 Proof of Proposition 1

We follow the same procedure as in the proof of Proposition 2 in the main paper to derive the futures market equilibrium at $t = 0$. We first conjecture the log-linear forms for the futures price and each island producer's long position in (IA.4) and (IA.5) with the coefficients $\tilde{h}_0$, $\tilde{h}_A$, $\tilde{h}_\theta$, $\tilde{l}_0$, $\tilde{l}_s$, and $\tilde{l}_F$ to be determined by equilibrium conditions.

Let z be a sufficient statistic of the information contained in F :

$$z \equiv \frac{\log F - \tilde{h}_0 - \tilde{h}_\theta \bar{\theta}}{\tilde{h}_A} = \log A + \frac{\tilde{h}_\theta}{\tilde{h}_A}\left(\theta - \bar{\theta}\right).$$

Then, conditional on observing s_i and F, producer i's expectation of $\log A$ is

$$\begin{aligned} E\left[\log A \mid s_i, \log F\right] &= E\left[\log A \mid s_i, z\right] = \frac{1}{\tau_A + \tau_s + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta}\left(\tau_A \bar{a} + \tau_s s_i + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta z\right) \\ &= c_0 + c_s s_i + c_F\left(\log F - \tilde{h}_0 - \tilde{h}_\theta \bar{\theta}\right), \end{aligned} \tag{IA.10}$$

where

$$\begin{aligned} c_0 &= \left(\tau_A + \tau_s + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}\left(\tau_A \bar{a} - \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta \frac{\tilde{h}_0 + \tilde{h}_\theta \bar{\theta}}{\tilde{h}_A}\right), \\ c_s &= \left(\tau_A + \tau_s + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}\tau_s, \\ c_F &= \left(\tau_A + \tau_s + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}\frac{\tilde{h}_A}{\tilde{h}_\theta^2}\tau_\theta. \end{aligned}$$

Producer i's conditional variance of $\log A$ is

$$\tilde{\tau}_{A,i} = Var\left[\log A \mid s_i, \log F\right] = \left(\tau_A + \tau_s + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}. \tag{IA.11}$$

By substituting equation (IA.5) into producer i's optimal production decision in equation (IA.2), we obtain

$$\begin{aligned}\log \tilde{X}_i &= \frac{1}{1-\phi(1-\eta)}\log\phi + \frac{\phi\eta}{1-\phi(1-\eta)}\tilde{l}_0 + \frac{1}{1-\phi(1-\eta)}\left(\phi\eta\tilde{l}_F - 1\right)\log F \\ &\quad + \left(\frac{1+\phi\eta\tilde{l}_s}{1-\phi(1-\eta)}\right)\left(c_0 + c_s s_i + c_F\frac{\log F}{\tilde{h}_A}\right) + \frac{\left(1+\phi\eta\tilde{l}_s\right)^2}{2(1-\phi(1-\eta))}\tilde{\tau}_{A,i} + \frac{\phi^2\eta^2\tilde{l}_s^2}{2(1-\phi(1-\eta))}\tau_s^{-1}.\end{aligned}$$

For the above equation to match the conjectured equilibrium position in equation (IA.5), the constant term and the coefficients on s_i and $\log F$ have to be identical:

$$\begin{aligned}\tilde{l}_0 &= \frac{\phi\eta}{1-\phi(1-\eta)}\tilde{l}_0 + \left(\frac{1+\phi\eta\tilde{l}_s}{1-\phi(1-\eta)}\right)c_0 + \frac{\left(1+\phi\eta\tilde{l}_s\right)^2}{2(1-\phi(1-\eta))}\tilde{\tau}_{A,i} \\ &\quad + \frac{\phi^2\eta^2\tilde{l}_s^2}{2(1-\phi(1-\eta))}\tau_s^{-1} + \frac{1}{1-\phi(1-\eta)}\log\phi, \qquad \text{(IA.12)} \\ \tilde{l}_s &= \left(\frac{1+\phi\eta\tilde{l}_s}{1-\phi(1-\eta)}\right)c_s, \qquad \text{(IA.13)} \\ \tilde{l}_F &= \frac{\phi\eta}{1-\phi(1-\eta)}\tilde{l}_F - \frac{1}{1-\phi(1-\eta)} + \left(\frac{1+\phi\eta\tilde{l}_s}{1-\phi(1-\eta)}\right)c_F. \qquad \text{(IA.14)}\end{aligned}$$

By substituting equation (IA.13) into (IA.14), we have

$$\tilde{l}_s = \frac{1+(1-\phi)\tilde{l}_F}{1-\phi(1-\eta)}\frac{\tilde{h}_\theta^2}{\tilde{h}_A}\tau_s\tau_\theta^{-1}. \tag{IA.15}$$

By manipulating equation (IA.13), we also have that

$$\tilde{l}_s = \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}\frac{\tau_s}{1-\phi(1-\eta)}. \tag{IA.16}$$

We now use market clearing of the futures market to determine three other equations for the coefficients. Aggregating equation (IA.5) gives the producers' aggregate position,

$$\int_{-\infty}^{\infty}\tilde{X}_i(s_i,F)\,d\Phi(\varepsilon_i) = \exp\left[\left(\tilde{l}_s + \tilde{l}_F\tilde{h}_A\right)\log A + \tilde{l}_F\tilde{h}_\theta\theta + \tilde{l}_0 + \tilde{l}_F\tilde{h}_0 + \frac{1}{2}\tilde{l}_s^2\tau_s^{-1}\right]. \tag{IA.17}$$

Equation (IA.3) gives $\tilde{X}_S$. Define

$$z_\theta \equiv \frac{\log F - \tilde{h}_0 - \tilde{h}_A \bar{a}}{\tilde{h}_\theta} = \frac{\tilde{h}_A}{\tilde{h}_\theta}(\log A - \bar{a}) + \theta.$$

The suppliers' conditional expection of θ is then

$$E[\theta \mid \log F] = E[\theta \mid z_\theta] = \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\left[\tau_\theta\bar{\theta} + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\left(\frac{\log F - \tilde{h}_0}{\tilde{h}_\theta} - \frac{\tilde{h}_A}{\tilde{h}_\theta}\bar{a}\right)\right],$$

with conditional variance $Var[\theta \mid \log F] = \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}$. Their conditional expectation of $\log A$ is

$$E[\log A \mid \log F] = E[\log A \mid z_\theta] = \left(\tau_A + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}\left[\tau_A\bar{a} + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\left(\frac{\log F - \tilde{h}_0 - \tilde{h}_\theta\bar{\theta}}{\tilde{h}_A}\right)\right],$$

with conditional variance $Var[\log A \mid \log F] = \left(\tau_A + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1}$. Thus, we obtain an expression for $\log \tilde{X}_S$ that is linear in $\log A$ and θ.

Next, the market-clearing condition $\log\left[e^{\kappa \log A + \theta}\int_{-\infty}^{\infty}\tilde{X}_i(s_i, F)\, d\Phi(\varepsilon_i)\right] = \log \tilde{X}_S$ requires that the coefficients on $\log A$ and θ and the constant term be identical on both sides:

$$\kappa + \tilde{l}_s + \tilde{l}_F\tilde{h}_A = k\tilde{h}_A + \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\left(\frac{\tilde{h}_\theta}{\tilde{h}_A}\tau_A + \kappa\tau_\theta\right), \tag{IA.18}$$

$$1 + \tilde{l}_F\tilde{h}_\theta = k\tilde{h}_\theta + \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\frac{\tilde{h}_\theta}{\tilde{h}_A}\left(\frac{\tilde{h}_\theta}{\tilde{h}_A}\tau_A + \kappa\tau_\theta\right), \tag{IA.19}$$

$$\begin{aligned}\tilde{l}_0 + \tilde{l}_F\tilde{h}_0 + \frac{1}{2}\tilde{l}_s^2\tau_s^{-1} &= k\tilde{h}_0 + \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\left(1 + \kappa\frac{\tilde{h}_\theta}{\tilde{h}_A}\right)\tau_\theta\bar{\theta} \\ &\quad - \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\frac{\tilde{h}_\theta}{\tilde{h}_A}\left(1 + \kappa\frac{\tilde{h}_\theta}{\tilde{h}_A}\right)\tau_A\bar{a} + \bar{\xi} - \sigma_\xi^2/2k \\ &\quad - \frac{\kappa^2}{2k}(1+2k)\left(\tau_A + \frac{\tilde{h}_A^2}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1} - \frac{1}{2k}(1+2k)\left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}.\end{aligned} \tag{IA.20}$$

Equation (IA.19) directly implies that

$$\tilde{l}_F = k + \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\left(\kappa\frac{\tilde{h}_\theta}{\tilde{h}_A} - 1\right)\tau_\theta\tilde{h}_\theta^{-1}. \tag{IA.21}$$

Equations (IA.18) and (IA.19) together imply that

$$\tilde{l}_s = \tilde{h}_\theta^{-1}\tilde{h}_A - \kappa.$$

By combining this equation with (IA.16), we arrive at

$$\tilde{l}_s^3 + 2\kappa \tilde{l}_s^2 + \left(\tau_\theta^{-1}\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_\theta^{-1}\tau_s + \kappa^2\right)\tilde{l}_s - \frac{\tau_\theta^{-1}\tau_s}{1-\phi(1-\eta)} = 0. \tag{IA.22}$$

By further making the convenient substitution $L_s = \tilde{l}_s + \frac{2}{3}\kappa$, called the Tschirnhaus transformation, we obtain the depressed cubic polynomial

$$L_s^3 + pL_s + q = 0,$$

where

$$\begin{aligned} p &= \tau_\theta^{-1}\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_\theta^{-1}\tau_s - \frac{1}{3}\kappa^2, \\ q &= -\frac{2}{3}\kappa\tau_\theta^{-1}\tau_A - \frac{2}{3}\kappa\frac{1-\phi}{1-\phi(1-\eta)}\tau_\theta^{-1}\tau_s - \frac{2}{27}\kappa^3 - \frac{\tau_\theta^{-1}\tau_s}{1-\phi(1-\eta)}. \end{aligned}$$

It is easy to verify that $\frac{q^2}{4} + \frac{p^3}{27} > 0$ and therefore L_s is a real root of this depressed cubic polynomial, which has one real and two complex roots. Following Cardano's method, the one real root of equation (IA.22) is given by

$$\tilde{l}_s = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} - \frac{2}{3}\kappa.$$

Since the coefficients of equation (IA.22) change sign only once, by Descartes' Rule of Signs the real root must be positive.

Since $\tilde{l}_s = \tilde{h}_\theta^{-1}\tilde{h}_A - \kappa$, we have

$$\tilde{h}_\theta = \left(\tilde{l}_s + \kappa\right)^{-1}\tilde{h}_A,$$

which, together with our expression for $\tilde{l}_s$ and equations (IA.15) and (IA.21), implies that

$$\tilde{h}_\theta = \left((1-\phi(1-\eta))\tau_s^{-1} + \frac{1-\phi}{\tau_\theta\left(\tilde{l}_s+\kappa\right)^2 + \tau_A}\right)\frac{\tau_\theta}{1+k(1-\phi)}\tilde{l}_s\left(\tilde{l}_s+\kappa\right) \tag{IA.23}$$

and therefore

$$\tilde{h}_A = \left((1-\phi(1-\eta))\tau_s^{-1} + \frac{1-\phi}{\tau_\theta\left(\tilde{l}_s+\kappa\right)^2 + \tau_A}\right)\frac{\tau_\theta}{1+k(1-\phi)}\tilde{l}_s\left(\tilde{l}_s+\kappa\right)^2. \tag{IA.24}$$

Since by equation (IA.22), $\tilde{l}_s > 0$, $\tilde{h}_A$ and $\tilde{h}_\theta$ must have the same sign. With $\tilde{h}_A$ and $\tilde{h}_\theta$ determined, $\tilde{l}_F$ is then given by equation (IA.21),

$$\tilde{l}_F = k + \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\left(\kappa\frac{\tilde{h}_\theta}{\tilde{h}_A} - 1\right)\tau_\theta \tilde{h}_\theta^{-1},$$

$\tilde{h}_0$ by equation (IA.12),

$$\begin{aligned}
\tilde{h}_0 \;=\; & \left(k - \tilde{l}_F + \frac{1-\phi(1-\eta)}{1-\phi}\tilde{l}_s\tau_s^{-1}\frac{\tilde{h}_A}{\tilde{h}_\theta^2}\tau_\theta\right)^{-1} \cdot \\
& \left(\frac{1}{1-\phi}\log\phi - \bar{\xi} + \sigma_\xi^2/2k + \frac{1}{2k}(1+2k)\left(1+\kappa^2\frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\right)\left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\right. \\
& + \frac{1}{2}\left(\tilde{l}_s + \frac{1-\phi(1-\eta)}{1-\phi}\left(1+\phi\eta\tilde{l}_s + \frac{\phi^2\eta^2\tilde{l}_s}{1-\phi(1-\eta)}\right)\right)\tilde{l}_s\tau_s^{-1} \\
& \left. + \left(\frac{1-\phi(1-\eta)}{1-\phi}\tilde{l}_s\tau_s^{-1} + \left(\tau_\theta\frac{\tilde{h}_A^2}{\tilde{h}_\theta^2} + \tau_A\right)^{-1}\left(\frac{\tilde{h}_A}{\tilde{h}_\theta} + \kappa\right)\right)\left(\tau_A\bar{a} - \left(\tilde{l}_s + \kappa\right)\tau_\theta\bar{\theta}\right)\right),
\end{aligned}$$

and $\tilde{l}_0$ by equation (IA.20),

$$\begin{aligned}
\tilde{l}_0 \;=\; & \left(k - \tilde{l}_F\right)\tilde{h}_0 + \bar{\xi} - \sigma_\xi^2/2k + \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\left(1 + \kappa\frac{\tilde{h}_\theta}{\tilde{h}_A}\right)\tau_\theta\bar{\theta} \\
& - \left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}\frac{\tilde{h}_\theta}{\tilde{h}_A}\left(1 + \kappa\frac{\tilde{h}_\theta}{\tilde{h}_A}\right)\tau_A\bar{a} - \frac{1}{2}\tilde{l}_s^2\tau_s^{-1} \\
& - \frac{1}{2k}(1+2k)\left(1+\kappa^2\frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\right)\left(\tau_\theta + \frac{\tilde{h}_\theta^2}{\tilde{h}_A^2}\tau_A\right)^{-1}.
\end{aligned}$$

We now derive the spot market equilibrium at $t = 1$. We again first conjecture that the spot price P_X and a goods producer's updated commodity demand take the log-linear forms given in equations (IA.6) and (IA.7) with the coefficients h_0, h_A, h_F, h_ξ, l_0, l_s, l_F, and l_P to be determined by equilibrium conditions.

The mean and variance of producer i's prior belief over $\log A$ carried from $t = 0$ is derived in (IA.10) and (IA.11). Define

$$z_p = \frac{\log P_X - h_0 - h_F\log F - h_\xi\bar{\xi}}{h_A} = \log A + \frac{h_\xi}{h_A}\left(\xi - \bar{\xi}\right).$$

Then, after observing the spot price P_X at $t = 1$, the producer's expectation of $\log A$ is

$$E\left[\log A \mid s_i, \log F, \log P_X\right] = E\left[\log A \mid s_i, \log F, z_p\right] = \frac{\tilde{\tau}_{A,i}c_s s_i + \frac{h_A^2}{h_\xi^2}\left(\frac{\log P_X - h_0 - h_F\log F - h_\xi\bar{\xi}}{h_A}\right)}{\tilde{\tau}_{A,i} + \frac{h_A^2}{h_\xi^2}\tau_\xi},$$

with conditional variance

$$\tau_{A,i} = Var\left[\log A \mid s_i, \log F, \log P_X\right] = \left(\tilde{\tau}_{A,i} + \frac{h_A^2}{h_\xi^2}\tau_\xi\right)^{-1}.$$

We use (IA.1) to compute $\log X_i$, and obtain a linear expression of s_i, $\log F$, and P_X. By matching the coefficients of this expression with the conjectured form in (IA.7), we obtain

$$\begin{aligned}
l_0 &= \frac{1}{1-\phi}\log\phi + \frac{(1+\phi\eta l_s)^2}{2(1-\phi)}\tau_{A,i} + \frac{1}{2(1-\phi)}\phi^2\eta^2 l_s^2\tau_s^{-1} - \frac{1+\phi\eta l_s}{1-\phi}\tau_{A,i}\frac{h_A}{h_\xi^2}\left(h_0 + h_\xi\bar{\xi}\right) \\
&\quad + \frac{1}{1-\phi}(1+\phi\eta l_s)\tau_{A,i}\tilde{\tau}_{A,i}\left(c_0 - c_F\left(\tilde{h}_0 + \tilde{h}_\theta\bar{\theta}\right)\right), \\
l_s &= \frac{\tilde{\tau}_{A,i}c_s}{(1-\phi(1-\eta))\tau_{A,i}^{-1} - \phi\eta\tilde{\tau}_{A,i}c_s}, \\
l_F &= \frac{1}{1-\phi}(1+\phi\eta l_s)\tau_{A,i}\left(\tilde{\tau}_{A,i}c_F - \frac{h_A}{h_\xi^2}h_F\right), && \text{(IA.25)} \\
l_P &= \frac{1}{1-\phi}(1+\phi\eta l_s)\tau_{A,i}\frac{h_A}{h_\xi^2} - \frac{1}{1-\phi}. && \text{(IA.26)}
\end{aligned}$$

Market clearing of the spot market requires $\int_{-\infty}^{\infty} X_i d\Phi(\varepsilon_i) = X_S$, which implies

$$(k - l_P)\log P_X = l_0 + \frac{1}{2}l_s^2\tau_s^{-1} + l_s\log A + l_F\log F - \xi.$$

By matching coefficients on both sides, we have

$$\begin{aligned}
(k-l_P)h_0 &= l_0 + \frac{1}{2}l_s^2\tau_s^{-1}, \\
(k-l_P)h_A &= l_s, && \text{(IA.27)} \\
(k-l_P)h_F &= l_F, && \text{(IA.28)} \\
(k-l_P)h_\xi &= -1. && \text{(IA.29)}
\end{aligned}$$

From equations (IA.27) and (IA.29), we have that $l_s = -\frac{h_A}{h_\xi}$, and given our expression for l_0 and l_F above, we also see that

$$\begin{aligned}
h_0 &= \left(k - l_P + \frac{1+\phi\eta l_s}{1-\phi}\tau_{A,i}\frac{h_A}{h_\xi^2}\right)^{-1}\cdot\left(\frac{1}{1-\phi}\log\phi + \frac{(1+\phi\eta l_s)^2}{2(1-\phi)}\tau_{A,i} + \frac{1}{2(1-\phi)}\phi^2\eta^2 l_s^2\tau_s^{-1}\right. \\
&\quad \left. + \frac{1}{1-\phi}(1+\phi\eta l_s)\tau_{A,i}\tilde{\tau}_{A,i}\left(c_0 - c_F\left(\tilde{h}_0 + \tilde{h}_\theta\bar{\theta}\right)\right) - \frac{1+\phi\eta l_s}{1-\phi}\tau_{A,i}\frac{h_A}{h_\xi^2}h_\xi\bar{\xi} + \frac{1}{2}l_s^2\tau_s^{-1}\right), \\
h_F &= \left(\frac{1-\phi}{1+\phi\eta l_s}\tau_{A,i}^{-1}(k-l_P) + \frac{h_A}{h_\xi^2}\right)^{-1}\tilde{\tau}_{A,i}c_F. && \text{(IA.30)}
\end{aligned}$$

From our expression for l_s above and $l_s = -h_A/h_\xi$, we have

$$l_s^3 + \tau_\xi^{-1}\left(\tilde{\tau}_{A,i} - \frac{\phi\eta\tilde{\tau}_{A,i}c_s}{1-\phi(1-\eta)}\right)l_s - \frac{\tau_\xi^{-1}\tilde{\tau}_{A,i}c_s}{1-\phi(1-\eta)} = 0. \tag{IA.31}$$

This is a depressed cubic polynomial whose unique real and positive root is given by

$$\begin{aligned} l_s \;=\;& \sqrt[3]{-\frac{1}{2}\frac{\tau_\xi^{-1}\tilde{\tau}_{A,i}a_s}{1-\phi(1-\eta)} + \sqrt{\frac{1}{4}\left(\frac{\tau_\xi^{-1}\tilde{\tau}_{A,i}a_s}{1-\phi(1-\eta)}\right)^2 + \frac{1}{27}\tau_\xi^{-3}\left(\left(\tilde{\tau}_{A,i} - \frac{\phi\eta\tilde{\tau}_{A,i}a_s}{1-\phi(1-\eta)}\right)\right)^3}} \\ &+ \sqrt[3]{-\frac{1}{2}\left(\frac{\tau_\xi^{-1}\tilde{\tau}_{A,i}a_s}{1-\phi(1-\eta)}\right) - \sqrt{\frac{1}{4}\left(\frac{\tau_\xi^{-1}\tilde{\tau}_{A,i}a_s}{1-\phi(1-\eta)}\right)^2 + \frac{1}{27}\tau_\xi^{-3}\left(\left(\tilde{\tau}_{A,i} - \frac{\phi\eta\tilde{\tau}_{A,i}a_s}{1-\phi(1-\eta)}\right)\right)^3}}. \end{aligned}$$

It follows that $l_s > 0$ and from equation (IA.29) that

$$h_A = \frac{(1-\phi)\,l_s + (1+\phi\eta l_s)\left(\tilde{\tau}_{A,i} + l_s^2\tau_\xi\right)^{-1} l_s^2}{1+(1-\phi)\,k} > 0,$$

and, since $l_s = -h_A/h_\xi > 0$,

$$h_\xi = -\frac{1-\phi + (1+\phi\eta l_s)\left(\tilde{\tau}_{A,i} + l_s^2\tau_\xi\right)^{-1} l_s}{1+(1-\phi)\,k} < 0.$$

We now prove that $l_F > 0$. Given the expression for l_F in (IA.25) and given $l_s > 0$, it is sufficient for $l_F > 0$ if

$$\tilde{\tau}_{A,i}c_F > \frac{h_A}{h_\xi^2}h_F.$$

Given the expression for h_F in (IA.30), and recognizing that $\tilde{\tau}_{A,i} > 0$ and $c_F > 0$, the above condition can be rewritten as

$$1 > \frac{h_A}{h_\xi^2}\left(\frac{1-\phi}{1+\phi\eta l_s}\tau_{A,i}^{-1}(k - l_P) + \frac{h_A}{h_\xi^2}\right)^{-1}.$$

Furthermore, from the expressions for $\tau_{A,i}$ and l_P, this condition can be further expressed as

$$\frac{1}{1+\phi\eta l_s}\left(1 + k(1-\phi)\right)\left(\tilde{\tau}_{A,i} + \frac{h_A^2}{h_\xi^2}\tau_\xi\right) > \frac{h_A}{h_\xi^2}.$$

Since $l_s = -\frac{h_A}{h_\xi}$, given our expression for $h_\xi < 0$, the condition reduces to

$$\frac{1-\phi}{1+\phi\eta l_s}\left(\tilde{\tau}_{A,i} + l_s^2\tau_\xi\right) > 0,$$

which is always satisfied. Therefore, $l_F > 0$. In addition, since $(k - l_P)\, h_A = l_s$ implies that $k > l_P$, we see from $(k - l_P)\, h_F = l_F$ that $h_F > 0$.

We now examine the sign of l_P. By substituting $l_s = -\frac{h_A}{h_\xi}$ and the expressions of $\tau_{A,i}$ and h_ξ into (IA.26), we have

$$l_P = -\frac{1}{h_\xi \left(1 + (1-\phi)\, k\right)} \left(\tilde{\tau}_{A,i} + l_s^2 \tau_\xi\right)^{-1} \left(k l_s - (\tau_\xi - k\phi\eta)\, l_s^2 - \tilde{\tau}_{A,i}\right).$$

Consequently, l_P can be positive or negative depending on the sign of $k l_s - (\tau_\xi - k\phi\eta)\, l_s^2 - \tilde{\tau}_{A,i}$.

D.2 Proof of Proposition 2

In (IA.8), $\log P_X$ is a linear expression of $\log A$, θ, and ξ. We need to show that as $\tau_\theta \to \infty$, the coefficients on $\log A$ and ξ converge to their corresponding values in the perfect-information benchmark (Proposition 3 of the main paper), and the variance of θ

$$V_\theta = h_F^2 \tilde{h}_\theta^2 \tau_\theta^{-1} \to 0.$$

We rewrite equation (IA.22) as

$$\left(\tilde{l}_s + \kappa\right)^2 \tilde{l}_s + \tau_\theta^{-1} \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right) \tilde{l}_s = \frac{\tau_\theta^{-1}\tau_s}{1-\phi(1-\eta)}.$$

As τ_θ becomes sufficiently large, the right-hand side converges to zero and therefore, since the cubic polynomial has a unique real solution, $\tilde{l}_s \to 0$. By substituting equation (IA.22) into our expression for $\tilde{h}_A$, one can express $\tilde{h}_A$ as

$$\tilde{h}_A = \frac{1-\phi(1-\eta)}{1+k(1-\phi)} \tau_s^{-1} \left(1 + \frac{(1-\phi)\,\tilde{l}_s}{1-(1-\phi)\,\tilde{l}_s}\right) \left(\frac{\tau_s}{1-\phi(1-\eta)} - \left(\tau_A + \frac{1-\phi}{1-\phi(1-\eta)}\tau_s\right) \tilde{l}_s\right).$$

As $\tau_\theta \to \infty$, $\tilde{l}_s \to 0$, and thus $\tilde{h}_A \to \frac{1}{1+k(1-\phi)}$. In addition, by substituting for c_s, we can rewrite (IA.31) as

$$\tau_\xi l_s^3 + \tilde{\tau}_{A,i} l_s = (1 + \phi\eta l_s)\, \frac{\tilde{\tau}_{A,i}^2 \tau_s}{1-\phi(1-\eta)}.$$

Since $\tau_\theta (l_s + \kappa)^2$ grows as τ_θ increases, $\tilde{\tau}_{A,i} = \left(\tau_s + \tau_A + \tau_\theta (l_s+\kappa)^2\right)^{-1} \to 0$ as $\tau_\theta \to \infty$. It then follows that $l_s \to 0$.

By substituting (IA.31) and our expression for c_s into our expression for h_ξ, we have

$$h_\xi = -\frac{1-\phi}{1+(1-\phi)\,k} - \frac{1-\phi(1-\eta)}{1+(1-\phi)\,k} \tau_s^{-1} \left(\tilde{\tau}_{A,i} l_s\right)^2.$$

As $\tau_\theta \to \infty$, $\tilde{\tau}_{A,i} l_s = (1+\phi\eta l_s)\frac{\tilde{\tau}_{A,i}^2 \tau_s}{1-\phi(1-\eta)} - \tau_\xi l_s^3 \to 0$, and therefore $h_\xi \to -\frac{1-\phi}{1+k(1-\phi)}$. Given that $l_s = -\frac{h_A}{h_\xi}$ and given our expression for l_P, we have that as $\tau_\theta \to \infty$, the coefficient of ξ in (IA.8) equals

$$l_P h_\xi = -\frac{1}{1-\phi}\left(1+\phi\eta l_s\right)\tau_{A,i} l_s - \frac{1}{1-\phi} h_\xi \to \frac{1}{1+k\left(1-\phi\right)},$$

which is its value in the perfect-information benchmark.

Since $l_s = -\frac{h_A}{h_\xi}$, and given that as $\tau_\theta \to \infty$, $l_s \to 0$ and $h_\xi \to -\frac{1-\phi}{1+k(1-\phi)}$, we have $h_A \to 0$. By substituting for $\tau_{A,i}$, c_F, l_P, and $\tilde{h}_A/\tilde{h}_\theta$, we can rewrite $h_F \tilde{h}_A$ as

$$\begin{aligned} h_F \tilde{h}_A &= \frac{1-\phi\left(1-\eta\right)}{1+k\left(1-\phi\right)}\tau_s^{-1}\tau_\theta\left(l_s+\kappa\right)^2 l_s \\ &= \frac{1-\phi\left(1-\eta\right)}{1+k\left(1-\phi\right)}\tau_s^{-1}\left(\left(1+\phi\eta l_s\right)\frac{\tau_s}{1-\phi\left(1-\eta\right)} - \tau_\xi\left(\tilde{\tau}_{A,i}^{-1} l_s^{3/2}\right)^2 - \left(\tau_s+\tau_A\right) l_s\right), \end{aligned}$$

where we use substitution with equation (IA.31). As $\tau_\theta \to \infty$, $l_s \to 0$, and $\left(\tilde{\tau}_{A,i}^{-1} l_s^{3/2}\right)^2 \to 0$, the coefficient on $\log A$ in (IA.8) $h_A + h_F \tilde{h}_A \to \frac{1}{1+k(1-\phi)}$, which is its value in the perfect-information benchmark.

By using the expressions of h_F, l_P, l_s, c_F, $\tau_{A,i}$, $\tilde{l}_s$, and $\tilde{h}_\theta$ in Proposition 1 and by manipulating terms, we have

$$h_F \tilde{h}_\theta = \frac{1-\phi\left(1-\eta\right)}{1+k\left(1-\phi\right)}\tau_s^{-1} l_s\left(\tilde{l}_s+\kappa\right)\tau_\theta.$$

Consequently, we can write V_θ as

$$V_\theta = \left(\frac{1-\phi\left(1-\eta\right)}{1+k\left(1-\phi\right)}\tau_s^{-1}\right)^2 l_s^2\left(\tilde{l}_s+\kappa\right)^2\tau_\theta.$$

We can rewrite equation (IA.22) as

$$\tau_\theta\left(\tilde{l}_s+\kappa\right)^2 = \frac{\tau_s}{1-\phi\left(1-\eta\right)}\tilde{l}_s^{-1} - \left(\tau_A + \frac{1-\phi}{1-\phi\left(1-\eta\right)}\tau_s\right).$$

By applying the Implicit Function Theorem to equation (IA.22),

$$\frac{\partial \tilde{l}_s}{\partial \tau_\theta} = -\frac{\left(\tilde{l}_s+\kappa\right)^2 \tilde{l}_s^2}{2\tau_\theta\left(\tilde{l}_s+\kappa\right)\tilde{l}_s^2 + \frac{\tau_s}{1-\phi(1-\eta)}} < 0.$$

Consequently, $\tau_\theta\left(\tilde{l}_s+\kappa\right)^2$ is growing in τ_θ. Now we can rewrite equation (IA.31) by substituting for $\tilde{\tau}_{A,i}$ and c_s as

$$\left(\tau_A+\tau_s+\left(\tilde{l}_s+\kappa\right)^2\tau_\theta\right)\sqrt{\frac{\tau_\xi l_s^3 + \left(\tau_A+\tau_s+\left(\tilde{l}_s+\kappa\right)^2\tau_\theta\right)^{-1} l_s}{1+\phi\eta l_s}} = \sqrt{\frac{\tau_s}{1-\phi\left(1-\eta\right)}}.$$

As $\tau_\theta \to \infty$, $l_s \to 0$. Thus, for this equation to hold, we must have $\tau_\theta (l_s + \kappa)^2 \to \infty$. The left-hand side (LHS) of the above equation can then be expressed as

$$\begin{aligned}
& \left(\tau_A + \tau_s + \left(\tilde{l}_s + \kappa\right)^2 \tau_\theta\right) \sqrt{\frac{\tau_\xi l_s^3 + \left(\tau_A + \tau_s + \left(\tilde{l}_s + \kappa\right)^2 \tau_\theta\right)^{-1} l_s}{1 + \phi \eta l_s}} \\
\approx \;& \tau_\theta \left(\tilde{l}_s + \kappa\right)^2 l_s^{3/2} \sqrt{\frac{\tau_\xi}{1 + \phi \eta l_s}} + o\left(\tau_\theta^{-1} \left(\tilde{l}_s + \kappa\right)^{-2}\right).
\end{aligned}$$

This suggests that $l_s^{3/2}$ must be shrinking at approximately the same rate as $\tau_\theta \left(\tilde{l}_s + \kappa\right)^2$ is growing for the LHS to remain finite. Therefore, l_s^2 must be shrinking at a faster rate and $V_\theta \to 0$ as $\tau_\theta \to \infty$.

In summary, we have shown that as $\tau_\theta \to \infty$, $\log P_X$ converges with its counterpart in the perfect-information benchmark. We can similarly prove that the producers' aggregate demand also converges.

References

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