

Multi-market Delegated Asset Management*

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This Draft: December 2008

Abstract

This paper studies optimal contracting in delegated asset management when a fund manager can exert unobservable effort and take unobservable investment positions in multiple markets. A key insight is that while giving the manager flexibility to invest in multiple markets increases investment efficiency, it weakens the link between fund performance and the manager's effort in his designated market, thus increasing agency cost. Building on this tradeoff, our model explains the existence of funds with narrow investment mandates, and provides a set of testable implications for a varying degree of investment flexibility across funds. These results shed light on capital immobility in financial markets, market segmentation, and the architecture of financial institutions.

Keywords: Institutional Frictions, Capital Immobility, Market Segmentation, Architecture of Financial Institutions.

*We thank Doug Diamond, Peter DeMarzo, Mike Fishman, Bill Janeway, Arvind Krishnamurthy, Adriano Rampini, Jeremy Stein, and seminar participants at University of Chicago, Duke University, Imperial College, INSEAD, London School of Economics, University of North Carolina, University of Vienna, and 2008 Cambridge-Princeton Conference on Finance for helpful comments. We also thank Ing-Haw Cheng and Adam Zawadowski for helpful editorial assistance.

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1 Introduction

Motivated by the widely spread agency frictions in the asset management industry, there is a large academic literature modeling how investors can use optimal incentive contracts to screen fund managers with different ability and to motivate unobservable effort from managers, and studying the impact of the agency frictions on the efficiency of financial markets.¹ This literature mostly focuses on one-market settings, while in reality fund managers often face investment opportunities in many markets. As such, investors need to motivate not only the right amount of effort from a fund manager, but also effort and investment allocation choices across the markets. In this paper, we analyze this important, but largely ignored, problem based on a model of delegated asset management with multiple markets.

There is a vast diversity in funds' investment mandates. As we illustrate in Section 4.1 using specific fund examples, some funds have narrowly defined investment mandates, such as only investing in Asian or Pacific Basin companies, while others are allowed to pursue a wide range of strategies. Those narrowly defined investment mandates appear suboptimal since they largely constrain the fund managers from exploiting opportunities in other markets. But, they are nevertheless common in many funds, either explicitly stated in fund prospectuses or implicitly implemented through relative-performance based compensation schemes.²

Why should these narrow mandates exist? After all, a typical expertise related argument implies that managers with superior expertise in one area will voluntarily choose to invest in their specialized area without the aid of mandates. A plausible explanation to this phenomenon, which is explored in our model, is that investment flexibility could adversely affect the delegation process. In addition, by understanding funds' investment flexibility, our model sheds light on capital immobility in financial markets, market segmentation, and the architecture of financial institutions.

We analyze a one-period model with a risk neutral principal delegating capital to a risk averse fund manager, who faces investment opportunities in two markets, A and B . Our model builds on two important ingredients. First, by exerting unobservable effort at a personal cost, the manager can improve the precision of his private signal about asset returns in each of the two markets. The second ingredient is the realistic assumption that

¹See, for example, Bhattacharya and Pfleiderer (1985), Heinkel and Stoughton (1994), Shleifer and Vishny (1997), Ou-Yang (2003), Stein (2005) and Carpenter, Dybvig, and Farnsworth (2006).

²According to the survey in a report by the Bank for International Settlements (BIS, 2003), many fund managers' bonus depends on their information ratios, which are measured by the funds' excess returns relative to their designated indices divided by the tracking errors. This compensation scheme implicitly discourages fund managers from exploiting opportunities outside their designated markets.

the incentive contract cannot be contingent upon the fund’s investment position.³ Like other delegated asset management models, the principal in our model uses an incentive contract based on fund performance and returns of the two markets to motivate the manager. The efficiency of the incentive contract, however, depends on the principal’s ability to differentiate performance driven by effort from that driven by plain luck.

A key insight of our model is that in multi-market delegated asset management, there is a conflict between ex post investment efficiency and ex ante incentive provision efficiency. To illustrate this conflict, suppose that the manager is incentivized to exert effort in market A . After exerting effort, the manager obtains a precise signal about market A , and he will invest in A if the signal is favorable. An interesting situation arises when the signal is unfavorable. In our model, independent of his effort, the manager always receives an endowed signal about market B . This signal, though less precise than the signal about market A , is informative. Should the manager invest in market B if his signal about B is favorable? The answer seems obvious from the investment efficiency perspective—he should. However, the flexibility to invest in B weakens the link between the fund performance and the manager’s unobservable effort on market A . To be precise, the investment flexibility makes it more difficult for the principal to judge from the fund performance whether the manager has exerted effort on market A , because good fund performance can be generated either by the manager’s effort on the designated market A or by random luck (a non-effort related opportunity) in the secondary market B . In the language of Holmstrom (1979) and Holmstrom and Milgrom (1987), the investment flexibility makes benchmarking more difficult, which reduces the ex ante incentive provision efficiency. As a result, the investment gain from giving the manager more investment flexibility is accompanied by an increased agency cost.

Building on this tradeoff, we show that the optimal incentive contract should implement different effort and investment strategies with a varying degree of investment flexibility, depending on the manager’s cost of effort and investment opportunities. Not surprisingly, for sufficiently low cost of effort, a *combined markets strategy* is optimal, in which the manager exerts effort on both markets and then invests the fund capital in the market with a better opportunity.

When the manager’s cost of effort is relatively high, it is optimal to induce the manager to work only on one (primary) market A with two alternative investment strategies. One is a

³Many institutions, such as hedge funds, strictly keep their investment positions at secret from the public. While some other institutions such as mutual funds regularly disclose their investment positions, their managers still maintain ample freedom to withhold the information regarding their positions between disclosures. This implies that if a fund chooses to compensate its manager based on the fund’s positions at some points, the manager’s ability of window dressing would undermine any intended purpose.

two-tiered strategy, in which the manager *maintains the flexibility to invest in the secondary market B*. More precisely, if he obtains a negative signal about market *A*, while the other less precise signal about market *B* is positive, the manager has the flexibility to take advantage of the opportunity in market *B*. The alternative is a *single market strategy*, in which the manager *only invests in the primary market*. These two strategies incur the same effort cost, but the two-tiered strategy is more flexible and offers higher investment efficiency. However, from our earlier discussion, the investment flexibility is accompanied by a higher agency cost. As a result, the more restrictive single market strategy dominates the two-tiered strategy when the manager's effort cost is above a threshold, or when his endowed investment opportunity in the secondary market is below a threshold.

Thus, our model provides an agency based explanation for funds with narrow investment mandates, together with a set of testable implications for the varying degree of investment flexibility across funds. For example, funds tend to face more stringent capital confinement when their managers have lower ability or when they work in more obscure markets that are difficult to analyze. Our model also predicts that managers with more investment flexibility tends to have high powered incentives than managers with less flexibility.

Our results help understand why capital often fails to flow to liquidity distressed markets that offer profitable opportunities.⁴ According to our hypothesis, once investors distribute their capital into different market segments through institutionally managed funds, agency considerations largely confine capital in its initial market segments. Even if one segment runs out of capital later and ends up in a liquidity crisis, capital in other segments may not be able to flow in to take advantage of the profitable opportunities. The existing literature typically attributes the immobility of capital to information barriers about asset fundamentals at the *receiving* end of capital flow. In contrast, our hypothesis builds on institutional constraints at the *originating* end. One prediction is that during a liquidity crisis, outside capital is more likely to flow in from funds that face less stringent relative performance evaluation and less severe limits on tracking errors.

Our model points out agency frictions as a possible explanation for market segmentation, a widely observed phenomenon in financial markets. For example, even in the absence of explicit capital controls, many emerging markets are segmented from the global financial markets in the sense that they offer more profitable investment opportunities than developed countries.⁵ Many argue that the increasing presence of institutionally managed funds can

⁴See, for example, Froot and O'Connell (1999), Gabaix, Krishnamurthy, and Vigneron (2007), Coval and Stafford (2007), Mitchell, Pedersen, and Pulvino (2008), and Da and Gao (2008) for a series of related evidence.

⁵See, for example, a recent study by Bekaert, Harvey, Lundblad, and Siegel (2008).

help global investors overcome the information barriers they face in investing in emerging markets, and therefore should have substantially reduced the segmentation of these markets. However, our model shows that, due to agency issues, the information opaqueness of these markets makes it necessary for global investors to impose a narrow investment mandate on the manager. Consequently, the manager's pricing kernel is mostly exposed to the idiosyncratic risk of his designated market, despite that he works for well-diversified global investors.

By highlighting the inefficiency in the one-to-one relationship between a principal and a fund manager, our model suggests that there is a potential gain for investors to employ a more complex institutional architecture. One possible design is to hire a team of two managers, one for each market and with a separate performance measure, and to allow capital transfer to the market with the most profitable opportunity. This team design resembles the internal capital markets used by firms for more efficient allocations of resources among different projects (e.g., Stein, 1997). Relative to the one-manager design we analyze, the team design generates an apparent efficiency gain, but at the additional cost of compensating one more manager. While the tradeoff remains to be analyzed in the future research, our model suggests that agency theory is a promising route to explain the ongoing trend in the asset management industry where strategic asset allocation has been increasingly delegated back to fund owners, leaving day-to-day tactical asset management in the hands of professional asset managers.⁶

This paper adds to the literature on agency frictions as the source of financial market inefficiency. The extant studies, following Shleifer and Vishny (1997), focus on the agency risk of arbitrage trading in dynamic settings—fund managers are reluctant to take on arbitrage positions because when asset prices deviate further away from fundamentals in the future, investors would withdraw money, causing forced liquidation. In contrast, our paper emphasizes that in a multi-market setting, agency frictions can lead to capital confinement, which limits fund managers' ability to take advantage of profitable opportunities elsewhere.

While our model belongs to the broad literature on multi-task agency problems, it uses a new mechanism based on efficiency of benchmarking to explain capital confinement in delegated asset management. As reviewed by Dewatripont, Jewitt, and Tirole (2000), there are three approaches in extant studies pointing to the cost of multiple tasks and the advantage of splitting them between several agents. First, because of unobservable substitution of effort allocations among various tasks (i.e., working on task 1 makes it more exhausting for an agent to work on task 2), Holmstrom and Milgrom (1991) show that it might be optimal to exclude

⁶See, for example, the BIS report (2003, page 19).

an agent from pursuing certain tasks (task exclusion). This effort substitution effect is absent from our model because of the fund manager’s additive effort cost across different markets. Second, Dewatripont and Tirole (1999) argue that the optimality of task separation arises naturally in the presence of direct conflicts between tasks (i.e., looking for ‘pros’ and ‘cons’ of a given decision). In our model, investment flexibility does not directly conflict with exerting effort in any market. Third, using a career concern framework, which is different from the explicit contracting framework that we employ, Dewatripont, Jewitt, and Tirole (1999) show that undertaking less tasks could induce a higher effort level from the agent, because adding up outcomes from more tasks leads to noisier aggregate performance (lower signal-to-noise ratio). In contrast, our model builds on the efficiency of benchmarking, which plays no role in the three aforementioned approaches. Given the pervasive use of benchmarking in the asset management industry (e.g., the recent BIS report, 2003), our approach is particularly relevant for addressing issues related to the architecture of financial institutions.

The paper is organized as follows. We present the model in Section 2. Section 3 studies the optimal fund design, and Section 4 discusses related implications. Section 5 concludes. All proofs are in the appendix.

2 The Model

2.1 Setup

We consider a single-period principal-agent model where a risk-neutral principal delegates capital to a risk-averse fund manager (the agent).⁷ The manager’s utility function over consumption $U(\cdot)$ satisfies $U(0) = 0$, $U'(\cdot) > 0$, and $U''(\cdot) < 0$.

The fund faces risky investment opportunities in two markets, A and B , with i.i.d. returns \tilde{r}_A and \tilde{r}_B . For simplicity, we assume that each market return can only take two possible values, a positive value r or a negative one $-r$, with equal probability:

$$\tilde{r}_i = \begin{cases} r & \text{with probability } \frac{1}{2} \\ -r & \text{with probability } \frac{1}{2} \end{cases}, i \in \{A, B\}.$$

The individual market considered here admits broad interpretations. It could be a specific market sector, such as the treasury bond market, the mortgage bond market, the U.S. stock market, a regional stock market, or the financial derivatives market. It could also be a certain

⁷The risk neutrality of the principal implies that he is only interested in maximizing expected fund return. This assumption rules out various hedging and diversification needs of the principal, and allows us to focus on effects of agency frictions on the delegation process. See Massa (2003) and Mamaysky and Spiegel (2002) for studies of how heterogeneity among individual investors in terms of investment horizon and risk preferences can motivate mutual fund families to offer funds specializing in different markets or strategies.

trading strategy, such as a long-short equity strategy, fixed income arbitrage or convertible bond arbitrage. Other than these two risky markets, the manager can also place his capital in a risk-free asset. We normalize the risk free interest rate to be 0.

Before making an investment decision, the manager—who possesses certain expertise that normal investors lack—obtains some *private* information regarding the likelihood of each market going up or down. More specifically, we assume that the manager receives a signal s_i about market i , where $i \in \{A, B\}$. The signal takes two possible values 1 or -1 . If the return in the market is positive (or negative), the signal is more likely to take the value 1 (or -1):

$$\Pr(s_i = 1 | \tilde{r}_i = r) = \Pr(s_i = -1 | \tilde{r}_i = -r) = \frac{1}{2} + \Delta + \theta_i.$$

If the probabilities are $\frac{1}{2}$, then the signal is uninformative. The term $\Delta + \theta_i > 0$ measures the precision of the signal s_i in revealing the market return. Denote by θ_i the manager’s effort in acquiring information. It can take two values, 0 and e , corresponding to “shirking” and “working” respectively. By working hard (e.g., conducting a thorough analysis), the manager improves the signal precision by e . The manager’s effort is unobservable to the principal and incurs a private cost to the manager. Moreover, even if the manager does not exert effort, the signal still contains some information about the return of market i , as reflected by the Δ component of the signal precision. This free information represents investment opportunity endowed to the manager because of his investment expertise. For simplicity, we also assume that the manager exerts effort before he receives any signal.⁸ In sum, the manager is endowed with some free information about each market, and he can improve the information precision by exerting unobservable effort.

Denote $\theta = \{\theta_A, \theta_B\}$ as the manager’s effort choice in the two markets and $\Theta = \{0, e\} \times \{0, e\}$ as the set of possible choices. The manager has an additive utility function over consumption and effort, i.e.,

$$\bar{U}(c, \theta) = U(c) - g(\theta),$$

where $g(\theta) \equiv k \left(\sum_i \frac{\theta_i}{e} \right)$. That is, the manager incurs a private cost of k (in his utility term) for each market that he exerts effort on, and the cost is $2k$ if he covers both markets.⁹

Ex ante, the manager’s two signals about markets A and B are equally likely to be positive or negative, and they are independent. After receiving a good signal about market

⁸We rule out the possibility that the manager makes his effort choice after he observes a free signal about each market. Such a sequential setup complicates the analysis, but does not add much to the economic insight.

⁹Because of the linear effort cost, our model does not admit the effort substitution effect illustrated by Holmstrom and Milgrom (1991).

i , the manager raises the conditional probability of market i having a positive return to be

$$\Pr(\tilde{r}_i = r | s_i = 1) = \frac{1}{2} + \Delta + \theta_i. \quad (1)$$

We assume $\Delta + e \leq \frac{1}{2}$ throughout the paper to make the probability meaningful.

The fund has one unit of initial capital. To deliver the key insight without getting into unnecessary complications, we make two simplifying assumptions. First, the manager cannot short sell any asset and cannot borrow either. Second, the manager can only invest all of the fund in one of the two risky markets or the risk free asset.¹⁰ Relaxing these simplifying assumptions will not eliminate our result. We denote the manager's investment choice by

$$x = \{x_A, x_B, x_0\},$$

where $x_i \in \{0, 1\}$ indicates the manager's investment position in market i with $i \in \{A, B\}$, and $x_0 \in \{0, 1\}$ is his position in the risk free asset. The borrowing constraint requires that $x_A + x_B + x_0 = 1$. We denote the set of all feasible investment choices by $X = \{x\}$. The fund's return \tilde{r}_F can take three possible values, i.e., $\tilde{r}_F \in \{r, 0, -r\}$.

2.2 Incentive Contract

The principal writes a compensation contract to induce a certain combination of effort and investment strategies from the manager. For efficient incentive provision, benchmarking the manager's performance to the two market returns, i.e., using relative performance evaluation, is beneficial. Therefore, in addition to the fund performance \tilde{r}_F , the incentive contract should include \tilde{r}_A and \tilde{r}_B .

Furthermore, we make a realistic assumption that the incentive contract cannot be contingent upon the fund's investment position. It is unrealistic to contract on fund positions for several reasons. First, fund managers often express concerns that excessive reporting to the public would reveal their investment advantage.¹¹ Second, it is difficult to find a single measure to summarize the investment positions taken by a real-life fund, which typically holds many positions with different characteristics in many dimensions. Third, the infeasibility of continuous monitoring in practice implies that any reporting of fund position can only take

¹⁰As we will show later, limited liability makes the optimal incentive contract convex with respect to fund performance. As a result, the manager would voluntarily invest only in one of the markets despite his risk aversion. However, allowing a fractional fund position across markets in our stylized model with discrete market returns opens up an unrealistic channel for the principal to perfectly identify the fund position through the realized fund return. Such identification breaks down in practice because of the continuous distribution of returns. To avoid this unnecessary complication, we make the simplifying assumption that the manager can only invest all fund in one market.

¹¹In practice, hedge funds always keep their positions at secret from the public, and mutual funds only file quarterly or semi-annual reports of their positions to investors.

place at discrete intervals. Basing incentive contracts on snapshots of fund positions will induce window dressing by fund managers to game the compensation scheme, invalidating the intended incentive provision.

Thus, an incentive contract Π is a mapping from the information set Ω generated by \tilde{r}_F , \tilde{r}_A and \tilde{r}_B to contingent non-negative payments to the manager:

$$\Pi : \Omega \equiv \{u, 0, d\} \times \{u, d\}^2 \rightarrow \mathbb{R}_+.$$

Due to limited liability, we rule out negative wages to the manager.¹² The fund return can take three possible values, u (up with a return of r), 0 , or d (down with a return of $-r$). Each market return can take on two possible values, u (up) or d (down). There are at most 12 possible outcomes. Because the fund cannot generate return u (d) when both markets go down (up), there are only 10 feasible outcomes. Therefore the contract only needs to specify 10 contingent payments to the manager.

Denote $\omega = \begin{pmatrix} \tilde{r}_F \\ \tilde{r}_A, \tilde{r}_B \end{pmatrix} \in \Omega$ as a possible outcome. It is easier to work with the payment in terms of the manager's utility (π_ω) than in terms of dollars (c_ω). These terms are equivalent because of the monotone relation $\pi_\omega = U(c_\omega)$. We write the contract as

$$\Pi = \{ \pi_{u,u}^u, \pi_{u,u}^0, \pi_{u,d}^u, \pi_{u,d}^0, \pi_{u,d}^d, \pi_{d,u}^u, \pi_{d,u}^0, \pi_{d,u}^d, \pi_{d,d}^0, \pi_{d,d}^d \}. \quad (2)$$

For instance, $\pi_{u,u}^u$ is the manager's utility when both markets are up and the fund return is also up. Then, $c_{u,u}^u = U^{-1}(\pi_{u,u}^u)$ is the cost of compensating the manager for this outcome.

For a given contract $\Pi = \{\pi_\omega\}$, the fund manager maximizes his expected utility by making his optimal effort and investment choices:

$$\max_{\theta \in \Theta, x \in X} \sum_{\omega \in \Omega} p_\omega(\theta, x) \pi_\omega - g(\theta),$$

where p_ω is the probability of outcome ω , and $g(\theta)$ is the effort cost associated with effort choice θ . The manager's effort and investment choices θ and x determine the outcome probability $\{p_\omega\}$. We write $\theta^*(\Pi)$ and $x^*(\Pi)$ as the manager's optimal effort and investment choices, respectively, in response to a given contract Π .

Thus, by using different incentive contracts, the principal can induce different effort and investment choices from the manager. The default strategy, which we call the "no effort

¹²We assume the manager to be both risk-averse and protected by limited liability. Limited liability is sufficient for generating a non-zero agency cost even if the manager is risk neutral. However, as will become clear later, the agency cost in this case is not affected by benchmarking difficulty in implementing different effort and investment strategies. The reason is that risk neutrality makes it efficient for the principal to shift all positive rewards to one good outcome that permits perfect identification of the manager's investment position through comparing the fund return to the market returns (e.g., $\tilde{r}_F = 0$ and both markets go down.)

strategy”, is to induce no effort and simply let the manager invest according to the imprecise signals endowed to him.

In Section 3, we will evaluate three other strategies to motivate some effort from the manager. Since the principal is risk neutral, when the effort cost is sufficiently low, the first-best combination of effort and investment strategies is for the manager to exert effort on both markets, and then to invest the fund capital in the market with better opportunity. This is what we call a “combined markets” strategy, an object we study in Section 3.3.

When it is too costly to induce the manager to cover both markets, there are two alternative strategies to consider. One is to pick one of the markets, say A , as the primary market, and to only motivate the manager to exert effort in the primary market. The manager invests in the primary market if it offers a good opportunity; otherwise, he chooses whether to invest in market B , the secondary market, depending on his endowed free signal about the market. We call this a “two-tiered” strategy, which is analyzed in Section 3.2. In contrast, Section 3.1 considers a “single market” strategy which induces the manager to exert effort and invest only in one market, say market A . More precisely, this strategy fully ignores the manager’s free, but informative, signal about market B , even when he finds a poor opportunity in market A . Relative to the two-tiered strategy, this strategy requires the same effort cost but forgoes a valuable investment opportunity in market B . As we will show later, this seemingly inferior strategy, because of its more efficient incentive provision, dominates the two-tiered strategy under certain conditions.

The manager has a reservation utility of \underline{U} , which represents his forgone outside opportunity cost by accepting the job of managing this fund. The participation constraint requires that:

$$\sum_{\omega \in \Omega} p_{\omega}(\theta^*(\Pi), x^*(\Pi)) \pi_{\omega} - g(\theta^*(\Pi)) \geq \underline{U}.$$

For simplicity, throughout this paper we assume that \underline{U} is sufficiently small so that the manager’s participation constraint is not binding.

The principal’s payoff from each outcome ω is the portfolio return minus the compensation cost:

$$W_{\omega} = 1 + \tilde{r}_F(\omega) - U^{-1}(\pi_{\omega}). \quad (3)$$

The principal maximizes the expected payoff from the fund by choosing an optimal incentive contract, i.e.,

$$V = \max_{\Pi} \sum_{\omega \in \Omega} p_{\omega}(\theta^*(\Pi), x^*(\Pi)) W_{\omega},$$

subject to the manager’s participation and incentive compatibility constraints.

We can further decompose the principal’s expected payoff into two components:

$$V = \sum_{\omega \in \Omega} p_{\omega} (1 + \tilde{r}_F(\omega)) - \sum_{\omega \in \Omega} p_{\omega} U^{-1}(\pi_{\omega}), \quad (4)$$

where the first part is the expected fund return, which is determined by the manager’s effort and investment strategy, and the second part is the expected cost of compensating the fund manager. This decomposition suggests the following two-step method to solve for the optimal contract: First, find the least costly contract to implement each of the three effort and investment strategies; then, compare these least costly contracts to determine the optimal contract that offers the highest expected net payoff to the principal.

It is important to note that our optimal contracting approach is consistent with the substantial relative-performance based bonus in individual fund managers’ compensation. While many funds, especially mutual funds, charge their investors a fixed fee based on Assets Under Management (AUM), individual fund managers’ incentive tends to be explicit. According to the recent BIS report (2003), the majority part of their compensation is relative-performance based bonus.¹³

3 Optimal Fund Design and Incentive Contract

To study the optimal fund design, we analyze the three strategies that involve the manager’s costly effort—the single market strategy, the two-tiered strategy, and the combined market strategy—in the order of increasing complexity. We then compare the efficiency of implementing these strategies in Section 3.5.

3.1 Single Market Strategy

We start with analyzing the least costly contract for implementing a single market strategy in market A . The contract induces the following effort and investment choices from the fund manager: the manager exerts effort only in market A ; after receiving the signal s_A^e , he invests all the fund capital in market A if the signal is positive, and invests in the risk free asset otherwise, regardless of his endowed signal s_B^0 about market B . Note that there is an opportunity loss when the manager’s signals suggest that market A lacks a good investment opportunity while market B offers a good one ($s_A^e = -1, s_B^0 = 1$).

¹³ “The size of the bonus component in individual asset managers’ compensation varies considerably across countries. However, at least in some countries, there seems to be a general trend towards a higher share of variable compensation in total pay over recent years.... US managers can earn average bonuses of 100% and higher. In the United Kingdom, where the median fund manager will get a bonus of about 100%, exceptional asset managers can earn as much as six times their base salary in the form of bonuses.” (the BIS report, 2003, page 23)

Because this strategy only involves one risky market, we can consider the class of contracts that specify four payments depending on \tilde{r}_F and \tilde{r}_A :

$$\left\{ \pi_{\tilde{r}_A}^{\tilde{r}_F} \right\} = \left\{ \pi_u^u, \pi_u^0, \pi_d^0, \pi_d^d \right\},$$

where the subscript of each payment denotes the return of market A , and the superscript denotes the fund return. In relating this contract to the fully specified one in (2), we can simply let $\pi_{\tilde{r}_A, \tilde{r}_B}^{\tilde{r}_F} = \pi_{\tilde{r}_A}^{\tilde{r}_F}$. This still leaves two off-equilibrium payments, $\pi_{u,d}^d$ and $\pi_{d,u}^u$, unspecified. These payments are important since they affect the manager's incentive compatibility constraint regarding a deviation strategy of investing in market B . In Section 3.1.2, we will first determine the equilibrium payments and then identify these two off-equilibrium payments.

3.1.1 Incentive Compatibility

The fund manager has two unobservable actions: exerting effort to obtain a precise signal and making the investment choice. In contrast to the costly effort on information acquisition, the investment choice per se does not involve any personal cost, and the incentive compatibility constraint regarding the investment choice is slack (which we will verify ex post). Here, we discuss the manager's incentive compatibility constraint regarding his effort choice. Taking the manager's investment choice as given, his expected utility from exerting effort on acquiring a precise signal about market A is:

$$\begin{aligned} & \mathbb{E} \left[\bar{U}(c, \theta) \mid \text{exerting effort and obtain } s_A^e \right] \\ &= \frac{1}{2} \left[\left(\frac{1}{2} + \Delta + e \right) \pi_u^u + \left(\frac{1}{2} - \Delta - e \right) \pi_u^0 + \left(\frac{1}{2} + \Delta + e \right) \pi_d^0 + \left(\frac{1}{2} - \Delta - e \right) \pi_d^d \right] - k. \end{aligned} \quad (5)$$

Take π_u^u for example. The probability of the outcome $\left(\begin{smallmatrix} \tilde{r}_F \\ \tilde{r}_A \end{smallmatrix} \right) = \begin{smallmatrix} (u) \\ (u) \end{smallmatrix}$ is the probability of state $\tilde{r}_A = u$, which is $\frac{1}{2}$, multiplied by the probability of the manager receiving a positive signal $s_A^e = 1$ conditional on $\tilde{r}_A = u$ and the manager exerting effort, which is $\left(\frac{1}{2} + \Delta + e \right)$. Similarly, the manager's expected utility from shirking is:

$$\begin{aligned} & \mathbb{E} \left[\bar{U}(c, \theta) \mid \text{shirking with } s_A^0 \right] \\ &= \frac{1}{2} \left[\left(\frac{1}{2} + \Delta \right) \pi_u^u + \left(\frac{1}{2} - \Delta \right) \pi_u^0 + \left(\frac{1}{2} + \Delta \right) \pi_d^0 + \left(\frac{1}{2} - \Delta \right) \pi_d^d \right]. \end{aligned} \quad (6)$$

Therefore, the manager's incentive compatibility constraint regarding exerting effort requires that the value of (5) is no less than that of (6), which is equivalent to

$$\frac{1}{2} \left[e\pi_u^u - e\pi_u^0 + e\pi_d^0 - e\pi_d^d \right] \geq k. \quad (7)$$

In condition (7), the coefficient of each utility term in the bracket gives the manager's incentive differential between “shirking” and “working” for a particular outcome ω . For instance, consider π_u^u . By working, the probability of getting π_u^u is $(\frac{1}{2} + \Delta + e) / 2$, while by shirking, the probability becomes $(\frac{1}{2} + \Delta) / 2$. The difference between these two probabilities is exactly the coefficient $\frac{1}{2}e$ in front of π_u^u in condition (7). The higher this coefficient, the more effective the payment π_u^u in motivating the manager to exert effort.

The least costly contract for implementing the single market strategy is determined by

$$\min_{\{\pi_u^u, \pi_u^0, \pi_d^0, \pi_d^d\} \in \mathbb{R}_+^4} \sum p_\omega U^{-1}(\pi_\omega) = \frac{1}{2} \left[\left(\frac{1}{2} + \Delta + e \right) U^{-1}(\pi_u^u) + \left(\frac{1}{2} - \Delta - e \right) U^{-1}(\pi_u^0) \right. \\ \left. + \left(\frac{1}{2} + \Delta + e \right) U^{-1}(\pi_d^0) + \left(\frac{1}{2} - \Delta - e \right) U^{-1}(\pi_d^d) \right],$$

subject to the incentive compatibility constraint in (7), which is binding in the solution.

3.1.2 The Least Costly Contract

Two outcomes $(\begin{smallmatrix} 0 \\ u \end{smallmatrix})$ and $(\begin{smallmatrix} d \\ d \end{smallmatrix})$, which represent poor performance relative to market A , have negative incentive differentials. Any payment to the manager for these outcomes is a reward for failure and thus should be minimized to zero ($\pi_u^0 = 0$ and $\pi_d^d = 0$). On the other hand, π_u^u and π_d^0 represent rewards for good performance in outcomes $(\begin{smallmatrix} u \\ u \end{smallmatrix})$ and $(\begin{smallmatrix} 0 \\ d \end{smallmatrix})$. Using the standard Lagrange method, the first order conditions provide that

$$U' [U^{-1}(\pi_u^u)] = U' [U^{-1}(\pi_d^0)] = \frac{(\frac{1}{2} + \Delta + e)}{\lambda e}, \quad (8)$$

where U^{-1} is the inverse function of the manager's utility function U , and λ is the Lagrange multiplier for the incentive compatibility constraint in (7). Combining this result with (7), we have $\pi_u^u = \pi_d^0 = \frac{k}{e}$.

We can describe the contract in a fully specified form, as in (2):

$$\begin{cases} \pi_{u,u}^u = \pi_{u,d}^u = \pi_{d,u}^0 = \pi_{d,d}^0 = \frac{k}{e}, \\ \pi_{u,u}^0 = \pi_{u,d}^0 = \pi_{d,u}^d = \pi_{d,d}^d = 0. \end{cases}$$

We also need to specify payments for two off-equilibrium outcomes $(\begin{smallmatrix} d \\ u,d \end{smallmatrix})$ and $(\begin{smallmatrix} u \\ d,u \end{smallmatrix})$ to prevent the manager from investing in the secondary market B :

$$\pi_{u,d}^d = \pi_{d,u}^u = 0.$$

It is straightforward to verify that under these terms, the manager will never deviate to invest in market B .¹⁴ The following proposition summarizes the contract derived above.

¹⁴The following conditions ensure that the manager will not deviate to invest in market B :

$$\begin{cases} q_1 (\frac{1}{2} - \Delta) \pi_{u,d}^d + \pi_{d,u}^u \leq (1 - (\frac{1}{2} + \Delta) q_1) \frac{k}{e} \\ q_2 \pi_{u,d}^d + \pi_{d,u}^u \leq \frac{k}{e} \end{cases},$$

Proposition 1 *The least costly contract for implementing the single market strategy requires the following payments:*

$$\begin{cases} \pi_{u,u}^u = \pi_{u,d}^u = \pi_{d,u}^0 = \pi_{d,d}^0 = \frac{k}{e}, \\ \pi_{u,u}^0 = \pi_{u,d}^0 = \pi_{d,u}^d = \pi_{d,d}^d = \pi_{u,d}^d = \pi_{d,u}^u = 0. \end{cases} \quad (9)$$

The principal's expected payoff from implementing this strategy is

$$V^{SM} = 1 + (\Delta + e)r - \left(\frac{1}{2} + \Delta + e\right)U^{-1}\left(\frac{k}{e}\right). \quad (10)$$

This contract benchmarks the manager's performance to the return of his designated market. The manager receives a positive reward if he secures the positive return of market A or avoids the negative return. Otherwise, he receives nothing. Consistent with the benchmarking idea, the same fund performance, 0, could lead to two different compensations, 0 or $\frac{k}{e}$, depending on whether the market return is positive or negative.

Another notable point is that the contract gives a zero payment for $(\frac{u}{d,u})$, the outcome in which the manager delivers good performance but the performance is chased to market B . This term represents a penalty if the manager generates a large tracking error, which is often used in practice according to the BIS report (2003). This penalty discourages the manager from investing in the secondary market and serves the role of confining the fund capital in its primary market.

3.1.3 Benchmarking and Cost to Incentive Ratio

As in Holmstrom (1979) and Holmstrom and Milgrom (1987), benchmarking plays a key role in our model. In other words, the optimal contract compares the manager's performance to market returns, and rewards the manager only for his effort-driven performance, but not for pure luck.¹⁵

We illustrate the impact of effort and luck on the optimal contract as follows. As in equation (8), the principal's optimization problem implies that any outcome ω with $\pi_\omega > 0$

where $q_1 \equiv \frac{\frac{1}{2}-\Delta-e}{\frac{1}{2}+\Delta+e}$, and $q_2 \equiv \left(\frac{\frac{1}{2}-\Delta}{\frac{1}{2}+\Delta}\right)^2$. The first condition arises from ruling out a deviation strategy of exerting effort on market A and then following the two-tiered investment strategy discussed in Section 3.2 (i.e., invest in market B when $s_A^e = 0$ and $s_B^0 = 1$); and the second condition comes from ruling out another deviation strategy of exerting no effort and following a two-tiered investment strategy.

¹⁵Our analysis focuses on incentive contracts based on the performance of a fund and the market returns. We can further show that *peer evaluation*, i.e., basing one manager's compensation on his relative performance to other funds trading in the same market, cannot help in optimal contracting. The reason is simple: In our model, *conditional on the true state of the market returns*, the signals are independent across managers. Thus, after the contract incorporates the realized market returns, it has already used the best information for relative performance evaluation. Other funds' performance, which is just a noisy version of the true state, does not offer any additional information. This result can be formally shown using the sufficient statistics argument in Holmstrom (1979).

warrants a positive payment satisfying

$$U'(U^{-1}(\pi_\omega)) = \frac{1}{\lambda} \frac{p_\omega}{Dp_\omega}, \quad (11)$$

where p_ω is the probability of the outcome ω , and Dp_ω is the incentive differential associated with the outcome, i.e., the coefficient of π_ω in the manager's incentive compatibility constraint (7). Since U' is a decreasing function, the condition (11) implies that the optimal payment π_ω decreases with the ratio $\frac{p_\omega}{Dp_\omega}$.

We call $\frac{p_\omega}{Dp_\omega}$ the cost to incentive ratio, which is first derived in Holmstrom (1979).¹⁶ The numerator p_ω captures a *cost* effect, i.e., the larger the probability of the outcome ω , the higher the expected cost of each dollar promised to this outcome. The denominator Dp_ω captures an *incentive* effect: the larger the incentive differential Dp_ω , the greater the manager's incentive to exert effort. This ratio thus determines the efficiency of incentive provision through rewarding the outcome ω .

The effort and investment strategy implemented by the manager determines the set of cost to incentive ratios $\left\{ \frac{p_\omega}{Dp_\omega} \right\}_{\omega \in \Omega}$. The difficulty of benchmarking in implementing certain strategies typically leads to a higher cost to incentive ratio $\frac{p_\omega}{Dp_\omega}$ for some outcomes, and therefore less efficient incentive provision. For instance, a luck component that cannot be filtered out by benchmarking will raise the probability p_ω , which makes compensation more costly. Moreover, Section 3.2.2 discusses an additional effect that rewarding luck from investing in other markets also dampens the incentive leverage Dp_ω for certain outcome, thus reducing the manager's incentive to exert effort.

3.2 Two-tiered Strategy

As we discussed before, implementing the single market strategy in market A imposes an efficiency loss by restricting the manager from investing in market B . This subsection studies a two-tiered strategy, which improves on this dimension: the manager only exerts effort on acquiring a precise signal s_A^e about market A ; if this signal is favorable, he invests in market A ; if s_A^e is unfavorable but his free signal s_B^0 on market B is favorable, he invests in market B ; otherwise, he invests in the risk free asset. Like the single market strategy, the two-tiered strategy induces manager's effort only in market A . However, in contrast to the single market strategy, the two-tiered strategy gives the manager flexibility to invest in market B when his signal on A is unfavorable but his signal on B is favorable.

¹⁶ $\frac{Dp_\omega}{p_\omega}$ corresponds to $\frac{f_a}{f}$ in Holmstrom (1979), where f is the probability density function of the performance, and f_a is the marginal impact of action a on the density function. Holmstrom points out that $\frac{f_a}{f}$ is the derivative of log likelihood, and interprets this measure as how strongly one is inclined to infer from the performance that the agent did not take the assumed action.

3.2.1 The Least Costly Contract

We derive the least costly contract for implementing the two-tiered strategy in a way similar to the single market strategy. We provide the derivation in the Appendix and state the result below.

Proposition 2 *The least costly contract for implementing the two-tiered strategy pays zero for the following poor performance*

$$\pi_{u,u}^0 = \pi_{u,d}^d = \pi_{u,d}^0 = \pi_{d,u}^d = \pi_{d,d}^d = 0,$$

and a modest reward for

$$\pi_{u,u}^u = \beta_1^{TT} > 0,$$

and the largest reward for

$$\pi_{u,d}^u = \pi_{d,u}^u = \pi_{d,d}^0 = \pi_{d,u}^0 = \beta_2^{TT} > \beta_1^{TT}. \quad (12)$$

The reward levels β_1^{TT} and β_2^{TT} are jointly determined by the following conditions:

$$\frac{U'(U^{-1}(\beta_2^{TT}))}{U'(U^{-1}(\beta_1^{TT}))} = M \quad \text{with} \quad M \equiv \frac{(\frac{1}{2} + \Delta + e)(\frac{1}{2} - \Delta)}{(\frac{1}{2} + \Delta + e) + (\frac{1}{2} - \Delta - e)(\frac{1}{2} + \Delta)} < 1, \quad (13)$$

and

$$\left(\frac{1}{2} - \Delta\right) \beta_1^{TT} + \left(\frac{5}{2} + \Delta\right) \beta_2^{TT} = 4\frac{k}{e}. \quad (14)$$

Finally, the principal's expected payoff from implementing the two-tiered strategy, is

$$\begin{aligned} \mathbb{E}[V^{TT}] &= 1 + \left(e + \frac{3}{2}\Delta\right) r \\ &\quad - \frac{1}{4} \left(\left(\frac{1}{2} + \Delta + e\right) + \left(\frac{1}{2} - \Delta - e\right) \left(\frac{1}{2} + \Delta\right) \right) U^{-1}(\beta_1^{TT}) \\ &\quad - \frac{1}{4} \left(\frac{1}{2} + \Delta + e\right) \left(\frac{5}{2} + \Delta\right) U^{-1}(\beta_2^{TT}). \end{aligned}$$

Consider a special case, in which the manager has a power utility:

$$U(c) = c^{1-\alpha}, \quad \text{with} \quad \alpha \in (0, 1)$$

Then, $U'(U^{-1}(\pi_\omega)) = (1-\alpha)\pi_\omega^{\frac{\alpha}{1-\alpha}}$. Solving equations (13) and (14) provides

$$\beta_1^{TT} = \frac{4M^{\frac{1-\alpha}{\alpha}}}{\left(\frac{1}{2} - \Delta\right) M^{\frac{1-\alpha}{\alpha}} + \left(\frac{5}{2} + \Delta\right)} \cdot \frac{k}{e}, \quad (15)$$

$$\beta_2^{TT} = \frac{4}{\left(\frac{1}{2} - \Delta\right) M^{\frac{1-\alpha}{\alpha}} + \left(\frac{5}{2} + \Delta\right)} \cdot \frac{k}{e}. \quad (16)$$

Our later discussions and proofs mostly build on this particular utility functional form.

The contract derived in Proposition 2 reflects the use of benchmarking in two aspects. First, the outcomes $(^u_{u,d})$, $(^u_{d,u})$, and $(^0_{d,d})$ clearly represent good fund performance relative to both markets A and B , and thus deserving a positive reward β_2^{TT} . Even though outcome $(^u_{u,u})$ is good performance in the absolute sense, the performance is rather modest relative to the good returns in both markets, thus only deserving a smaller positive reward β_1^{TT} (we will provide a detailed discussion of this term shortly). Second, the contract benchmarks the fund performance to the two markets in an asymmetric way, as shown by the difference in rewards $\pi_{u,d}^0$ and $\pi_{d,u}^0$. While the absolute fund performance is identical in these outcomes, the outcome $(^0_{u,d})$ indicates that the manager fails to capture the positive return in his primary market A , and thus represents a poor performance. In contrast, the outcome $(^0_{d,u})$ is a good performance deserving reward because the manager avoids the bad return in his primary market, even though he misses the positive return in market B . This asymmetry in performance evaluation is a natural implication of the two-tiered strategy.

3.2.2 Higher Agency Cost

Interestingly, while the two-tiered strategy initially appears superior, allowing the manager to take advantage of his endowed signal about market B exacerbates the agency problem. This is because that the additional investment flexibility, by introducing luck from market B into the performance, makes “benchmarking” more difficult, weakens the link between the fund performance and the manager’s effort in market A . This in turn leads to less efficient incentive provision.

This negative impact manifests itself in the outcome $\omega = (^u_{u,u})$. As reflected in the optimal payments in Proposition 2, the manager only receives a modest reward for this outcome. Following our discussion in Section 3.1.3, we investigate the cost to incentive ratio, defined in equation (11), of this outcome. The probability of $(^u_{u,u})$ is

$$p_\omega = \frac{1}{2} \cdot \frac{1}{2} \cdot \left[\left(\frac{1}{2} + \Delta + e \right) + \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} + \Delta \right) \right]. \quad (17)$$

Here, $\frac{1}{2} \cdot \frac{1}{2}$ is the ex ante probability of the state that both markets go up (i.e., $\tilde{r}_A = \tilde{r}_B = r$). In contrast to the single market strategy, there are two ways—by either investing in market A or B —for the manager to deliver a positive fund return $\tilde{r}_F = r$. In the square bracket of equation (17), the first term is the conditional probability of the manager receiving a positive signal about A ($s_A^e = 1$, so the manager invests in market A and $\tilde{r}_F = \tilde{r}_A = r$), and the second term is the conditional probability of the manager receiving a negative signal about

A and a positive signal about B ($s_A^e = -1$, $s_B^0 = 1$, so the manager invests in market B and $\tilde{r}_F = \tilde{r}_B = r$).

Equation (17) gives the manager's probability of receiving $\pi_{u,u}^u$ by working. If he shirks, the probability becomes

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \left[\frac{1}{2} + \Delta + \left(\frac{1}{2} - \Delta \right) \left(\frac{1}{2} + \Delta \right) \right]. \quad (18)$$

Thus, the incentive leverage of $\pi_{u,u}^u$ is the difference between equations (17) and (18):

$$Dp_\omega = \frac{e}{4} \left(\frac{1}{2} - \Delta \right).$$

As in our discussion regarding equation (11), the effectiveness of payment $\pi_{u,u}^u$ in incentivizing the manager is determined by the cost to incentive ratio:

$$\begin{aligned} \left(\frac{p}{Dp} \right)_{u,u}^u &= \frac{\frac{1}{2} \cdot \frac{1}{2} \cdot \left[\left(\frac{1}{2} + \Delta + e \right) + \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} + \Delta \right) \right]}{\frac{1}{4} e \left(\frac{1}{2} - \Delta \right)} \\ &= \frac{\frac{1}{2} + \Delta + e + \overbrace{\left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} + \Delta \right)}^{\text{increased } p_\omega \text{ due to luck from market } B}}{e - \underbrace{e \left(\frac{1}{2} + \Delta \right)}_{\text{reduced incentive}}}. \end{aligned}$$

We have decomposed each term in the fraction relative to $\frac{\frac{1}{2} + \Delta + e}{e}$, the corresponding cost to incentive ratio in equation (8) for implementing the single market strategy.

Compared to the single market strategy, there are two effects going on in $\left(\frac{p}{Dp} \right)_{u,u}^u$ due to the more flexible investment strategy. First, in the numerator, the probability of the outcome p_ω becomes larger. The manager can deliver a positive return not only through identifying a good opportunity in market A by effort, but also through investing in market B which is by luck independent of his effort. The contract cannot distinguish these two possibilities because the fund's position is not contractible. The increased luck component in p_ω raises the cost of compensating the manager.

Second, and more interestingly, allowing for investing in market B reduces the incentive strength $(Dp)_{u,u}^u$ in the denominator. To see this, consider the impact of the manager's effort on the two sources of $p_{u,u}^u$. As explained before, in the square bracket of equation (17), the first term $\frac{1}{2} + \Delta + e$, i.e., the conditional probability of delivering a positive return through market A , is the same as that in implementing the single market strategy. But the second term $\left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} + \Delta \right)$, i.e., the probability of getting good fund performance through market B , is decreasing in the manager's effort. It is this second term that dampens the

manager's ex ante incentive to exert effort in market A . Intuitively, the manager's effort in market A makes it more likely to spot the good opportunity in market A , which crowds out the need for a free luck from market B (i.e., when $s_A^e = -1$ which occurs with the probability of $\frac{1}{2} - \Delta - e$). This crowding out effect, which is at work only in implementing the two-tiered strategy, reduces the manager's gain from exerting his effort in market A , and therefore his ex ante working incentives.

Overall, in implementing the two-tiered strategy, the possibility of getting good performance from market B increases the cost to incentive ratio for the outcome $\omega = \begin{pmatrix} u \\ u, u \end{pmatrix}$.¹⁷ Thus, allowing the manager to invest in market B reduces the incentive provision efficiency and increases the agency cost. The following proposition formally states this result.

Proposition 3 *The expected compensation cost of implementing the single market strategy is lower than the two-tiered strategy, i.e., $K^{SM} < K^{TT}$.*

This proposition provides the crust of our model. This result suggests that in implementing the two-tiered strategy, the additional investment benefit $\frac{1}{2}\Delta r$ from allowing the flexibility of investing in market B , comes with an increased agency cost $K^{TT} - K^{SM}$. When the additional agency cost is sufficiently large, the principal would prefer to implementing the single market strategy, as formally shown in the following proposition.

Proposition 4 *The difference in expected compensation cost $K^{TT} - K^{SM}$ increases with the manager's cost of effort k . Consequently, the principal prefers to implement the single market strategy over the two-tiered strategy when k is higher than a threshold k^* , and vice versa.*

3.3 Combined Markets Strategy

Finally, we consider the most aggressive strategy—the combined markets strategy. Instead of only exerting effort on acquiring a precise signal about one market, the manager exerts effort to acquire precise signals about both markets A and B , at a personal cost of $2k$.¹⁸ Based on the signals, he invests in the market with the best opportunity. More specifically,

¹⁷For outcomes other than $\begin{pmatrix} u \\ u, u \end{pmatrix}$ and $\begin{pmatrix} d \\ d, d \end{pmatrix}$, investors can perfectly infer the manager's position through the fund return and the market returns. As a result, these outcomes share the same cost to incentive leverage ratio as in implementing the single market strategy. Finally, because the outcome $\begin{pmatrix} d \\ d, d \end{pmatrix}$ has a negative incentive differential, its compensation is always zero.

¹⁸One might argue for implementing a sequential strategy, in which the manager stops his effort after receiving a positive signal on one of the markets. While we only analyze a static model, in practice, a manager needs to continuously follow and analyze the markets to spot valuable investment opportunities. Thus, exerting effort sequentially on two markets is not realistic in this regard.

if only one of the signals is positive, he invests in the market that gives the positive signal; if both signals are negative, he invests in the risk free asset; and if both signals are positive, the manager randomizes between investing in A or B with equal probabilities.¹⁹

As we show in the Appendix, when e is relatively small, the binding incentive compatibility constraint is related to the manager's deviation strategy of exerting effort in a single market. We summarize the resulting contract in the following proposition.

Proposition 5 *When e is relatively small, the least costly contract for implementing the combined markets strategy takes the following form:*

$$\pi_{u,u}^u = \beta_1^{CM} < \pi_{u,d}^u = \pi_{d,u}^u = \beta_2^{CM} < \pi_{d,d}^0 = \beta_3^{CM},$$

and zero for all other outcomes. The three positive payoff levels, β_1^{CM} , β_2^{CM} , and β_3^{CM} , satisfy

$$\begin{aligned}\beta_1^{CM} &= \frac{k}{e} \frac{4M_1^{\frac{1-\alpha}{\alpha}}}{\left(\frac{1}{2} - \Delta - e\right) M_1^{\frac{1-\alpha}{\alpha}} + \left(\frac{1}{2} + \Delta + e\right) \left(1 + M_2^{\frac{1-\alpha}{\alpha}}\right)}, \\ \beta_2^{CM} &= \frac{k}{e} \frac{4M_2^{\frac{1-\alpha}{\alpha}}}{\left(\frac{1}{2} - \Delta - e\right) M_1^{\frac{1-\alpha}{\alpha}} + \left(\frac{1}{2} + \Delta + e\right) \left(1 + M_2^{\frac{\alpha-1}{\alpha}}\right)}, \\ \beta_3^{CM} &= \frac{k}{e} \frac{4}{\left(\frac{1}{2} - \Delta - e\right) M_1^{\frac{1-\alpha}{\alpha}} + \left(\frac{1}{2} + \Delta + e\right) \left(1 + M_2^{\frac{1-\alpha}{\alpha}}\right)},\end{aligned}$$

where

$$M_1 \equiv \frac{\frac{1}{2} - \Delta - e}{\frac{3}{2} - \Delta - e} \quad \text{and} \quad M_2 \equiv \frac{\frac{1}{2} + \Delta + e}{\frac{3}{2} + \Delta + e}$$

with $0 < M_1 < M_2 < 1$. The principal's expected payoff is

$$\begin{aligned}V^{CM} &= 1 + \frac{3}{2}(e + \Delta)r - \frac{1}{4} \left(\frac{1}{2} + \Delta + e\right) \left(\frac{3}{2} - \Delta - e\right) U^{-1}(\beta_1^{CM}) \\ &\quad - \frac{1}{4} \left(\frac{1}{2} + \Delta + e\right) \left(\frac{3}{2} + \Delta + e\right) U^{-1}(\beta_2^{CM}) - \frac{1}{4} \left(\frac{1}{2} + \Delta + e\right)^2 U^{-1}(\beta_3^{CM}).\end{aligned}$$

¹⁹Of course, randomization has to be incentive compatible for the manager. We also verify that in this situation, under the derived compensation contract, the manager prefers investing all fund capital in one market, rather than dividing the capital equally between the two markets. In other words, the half/half investment strategy is suboptimal. This finding is particularly interesting, because one might think that a risk-averse manager would prefer diversifying his fund investment. This argument is invalid because it ignores the functional form of the manager's performance-based payments. In our model, because the manager is protected by limited liability, the compensation scheme is convex, and this inherent convexity induces the manager to take a cornered (unobservable) investment decision. For instance, consider the case when the two market returns are (u, d) . By investing half/half in both markets, the manager will get the outcome of $\binom{0}{u,d}$, instead of $\binom{u}{u,d}$ or $\binom{d}{u,d}$ through randomizing. Since the contract specifies $\pi_{u,d}^0 = \pi_{u,d}^d = 0 < \pi_{u,d}^u$, the manager prefers to investing in one of the markets, rather than both.

The contract derived in Proposition 5 again reflects the use of benchmarking. In contrast to the contract for implementing the two-tiered strategy, it benchmarks the fund performance to the two markets in a symmetric way.

3.4 Incentive Slope

From a broad view, as in Holmstrom and Milgrom (1987), there are two key features to capture any compensation contract: the pay level, which is mainly tied to the manager's outside option; and the incentive slope, which measures the pay-performance sensitivity of a compensation contract. A contract's incentive slope is directly determined by incentive provision purposes. We now analyze the incentive slopes of the least costly contracts for implementing different strategies.

To facilitate the analysis of the nonlinear contracts in our framework, we define the manager's *incentive wedge* for different fund performance conditional on all possible exogenous market returns: (u, u) , (u, d) , (d, u) and (d, d) . In our model, for each pair of market returns, there are always two levels of compensation for rewarding good performance and penalizing poor performance.²⁰ We take the difference between these two levels to be the incentive wedge, which can be either utility-based (in terms of π) or dollar-based (in terms of c). Since each pair of market returns shares an equal probability of $\frac{1}{4}$, we simply measure the overall incentive slope of the contract by the ex ante expected incentive wedge. We prove the following proposition on the ranking of incentive slopes for implementing different strategies.

Proposition 6 *Among the least costly contracts for implementing different strategies, the expected incentive wedge, either utility-based or dollar-based, has the following declining order: the combined markets strategy, the two-tiered strategy, the single market strategy, and the no effort strategy.*

Implementing the no effort strategy has the lowest incentive slope because it does not require any incentive slope at all. Implementing the two-tiered strategy requires a higher incentive slope than the single market strategy. The reason is again rooted in inefficient benchmarking. Because of the manager's risk aversion, a minimal incentive wedge is desirable. As we discussed before, the increased investment flexibility in implementing the two-tiered strategy causes difficulty in benchmarking and increases the cost to incentive

²⁰Even when the market returns are (u, d) or (d, u) , the contract still specifies two levels in the compensation, although fund performance could be $u, 0$, or d . For instance, in the contract for implementing the two tiered strategy, $\pi_{u,d}^u = \beta_2^{TT} > 0$ and $\pi_{u,d}^0 = \pi_{u,d}^d = 0$, where the latter two outcomes represent poor fund performance. This two-level compensation structure originates from our binary effort-choice framework, in which the incentive contract is designed to induce effort in the most efficient way.

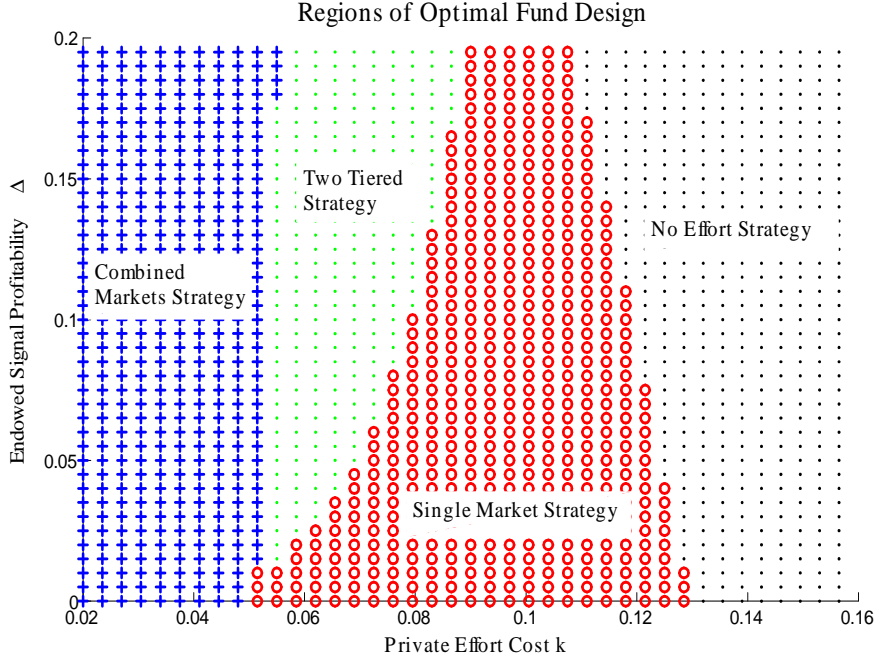


Figure 1: Optimal Fund Design. This figure plots the regions across two model parameters k and Δ , inside which the optimal contract implements the no effort, single market, two-tiered and combined markets strategies. The parameter k represents the manager’s private cost of effort, while the parameter Δ is the precision of the manager’s endowed information on each market’s return. Other parameters are set as $\alpha = 0.8$, $r = 0.4$, and $e = 0.2$.

ratio of the outcome $(u_{u,u})$. As a result, the least costly contract has to pay less for this outcome, relative to that in implementing the single market strategy. But in order to fulfill the incentive requirement, the contract has to raise payments for other outcomes, thus leading to a higher expected incentive slope. When implementing the combined market strategy, in addition to the difficulty of benchmarking, the required effort also doubles. Both elements contribute to the highest incentive slope in implementing the combined market strategy.

3.5 Optimal Fund Design

The optimal fund design amounts to comparing the principal’s expected payoff in equation (4) by implementing the four fund strategies—no effort, single market, two-tiered, and combined markets. To focus on the trade-off between ex ante agency cost and ex post investment efficiency, we examine two model parameters k and Δ . The parameter k represents the manager’s private cost of effort; the larger the k , the higher the ex ante agency cost. The parameter Δ is the precision of the manager’s endowed information independent of his effort. The higher the Δ , the greater the ex post investment benefit by allowing the fund

manager to employ a more flexible investment strategy.

Figure 1 plots four regions across Δ and k , inside which each of the no effort, single market, two-tiered and combined markets strategies becomes optimal. For k roughly below 0.05, it is optimal to implement the combined markets strategy, which is the most aggressive strategy for exploiting market opportunities. When k gets higher, the agency cost consideration becomes dominant. As a result, the two-tiered and single market strategies are more desirable than the combined markets strategy.

Figure 1 also shows that for a given value of k , the preference between the two-tiered and single market strategies is divided across a threshold in Δ . The single market strategy is preferred if Δ is lower than a threshold, since Δ represents the opportunity cost of restricting the manager to a single market, while the two-tiered strategy is preferred otherwise. More importantly, the threshold in Δ increases with k : when the agency problem worsens, the additional agency cost $K^{TT} - K^{SM}$ due to the lost efficiency in benchmarking rises accordingly (Proposition 4), and therefore the principal is willing to sacrifice a greater investment opportunity by confining the manager to a single market.

When the manager’s effort cost k goes up further, the no effort strategy becomes optimal. Figure 1 shows that given k , the no effort strategy is preferred if Δ is higher than a threshold. This is because when the manager already has a reasonable amount of investment opportunity without effort (a high Δ), inducing costly effort from the manager becomes less important.

To sum up, Figure 1 shows that agency considerations can lead to funds with narrow investment mandates, as well as funds with more investment flexibility, depending on the manager’s cost of effort and investment opportunities.

4 Implications and Discussions

4.1 Three Fund Examples

Each of the fund strategies that we analyze captures some basic features of real life contracts between investors and fund managers. To gain some perspectives, we extract the objectives and investment strategies of two mutual funds, *Putnam Asia Pacific Growth Fund* and *Vanguard Convertible Securities Fund*, and an anonymous hedge fund from their prospectuses,²¹ and summarize them in Table I.

Putnam Asia Pacific Growth Fund pursues a strategy closely resembling the single market strategy derived in our model. The resemblance is reflected in two aspects. First, the fund is

²¹We do not list the name of the hedge fund because of the fund’s nondisclosure policy to the public.

restricted to investing at least 85% of its capital in Asian or Pacific Basin companies, which can be interpreted as the single market in our model. Second, the fund can only temporarily use alternative strategies that are mainly designed to limit losses, including investing in the United States. The U.S. market plays the role of the risk free asset in our model.

Vanguard Convertible Securities Fund implements a strategy similar to the two-tiered strategy analyzed in our model. While the fund needs to invest at least 80% of its assets in its primary market—convertible securities, it may also invest in other securities to achieve its objective, such as nonconvertible corporate or U.S. government bonds, common stocks, and money market instruments. These securities serve the role of the secondary market in the two-tiered strategy of our model.

Finally, the anonymous hedge fund clearly pursues a combined markets strategy. The fund uses multiple arbitrage investment strategies that span securities in all asset classes, industry sectors and geographic boundaries. It is also worth noting that this hedge fund’s investment style is common in the hedge fund industry. Investors often impose much less trading restrictions on hedge funds than on mutual funds.

The investment covenants shown in Table 1 support the key result of our model that there is a varying degree of investment flexibility among funds: some funds impose restrictive capital confinement by specifying narrow investment mandates to their managers, some give full investment flexibility, and some others lie in between.

Note that a covenant alone cannot prevent the manager from violating the contract. It has to come with an auditing technology and a penalty scheme to enforce the covenant. Almazan et al. (2004, footnote 2) document how the SEC and fund advisors punish—i.e., “impose sanctions, fines, or censure on”—an individual fund manager if he is caught violating the fund’s investment covenants. One can interpret the optimal contract derived in our model as jointly using the realized fund return and two market returns as an imperfect auditing technology for spotting the manager’s violation of the intended investment strategy. The penalty for the spotted violation is simply to give a zero payment no matter whether the fund performance is good or bad.

In practice, another widely used method for enforcing a desirable degree of capital confinement in funds is through penalties and rewards based on funds’ tracking errors relative to the designated indices. For example, some funds impose penalty on managers who exceed certain limits on acceptable tracking errors. This penalty scheme corroborates well with the incentive contract derived in Proposition 1 for the single market strategy. In our model, to implement the narrow-mandated single market strategy, the incentive contract penalizes the manager when he delivers a fund performance opposite to the designated market return

Table I: Three Fund Examples

A: Putnam Asia Pacific Growth Fund

Fund objective: to seek capital appreciation.

Primary investment strategies: We invest mainly in common stocks of Asian or Pacific Basin companies. Under normal circumstances, we invest at least 85% of the fund's net assets in Asian or Pacific Basin companies.

Alternative investment strategies: Under normal market conditions, we keep the fund's portfolio fully invested, with minimal cash holdings. However, at times we may judge that market conditions make pursuing the fund's usual investment strategies inconsistent with the best interests of its shareholders. We then may temporarily use alternative strategies that are mainly designed to limit losses, including investing solely in the United States.

B: Vanguard Convertible Securities Fund

Fund objective: to provide current income and long-term capital appreciation.

Primary investment strategies: The Fund invests at least 80% of its assets in convertible securities, which are hybrid securities that combine the investment characteristics of bonds and common stocks. Convertible securities include corporate bonds and preferred stocks that are convertible into common stock, as well as debt securities with warrants or common stock attached.

Alternative investment strategies: Besides investing in convertible securities, the Fund may invest in nonconvertible corporate or U.S. government bonds, common stocks, or money market instruments, and may make other kinds of investments to achieve its objective. The Fund is authorized to invest up to 20% of its assets in foreign securities that are denominated in U.S. dollars. These securities may be traded on U.S. or foreign markets. The Fund may invest, to a limited extent, in derivatives.

C: An Anonymous Hedge Fund

Fund objective: to achieve absolute returns with minimal risk, rather than outperform a given benchmark or asset class.

Primary investment strategies: Our pursuit of multiple arbitrage investment strategies provides us with important flexibility in seeking profits as well as downside protection. We also emphasize diversity in asset classes, industry sectors and geographic boundaries — The fund may invest in domestic and foreign equity and debt securities, asset-backed securities, currencies, futures and forward contracts, options and other financial instruments.

($\pi_{u,d}^d = \pi_{d,u}^u = 0$), which corresponds to a large tracking error.²² Some other funds reward managers based on their information ratios—i.e., excess returns divided by tracking errors. This compensation scheme permits a manager to invest outside his designated market, but at the same time gives asymmetric rewards for performance from his designated and outside markets. This scheme reflects the main features of the incentive contract derived in Proposition 2.

4.2 Implications for Incentive Contracts

Our analysis provides several testable implications for incentive contracts in delegated asset management.

Implication 1: Fund managers with lower ability are more likely to be confined in trading a specific market sector or asset class; on the other hand, managers with higher ability tend to face less capital confinement.

In this implication, low ability of managers corresponds to a high cost of effort k in the model. This implication explains why hedge fund managers usually face less capital confinement than mutual fund managers. The reason is that hedge fund managers tend to be more talented,²³ and therefore the effort cost is lower.

Implication 2: Fund managers whose designated markets are more informationally opaque face more stringent capital confinement.

Information opaqueness corresponds to a low value of e , the amount of information the manager can acquire for a given amount of effort. In our model, the effect of e is exactly opposite to that of k . This implication is consistent with the casual observation that, mutual funds who invest in emerging markets, such as Putnam Asia Pacific Growth Fund mentioned above, usually face tight restrictions in their investment domain. One can directly test this implication by examining the link between the information environment of a fund's designated market, and the investment flexibility that the fund offers to its manager. The hypothesis is that the more opaque the designated market, the more restrictive the fund's investment strategy.

²²Goyal and Wahal (2008) study the selection and termination of the investment management firms by institutional plan sponsors (e.g., retirement plans, endowments), and find that plan sponsors tend to fire their investment management firms after poor performance measured by excess returns and information ratios. Information ratios also play a significant role in determining the fund flows which constitute the implicit portion of the fund manager's compensation. For instance, Del Guercio and Tkac (2002) find evidence that investors direct their money to funds with a higher Jensen's alpha and lower tracking errors, and this result is much stronger for pension funds than mutual funds.

²³See Kostovetsky (2008) for some evidence.

Implication 3: Investors are more likely to grant the manager flexibility to invest in a market that is more related to the fund’s designated market.

The example on Vanguard Convertible Securities Fund helps illustrate this implication. Since convertible securities are hybrid securities that combine the investment characteristics of bonds and common stocks, the manager can easily extract free information from his analysis of convertible securities about related stocks and bonds. In other words, because the manager’s Δ about these securities is likely to be large, it is more efficient to grant the fund some flexibility in investing in these related securities. Building on this example, our model leads to the following testable hypothesis: Funds that invest in hybrid securities tend to implement a two-tiered strategy using those securities related to the hybrid securities as the secondary market.

Implication 4: For a given set of model parameters, implementing different strategies involves pay-performance sensitivity in the following increasing order: the no effort strategy, the single market strategy, the two-tiered strategy, and the combined markets strategy.

This implication follows directly from Proposition 6. The increasing pattern is consistent with our casual observation that hedge funds tend to give high-powered incentive to their managers than mutual funds.²⁴

4.3 Implications for Capital Immobility

Our results help understand “capital immobility,” i.e., capital often fails to flow to distressed markets that offer profitable opportunities. There are numerous examples. Many pundits observe that capital immobility was a key factor leading to the 1998 financial market crisis when margin calls forced the hedge fund Long Term Capital Management to liquidate its large leveraged positions in fixed income securities, but not enough capital came to absorb its liquidation. Froot and O’Connell (1999) show that supply of capital in the catastrophe insurance market is inelastic because there are times during which the price of catastrophe insurance seems to be high and the capital of catastrophe insurers is low. Gabaix, Krishnamurthy, and Vigneron (2007) find that idiosyncratic prepayment risk in the mortgage-backed securities market carries a risk premium, which is inconsistent with perfectly elastic supply

²⁴To formally test this implication, one needs to bear in mind an endogeneity issue. While this implication compares the pay-performance sensitivity across different fund designs *by fixing the model parameters*, the observed fund contracts arise *endogenously* for potentially different parameters, as shown in Figure 1. As a result, without other exogenous identification assumptions, a simple regression analysis produces a downward biased estimator for the impact of fund design on pay-performance sensitivity received by fund managers.

of capital to this market. Other examples include the depressed convertible bond market after convertible hedge funds faced large redemption of capital from investors in 2005 (e.g., Mitchell, Pedersen, and Pulvino, 2008), the temporary price discount for stocks after fire sales by mutual funds (e.g., Coval and Stafford, 2007), and the distressed market for newly down-graded junk bonds (e.g., Da and Gao, 2008).

Our model suggests that agency considerations can cause investors to confine a fund in a designated market even when the fund manager has a better investment opportunity in another distressed market. Our earlier discussion about the comparison between the single market and two-tiered strategies already highlights this point. We can interpret Δ as the outside opportunity presented to a fund manager. Whether to grant the manager flexibility to take advantage of the outside opportunity depends on the manager's cost of effort k in his designated primary market. As shown by Figure 1, there exists a threshold $\Delta^*(k) > 0$ such that only when $\Delta > \Delta^*(k)$, i.e., only when the profitability of the outside opportunity is above the threshold, is the manager allowed to take advantage of the outside opportunity. Also note that the threshold $\Delta^*(k)$ increases with k , which means that managers with smaller agency cost have greater flexibility in moving capital outside their designated markets.

Our model thus leads to a new hypothesis for capital immobility during liquidity crises based on agency frictions at the *originating* end of capital flow. The economy could well have adequate capital. However, once investors distribute their capital into different market segments through institutionally managed funds, agency considerations will largely confine the capital in its initial market segments. Even if one segment runs out of capital later and ends up in a liquidity crisis, excess capital in other segments would not be able to flow in immediately. As the crisis deteriorates, the distressed segment will gradually attract capital from other segments, starting from funds that face less severe confinement, i.e., with lower confinement threshold $\Delta^*(k)$.

Our agency based hypothesis of capital immobility complements the growing literature that studies the impact of financial intermediaries' capital inside the crisis market under the premise that outside capital would not flow in, e.g., Kyle and Xiong (2001), Gromb and Vayanos (2002), Brunnermeier and Pedersen (2008), He and Krishnamurthy (2008), and Bolton, Santos, and Scheinkman (2008). These studies typically motivate this premise based on various information barrier arguments about the distressed market at the *receiving* end of capital flow, e.g., outside investors hesitate to invest in the crisis market because they cannot distinguish whether the price drop is driven by liquidity reasons or worsened fundamentals. Our hypothesis is also different from that based on search frictions, e.g., Duffie, Garleanu, and Pedersen (2005), who suggest that the speed of capital flow depends

on the rate of random matching between buyers and sellers.

Our hypothesis predicts that during a liquidity crisis, outside capital is more likely to flow in from funds that face less stringent relative performance evaluation and less severe limits on tracking errors. Moreover, our hypothesis predicts that the capital managed by funds whose designated markets are more transparent will be able to move into a distressed market earlier. These two novel empirical implications sharply distinguish our model from the aforementioned alternative arguments. One can directly test our hypothesis using those liquidity distress samples described at the beginning of this subsection.

4.4 Implications for Market Segmentation

By establishing the advantage of capital confinement in coping with agency issues in delegated asset management, our model provides an alternative explanation for market segmentation, a widely observed phenomenon in financial markets. For example, Bekaert, Harvey, Lundblad, and Siegel (2008) show that while stock markets of developed countries have become integrated since 1993, emerging markets are still segmented from the global financial markets. In particular, they find that after controlling financial leverage and earnings volatility, emerging markets display a significantly higher industrial earnings yield (the inverse of price to earnings ratio) than developed countries. This result suggests that even in absence of explicit capital controls, there is inadequate investment from developed countries to take advantage of the profitable opportunity in emerging markets.

Many factors, such as a country's financial and trade openness, political risk, and stock market development, could have contributed to the segmentation of emerging markets. Moreover, a common argument is that investors do not possess the capacity/ability to manage their investment in emerging markets because of the great information opaqueness in these markets. However, investors can hire professional managers to overcome the information barriers. Then, the increasing presence of institutionally managed funds should have substantially curtailed the segmentation of emerging markets.

Our model contributes a new insight to this debate. Even when global investors delegate their capital to a professional manager to invest in emerging markets, the information opaqueness of these markets makes it necessary to impose a narrow investment mandate on the manager in order to reduce agency cost involved in the delegation process. In other words, the manager has to invest primarily in a specific emerging market, say Russia, and his compensation is closely tied to his fund performance. Thus, even if this manager is the marginal investor in the Russian market, his pricing kernel is mostly exposed to the idiosyncratic risk of the Russian market despite the fact that he works for well-diversified global

investors. To sum up, our model suggests agency frictions as an additional indirect investment barrier for emerging markets. This mechanism also applies to explaining segmentation of other informationally opaque markets.

4.5 Discussions on Alternative Institutional Architecture

Our analysis shows that agency considerations can lead investors to confine their fund in a single market, even at a substantial cost of forgone investment opportunities in other markets. A key ingredient of our analysis is that while the core of the agency problem is the manager's unobservable actions (information acquisition effort and investment decisions) in two markets, investors can only use a one-dimensional performance measure—the fund return—to infer the manager's actions. The resulting information pooling, a practical friction in delegated asset management, is the ultimate source of inefficiency in our model.

By highlighting the inefficiency in the one-to-one relationship between a principal and a fund manager, our model suggests that there is a potential gain for investors to employ more complex institutional designs. To resolve the information pooling problem, a possibly superior design is to hire a team of two managers, one for each market and with a separate performance measure. This team design allows investors to efficiently provide incentives to induce effort from each manager in his designated market, and, at the same time, to maintain investment efficiency by requesting a manager without a profitable opportunity in his designated market to return capital for a redistribution to the other manager. This team design is similar to the internal capital markets employed by firms to efficiently allocate resources among different projects (e.g., Stein, 1997). Relative to the one-manager design we analyzed before, the team design generates an apparent efficiency gain, which is also accompanied by an additional cost. The team design doubles the compensation cost, but the expected investment gain is not doubled because the two managers' effort is repetitive when their signals about their respective markets are both positive. This suggests that the efficiency gain comes with an unbalanced increase in compensation cost. We will leave the analysis of this tradeoff to future research.

Interestingly, there is an ongoing trend in the asset management industry where strategic asset allocation has been increasingly delegated back to fund owners, leaving day-to-day tactical asset management in the hands of professional asset managers (e.g., the BIS report, 2003, page 19). While this two-layer architecture of asset management has many practical implications,²⁵ its microeconomic foundation is far from clear. By analyzing the agency

²⁵By taking this two-layer architecture as given, a recent study of van Binsbergen, Brandt, and Koijen (2008) uses a framework of optimal dynamic portfolio choice to study the cost of decentralized investment

frictions in multi-market delegated asset management, our model shows that agency theory provides a promising route to develop such a foundation.²⁶

5 Conclusion

We analyze a realistic delegated asset management problem, in which a principal hires a fund manager to invest his money in multiple asset markets. The existence of multiple markets implies that the principal needs to motivate not only the right amount of effort from the manager, but also effort and investment allocation choices across the markets. We highlight a key insight that while giving the manager flexibility to invest in multiple markets increases investment efficiency, it weakens the link between fund performance and the manager's effort in his designated market, thus increasing agency costs. Building on this tradeoff between ex post investment efficiency and ex ante incentive provision efficiency, our model explains the existence of funds with narrow investment mandates, and provides a set of testable implications for varying degree of investment flexibility across funds. These results shed light on a series of practical issues in financial markets, including immobility of capital flow to liquidity distressed markets, segmentation of emerging markets from the global financial markets, and the increasingly popular two-layer architecture of delegated asset management.

A Appendix

A.1 Proof of Proposition 2

We focus on the key incentive compatibility constraint regarding effort, and verify later that the associated investment strategy is indeed incentive compatible.

management. They also investigate the value of optimally designed benchmark.

²⁶In a related line of research, Gervais, Lynch, and Musto (2005) show that it is beneficial for individual fund managers to work for a fund family, rather than directly for investors, because fund families have information advantage regarding an individual manager's skill relative to normal investors.

The manager's expected utility from exerting effort in market A is

$$\begin{aligned}
& \mathbb{E} [\bar{U}(c, \theta) | \text{exerting effort and obtain } s_A^e] \\
&= \frac{1}{4} \left[\left(\frac{1}{2} + \Delta + e \right) + \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} + \Delta \right) \right] \pi_{u,u}^u + \frac{1}{4} \left(\frac{1}{2} + \Delta + e \right) \pi_{u,d}^u \\
&+ \frac{1}{4} \left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} + \Delta \right) \pi_{d,u}^u + \frac{1}{4} \left[\left(\frac{1}{2} - \Delta - e \right) + \left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} - \Delta \right) \right] \pi_{d,d}^d \\
&+ \frac{1}{4} \left(\frac{1}{2} - \Delta - e \right) \pi_{d,u}^d + \frac{1}{4} \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} - \Delta \right) \pi_{u,d}^d \\
&+ \frac{1}{4} \left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} + \Delta \right) \pi_{d,d}^0 + \frac{1}{4} \left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} - \Delta \right) \pi_{d,u}^0 \\
&+ \frac{1}{4} \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} + \Delta \right) \pi_{u,d}^0 + \frac{1}{4} \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} - \Delta \right) \pi_{u,u}^0 - k. \tag{19}
\end{aligned}$$

The manager's expected utility from shirking is

$$\begin{aligned}
& \mathbb{E} [\bar{U}(c, \theta) | \text{shirking and obtain } s_A^0] \\
&= \frac{1}{4} \left[\left(\frac{1}{2} + \Delta \right) + \left(\frac{1}{2} - \Delta \right) \left(\frac{1}{2} + \Delta \right) \right] \pi_{u,u}^u + \frac{1}{4} \left(\frac{1}{2} + \Delta \right) \pi_{u,d}^u \\
&+ \frac{1}{4} \left(\frac{1}{2} + \Delta \right) \left(\frac{1}{2} + \Delta \right) \pi_{d,u}^u + \frac{1}{4} \left[\left(\frac{1}{2} - \Delta \right) + \left(\frac{1}{2} + \Delta \right) \left(\frac{1}{2} - \Delta \right) \right] \pi_{d,d}^d \\
&+ \frac{1}{4} \left(\frac{1}{2} - \Delta \right) \pi_{d,u}^d + \frac{1}{4} \left(\frac{1}{2} - \Delta \right) \left(\frac{1}{2} - \Delta \right) \pi_{u,d}^d \\
&+ \frac{1}{4} \left(\frac{1}{2} + \Delta \right) \left(\frac{1}{2} + \Delta \right) \pi_{d,d}^0 + \frac{1}{4} \left(\frac{1}{2} + \Delta \right) \left(\frac{1}{2} - \Delta \right) \pi_{d,u}^0 \\
&+ \frac{1}{4} \left(\frac{1}{2} - \Delta \right) \left(\frac{1}{2} + \Delta \right) \pi_{u,d}^0 + \frac{1}{4} \left(\frac{1}{2} - \Delta \right) \left(\frac{1}{2} - \Delta \right) \pi_{u,u}^0. \tag{20}
\end{aligned}$$

The manager exerts efforts if and only if (19) dominates (20), which is equivalent to:

$$\begin{aligned}
& \frac{e}{4} \left[\left(\frac{1}{2} - \Delta \right) \pi_{u,u}^u + \pi_{u,d}^u + \left(\frac{1}{2} + \Delta \right) \pi_{d,u}^u + \left(\frac{1}{2} + \Delta \right) \pi_{d,d}^0 + \left(\frac{1}{2} - \Delta \right) \pi_{d,u}^0 \right] \\
&- \left(\frac{1}{2} - \Delta \right) \pi_{u,u}^0 - \left(\frac{1}{2} - \Delta \right) \pi_{u,d}^d - \pi_{d,u}^d - \left(\frac{1}{2} + \Delta \right) \pi_{d,d}^d - \left(\frac{1}{2} + \Delta \right) \pi_{u,d}^0 \geq k. \tag{21}
\end{aligned}$$

The expected cost of compensation to the manager, $\mathbb{E} [U^{-1}(\pi_\omega)]$, is:

$$\begin{aligned}
& \frac{1}{4} \left[\left(\frac{1}{2} + \Delta + e \right) + \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} + \Delta \right) \right] U^{-1}(\pi_{u,u}^u) + \frac{1}{4} \left(\frac{1}{2} + \Delta + e \right) U^{-1}(\pi_{u,d}^u) \\
&+ \frac{1}{4} \left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} + \Delta \right) U^{-1}(\pi_{d,u}^u) + \frac{1}{4} \left[\left(\frac{1}{2} - \Delta - e \right) + \left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} - \Delta \right) \right] U^{-1}(\pi_{d,d}^d) \\
&+ \frac{1}{4} \left(\frac{1}{2} - \Delta - e \right) U^{-1}(\pi_{d,u}^d) + \frac{1}{4} \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} - \Delta \right) U^{-1}(\pi_{u,d}^d) \\
&+ \frac{1}{4} \left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} + \Delta \right) U^{-1}(\pi_{d,d}^0) + \frac{1}{4} \left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} - \Delta \right) U^{-1}(\pi_{d,u}^0) \\
&+ \frac{1}{4} \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} + \Delta \right) U^{-1}(\pi_{u,d}^0) + \frac{1}{4} \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} - \Delta \right) U^{-1}(\pi_{u,u}^0). \tag{22}
\end{aligned}$$

All payment terms in (22) represent positive cost to the principal. However, in condition (21) five of them have negative incentive differentials $Dp_\omega < 0$. Payments for these five outcomes are rewards for shirking and should be minimized to zero:

$$\pi_{u,u}^0 = \pi_{u,d}^d = \pi_{u,d}^0 = \pi_{d,u}^d = \pi_{d,d}^d = 0.$$

For the remaining five outcomes, $\omega = \begin{pmatrix} u \\ u,d \end{pmatrix}, \begin{pmatrix} u \\ d,u \end{pmatrix}, \begin{pmatrix} 0 \\ d,d \end{pmatrix}, \begin{pmatrix} 0 \\ d,u \end{pmatrix}$ and $\begin{pmatrix} u \\ u,u \end{pmatrix}$, the payment π_ω is determined by minimizing expected compensation cost, subject to the manager's incentive compatibility constraint in (21). The first order condition is already given in (11), which determines π_ω through the cost to incentive ratio $\frac{p_\omega}{Dp_\omega}$, which yields the optimal payments summarized in Proposition 2. We can also verify that the specified investment strategy is incentive compatible under the derived contract.

A.2 Proof of Proposition 3

The expected compensation cost of implementing the two-tiered strategy is

$$K^{TT} = \frac{1}{4} \left(\frac{1}{2} + \Delta + e \right) \left(\frac{5}{2} + \Delta \right) U^{-1}(\beta_2^{TT}) + \frac{1}{4} \left[\left(\frac{1}{2} + \Delta + e \right) + \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} + \Delta \right) \right] U^{-1}(\beta_1^{TT}).$$

Plugging in β_1^{TT} and β_2^{TT} from equations (15) and (16) and letting

$$F(M) \equiv \left(\frac{1}{2} - \Delta \right) + \left(\frac{5}{2} + \Delta \right) M^{\frac{\alpha-1}{\alpha}},$$

we can further express the expected compensation cost as

$$\begin{aligned} K^{TT} &= \frac{1}{4} \left(\frac{1}{2} + \Delta + e \right) \left[\left(\frac{5}{2} + \Delta \right) M^{\frac{-1}{\alpha}} + \left(\frac{1}{2} - \Delta \right) M^{-1} \right] \left(\frac{4}{F(M)} \right)^{\frac{1}{1-\alpha}} U^{-1} \left(\frac{k}{e} \right) \\ &= \frac{1}{4} \left(\frac{1}{2} + \Delta + e \right) M^{-1} F(M) \left(\frac{4}{F(M)} \right)^{\frac{1}{1-\alpha}} U^{-1} \left(\frac{k}{e} \right) \\ &= \frac{1}{M} \left[\frac{4}{F(M)} \right]^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{2} + \Delta + e \right) U^{-1} \left(\frac{k}{e} \right). \end{aligned} \tag{23}$$

Since $M < 1$ and $\frac{\alpha-1}{\alpha} < 0$, we have $M^{\frac{\alpha-1}{\alpha}} > 1$. As a result, $\frac{1}{2} - \Delta < \left(\frac{3}{2} - \Delta \right) M^{\frac{\alpha-1}{\alpha}}$ and then $F(M) < 4M^{\frac{\alpha-1}{\alpha}}$, which is equivalent to

$$\frac{1}{M} \left[\frac{4}{F(M)} \right]^{\frac{\alpha}{1-\alpha}} > 1.$$

This further implies that $K^{TT} > \left(\frac{1}{2} + \Delta + e \right) U^{-1} \left(\frac{k}{e} \right)$. Since the expected compensation cost of implementing the single market is

$$K^{SM} = \left(\frac{1}{2} + \Delta + e \right) U^{-1} \left(\frac{k}{e} \right), \tag{24}$$

we have $K^{TT} > K^{SM}$.

A.3 Proof of Proposition 4

Based on equations (23) and (24), the reduction in expected compensation cost by implementing a single market strategy is

$$K^{TT} - K^{SM} = \left\{ \frac{1}{M} \left[\frac{4}{F(M)} \right]^{\frac{\alpha}{1-\alpha}} - 1 \right\} \left(\frac{1}{2} + \Delta + e \right) U^{-1} \left(\frac{k}{e} \right).$$

Since $U(c) = c^{1-\alpha}$, $U^{-1}(\cdot)$ is a monotonically increasing function with the range $[0, \infty)$. Thus, the cost reduction increases monotonically with the manager's effort cost k , and there exists a threshold k^* , at which the cost reduction is equal to the expected fund return gain $\frac{1}{2}\Delta r$ by the two-tiered strategy. When $k > k^*$ ($k < k^*$), the cost reduction is higher (lower) than the increase in return, making the single market (two-tiered) strategy preferable.

A.4 Proof of Proposition 5

To discuss the manager's incentive compatibility constraints, we first write down the manager's expected payoff by implementing the combined markets strategy, together with two other relevant deviation strategies, in which the manager shirks.

1. The manager exerts effort on both markets. If $s_A^e \neq s_B^e$, he invests in the market with better opportunity; if $s_A^e = s_B^e = -1$, he invests in the risk free asset; if $s_A^e = s_B^e = 1$, the manager randomizes between investing in market A and B . His expected utility is

$$\begin{aligned} V^{2e} = & \frac{1}{4} \left[\left(\frac{1}{2} + \Delta + e \right) \left(\frac{3}{2} - \Delta - e \right) \pi_{u,u}^u + \left(\frac{1}{2} + \Delta + e \right) \left(\frac{3}{4} + \frac{\Delta}{2} + \frac{e}{2} \right) (\pi_{u,d}^u + \pi_{d,u}^u) \right] \\ & + \frac{1}{4} \left[\left(\frac{1}{2} - \Delta - e \right) \left(\frac{3}{4} - \frac{\Delta}{2} - \frac{e}{2} \right) (\pi_{u,d}^d + \pi_{d,u}^d) + \left(\frac{1}{2} - \Delta - e \right) \left(\frac{3}{2} + \Delta + e \right) \pi_{d,d}^d \right] \\ & + \frac{1}{4} \left[\left(\frac{1}{2} + \Delta + e \right)^2 \pi_{d,d}^0 + \left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} - \Delta - e \right) \pi_{d,u}^0 \right] \\ & + \frac{1}{4} \left[\left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} - \Delta - e \right) \pi_{u,d}^0 + \left(\frac{1}{2} - \Delta - e \right)^2 \pi_{u,u}^0 \right] - 2k. \end{aligned}$$

2. The manager exerts effort only in one market. He first randomizes the market (say A) to work on. If $s_A^e = 1$, he invests in market A ; if $s_A^e = -1$ and $s_B^0 = 1$, he invests in market B ; and if $s_A^e = s_B^e = -1$, he invests in the risk free asset. His expected utility

is

$$\begin{aligned}
V^{1e} &= \frac{1}{4} \left[\left(\frac{1}{2} + \Delta + e \right) + \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} + \Delta \right) \right] \pi_{u,u}^u \\
&+ \frac{1}{4} \cdot \frac{1}{2} \left[\left(\frac{1}{2} + \Delta + e \right) \left(\frac{3}{2} + \Delta \right) (\pi_{u,d}^u + \pi_{d,u}^u) + \left(\frac{1}{2} - \Delta - e \right) \left(\frac{3}{2} - \Delta \right) (\pi_{u,d}^d + \pi_{d,u}^d) \right] \\
&+ \frac{1}{4} \left[\left(\frac{1}{2} - \Delta - e \right) + \left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} - \Delta \right) \right] \pi_{d,d}^d \\
&+ \frac{1}{4} \left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} + \Delta \right) \pi_{d,d}^0 + \frac{1}{4} \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} - \Delta \right) \pi_{u,u}^0 \\
&+ \frac{1}{4} \cdot \frac{1}{2} \left[\left(\frac{1}{2} + \Delta + e \right) \left(\frac{1}{2} - \Delta \right) + \frac{1}{4} \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} + \Delta \right) \right] (\pi_{d,u}^0 + \pi_{u,d}^0) - k
\end{aligned}$$

3. The managers exerts no effort on information acquisition at all and follows the same randomizing strategy as in case 1. His expected utility is

$$\begin{aligned}
V^{0e} &= \frac{1}{4} \left[\left(\frac{1}{2} + \Delta \right) \left(\frac{3}{2} - \Delta \right) \pi_{u,u}^u + \left(\frac{1}{2} + \Delta \right) \left(\frac{3}{4} + \frac{\Delta}{2} \right) (\pi_{u,d}^u + \pi_{d,u}^u) \right] \\
&+ \frac{1}{4} \left[\left(\frac{1}{2} - \Delta \right) \left(\frac{3}{4} - \frac{\Delta}{2} \right) (\pi_{u,d}^d + \pi_{d,u}^d) + \left(\frac{1}{2} - \Delta \right) \left(\frac{3}{2} + \Delta \right) \pi_{d,d}^d \right] \\
&+ \frac{1}{4} \left[\left(\frac{1}{2} + \Delta \right)^2 \pi_{d,d}^0 + \left(\frac{1}{2} + \Delta \right) \left(\frac{1}{2} - \Delta \right) \pi_{d,u}^0 \right] \\
&+ \frac{1}{4} \left[\left(\frac{1}{2} + \Delta \right) \left(\frac{1}{2} - \Delta \right) \pi_{u,d}^0 + \left(\frac{1}{2} - \Delta \right)^2 \pi_{u,u}^0 \right]
\end{aligned}$$

When e is relatively small, it is sufficient to consider the incentive compatibility constraint regarding the dominance of working in both markets versus working in only one market:

$$\begin{aligned}
0 \leq IC_{2e-1e} &= V^{2e} - V^{1e} \\
&\propto -4k + e \left(\frac{1}{2} - \Delta - e \right) \pi_{u,u}^u + \frac{e}{2} \left(\frac{1}{2} + \Delta + e \right) (\pi_{u,d}^u + \pi_{d,u}^u) \\
&\quad - \frac{e}{2} \left(\frac{1}{2} - \Delta - e \right) (\pi_{u,d}^d + \pi_{d,u}^d) - e \left(\frac{1}{2} - \Delta - e \right) \pi_{d,d}^d + e \left(\frac{1}{2} + \Delta + e \right) \pi_{d,d}^0 \\
&\quad - e(\Delta + e) (\pi_{d,u}^0 + \pi_{u,d}^0) - \left(\frac{1}{2} - \Delta - e \right) \left(\frac{1}{2} - \Delta \right) \pi_{u,u}^0.
\end{aligned}$$

Similar to the previous analysis, when this constraint is binding, the most efficient contract has the following properties: $\pi_{u,u}^u > 0$, $\pi_{u,d}^u = \pi_{d,u}^u > 0$, and $\pi_{d,d}^0 > 0$; payoffs for other outcomes are all zero. By comparing the ratio of probability to incentive differential, we find

$$\pi_{u,u}^u \equiv \beta_1^{CM} < \pi_{u,d}^u = \pi_{d,u}^u \equiv \beta_2^{CM} < \pi_{d,d}^0 \equiv \beta_3^{CM}.$$

The three positive payoff levels β_1^{CM} , β_2^{CM} and β_3^{CM} satisfy

$$\begin{aligned} & U' (U^{-1} (\beta_1^{CM})) : U' (U^{-1} (\beta_2^{CM})) : U' (U^{-1} (\beta_3^{CM})) \\ &= \frac{(\frac{1}{2} + \Delta + e) (\frac{3}{2} - \Delta - e)}{\frac{1}{2} - \Delta - e} : \frac{3}{2} + \Delta + e : \frac{1}{2} + \Delta + e \\ &= \frac{1}{M_1} : \frac{1}{M_2} : 1 \end{aligned}$$

where

$$M_1 \equiv \frac{\frac{1}{2} - \Delta - e}{\frac{3}{2} - \Delta - e}, M_2 \equiv \frac{\frac{1}{2} + \Delta + e}{\frac{3}{2} + \Delta + e}$$

Finally, the binding constraint implies

$$e \left(\frac{1}{2} - \Delta - e \right) \beta_1^{CM} + e \left(\frac{1}{2} + \Delta + e \right) (\beta_2^{CM} + \beta_3^{CM}) = 4k. \quad (25)$$

Under the power utility, the closed-form solution is

$$\begin{aligned} \beta_3^{CM} &= \frac{4k}{e \left(\frac{1}{2} - \Delta - e \right) M_1^{\frac{1-\alpha}{\alpha}} + e \left(\frac{1}{2} + \Delta + e \right) \left(1 + M_2^{\frac{1-\alpha}{\alpha}} \right)}, \\ \beta_2^{CM} &= M_2^{\frac{1-\alpha}{\alpha}} \beta_3^{CM}, \text{ and } \beta_1^{CM} = M_1^{\frac{1-\alpha}{\alpha}} \beta_3^{CM}. \end{aligned}$$

Given this contract, we now check the IC constraint that prevents the manager from not exerting effort in any market at all:

$$\begin{aligned} 0 \leq IC_{2e-0e} &= V^{2e} - V^{0e} \\ &\propto e(1 - 2\Delta - e) \beta_1^{CM} + e(2 + 2\Delta + e) \beta_2^{CM} + e(1 + 2\Delta + e) \beta_3^{CM} - 8k \\ &= e^2 \beta_1^{CM} + e \left[(1 - e) \beta_2^{CM} - e \beta_3^{CM} \right] \\ &= e \left[e \beta_1^{CM} + (1 - e) \beta_2^{CM} - e \beta_3^{CM} \right]. \end{aligned} \quad (26)$$

We have used equation (25) in deriving the last equation. When e is small, and β_3^{CM} does not exceed β_2^{CM} and β_1^{CM} too much, this incentive compatibility constraint holds.

We have also verified incentive-compatibility constraints for other deviation strategies, including investing half in each market when $s_A^e = s_B^e = 1$.

A.5 Proof of Proposition 6

We normalize $\frac{k}{e}$ to be 1 in this proof. Focus on the utility-based incentive wedge first. Implementing the single market strategy requires a baseline incentive wedge of 1. To implement the two-tiered strategy, the required incentive wedge is

$$\frac{1}{4} \beta_1^{TT} + \frac{3}{4} \beta_2^{TT} = \frac{M^{\frac{1-\alpha}{\alpha}} + 3}{\left(\frac{1}{2} - \Delta \right) M^{\frac{1-\alpha}{\alpha}} + \left(\frac{5}{2} + \Delta \right)} > 1.$$

To implement the combined market strategy, the required incentive wedge is

$$\begin{aligned}
& \frac{1}{4}\beta_1^{CM} + \frac{1}{2}\beta_2^{CM} + \frac{1}{4}\beta_3^{CM} \\
&= \frac{1}{4}\beta_1^{CM} + \frac{1-\rho}{2}\beta_2^{CM} + \frac{1}{4}\beta_3^{CM} + \frac{\rho}{2}\beta_2^{CM} \\
&> \frac{1}{4}\beta_1^{CM} + \frac{1-\rho}{2}\beta_2^{CM} + \frac{1}{4}\beta_3^{CM} + \frac{\rho e}{2} \frac{\beta_3^{CM} - \beta_1^{CM}}{1-e} \\
&= \left(\frac{1}{4} - \frac{\rho e}{2(1-e)} \right) \beta_1^{CM} + \frac{1-\rho}{2} \beta_2^{CM} + \left(\frac{1}{4} + \frac{\rho e}{2(1-e)} \right) \beta_3^{CM} \\
&= \frac{\left(1 - \frac{2\rho e}{1-e}\right) M_1^{\frac{1-\alpha}{\alpha}} + 2(1-\rho) M_2^{\frac{1-\alpha}{\alpha}} + \left(1 + \frac{2\rho e}{1-e}\right)}{\left(\frac{1}{2} - \Delta - e\right) M_1^{\frac{1-\alpha}{\alpha}} + \left(\frac{1}{2} + \Delta + e\right) \left(1 + M_2^{\frac{1-\alpha}{\alpha}}\right)}
\end{aligned} \tag{27}$$

where $\rho \in (0, 1)$ is any chosen constant. Note that the step in (27) uses the condition (26). To show that implementing the combined strategy requires a higher incentive slope than the two-tiered strategy, we need to show that

$$\frac{M^{\frac{1-\alpha}{\alpha}} + 3}{\left(\frac{1}{2} - \Delta\right) M^{\frac{1-\alpha}{\alpha}} + \left(\frac{5}{2} + \Delta\right)} < \frac{\left(1 - \frac{2\rho e}{1-e}\right) M_1^{\frac{1-\alpha}{\alpha}} + 2(1-\rho) M_2^{\frac{1-\alpha}{\alpha}} + \left(1 + \frac{2\rho e}{1-e}\right)}{\left(\frac{1}{2} - \Delta - e\right) M_1^{\frac{1-\alpha}{\alpha}} + \left(\frac{1}{2} + \Delta + e\right) \left(1 + M_2^{\frac{1-\alpha}{\alpha}}\right)}. \tag{28}$$

Let $M' = M^{\frac{1-\alpha}{\alpha}}$, $M'_1 = M_1^{\frac{1-\alpha}{\alpha}}$ and $M'_2 = M_2^{\frac{1-\alpha}{\alpha}}$. One can show that $0 < M_1 < M < M_2 < 1$, therefore $0 < M'_1 < M' < M'_2 < 1$ because $\alpha \in (0, 1)$. Inequality in (28) is equivalent to

$$\begin{aligned}
& \left[1 - 2\Delta - 3e + \frac{\rho e}{1-e} (5 + 2\Delta) \right] + \left[\left(\frac{7}{2} - 5\rho - (1 + 2\rho) \Delta - 3e \right) M'_2 \right. \\
& \left. - \left(2\Delta + e - \frac{\rho e}{1-e} (1 - 2\Delta) \right) M' + \left(\frac{1}{2} - \rho - (3 - 2\rho) \Delta - e \right) M' M'_2 \right] \\
& + \left(1 + 4\Delta + 3e - \frac{\rho e}{1-e} (5 + 2\Delta) \right) M'_1 + \left(e - \frac{\rho e}{1-e} (1 - 2\Delta) \right) M' M'_1 > 0.
\end{aligned}$$

We let $\rho = \frac{1}{6}$. The first square bracket is positive if

$$\begin{aligned}
0 &< (1-e)(1-2\Delta-3e) + (5+2\Delta)\rho e \\
&= (1-e-\rho e) - 2(1-e-\rho e)\Delta - 2(1-e-\rho e)e + 6\rho e - 2\rho e^2 - e(1-e).
\end{aligned}$$

A sufficient condition of this inequality is $\rho > \frac{1-e}{6-2e}$, which is satisfied when $\rho = \frac{1}{6}$. Regarding the second square bracket, since $1 > M'_2 = M' > 0$, it is sufficient to ensure that the sum of coefficients (in front of M'_2 , M'_2 and $M' M'_2$) in the square bracket is positive if $4 - 6\rho - 6\Delta - 5e > 0$, which is again satisfied when $\rho = \frac{1}{6}$ (note that $1 > 2(\Delta + e)$). Finally, it is easy to show that both terms on the third line are positive. The claim thus follows.

Next, we show the order of incentive slopes in dollars for these three fund designs. It is straightforward to show the relationship between the single market strategy and the two-tiered strategy. The challenging part is to show that the combined markets strategy has a higher expected dollar incentive slope than the two-tiered strategy, which says that

$$G \equiv \frac{1}{4}U^{-1}(\beta_1^{CM}) + \frac{2}{4}U^{-1}(\beta_2^{CM}) + \frac{1}{4}U^{-1}(\beta_3^{CM}) \\ - \frac{1}{4}U^{-1}(\beta_1^{TT}) + \frac{2}{4}U^{-1}(\beta_2^{TT}) + \frac{1}{4}U^{-1}(\beta_2^{TT}) > 0$$

Note that U^{-1} is increasing and convex, and it is easy to show that $\beta_3^{CM} > \beta_3^{TT}$. Our proof proceeds in two steps.

Step 1: If $\beta_1^{CM} \leq \beta_1^{TT}$, then

$$G = \frac{1}{4}[U^{-1}(\beta_1^{CM}) - U^{-1}(\beta_1^{TT})] + \frac{2}{4}[U^{-1}(\beta_2^{CM}) - U^{-1}(\beta_2^{TT})] \\ + \frac{1}{4}[U^{-1}(\beta_3^{CM}) - U^{-1}(\beta_2^{TT})] \\ \geq \frac{1}{4}U^{-\nu}(\beta_1^{TT})[\beta_1^{CM} - \beta_1^{TT}] + \frac{2}{4}U^{-\nu}(\beta_2^{TT})[\beta_2^{CM} - \beta_2^{TT}] \\ + \frac{1}{4}U^{-\nu}(\beta_2^{TT})[\beta_3^{CM} - \beta_2^{TT}] \\ > U^{-\nu}(\beta_2^{TT}) \left[\frac{\beta_1^{CM} + 2\beta_2^{CM} + \beta_3^{CM}}{4} - \frac{\beta_1^{TT} + 3\beta_2^{TT}}{4} \right] > 0$$

where the second inequality uses the convexity of U^{-1} , the second inequality is due to $U^{-\nu}(\beta_1^{TT}) < U^{-\nu}(\beta_2^{TT})$ and $\beta_1^{CM} - \beta_1^{TT} \leq 0$, and the last inequality is just the result of utility-based incentive wedge.

Step 2: If $\beta_1^{CM} > \beta_1^{TT}$, then

$$G > \frac{2}{4}[U^{-1}(\beta_2^{CM}) - U^{-1}(\beta_2^{TT})] + \frac{1}{4}[U^{-1}(\beta_3^{CM}) - U^{-1}(\beta_2^{TT})].$$

We first show that $2\beta_2^{CM} + \beta_3^{CM} > 3\beta_2^{TT}$. Using a similar technique as before,

$$\frac{1}{2}\beta_2^{CM} + \frac{1}{4}\beta_3^{CM} > \frac{-\frac{2\rho e}{1-e}M'_1 + 2(1-\rho)M'_2 + (1 + \frac{2\rho e}{1-e})}{(\frac{1}{2} - \Delta - e)M'_1 + (\frac{1}{2} + \Delta + e)(1 + M'_2)}.$$

Then, we only need to show that

$$\frac{-\frac{2\rho e}{1-e}M'_1 + 2(1-\rho)M'_2 + (1 + \frac{2\rho e}{1-e})}{(\frac{1}{2} - \Delta - e)M'_1 + (\frac{1}{2} + \Delta + e)(1 + M'_2)} > \frac{3}{(\frac{1}{2} - \Delta)M' + (\frac{5}{2} + \Delta)}.$$

By setting $\rho = \frac{1}{6}$ we can obtain the result above. Now given that $2\beta_2^{CM} + \beta_3^{CM} > 3\beta_2^{TT}$ and that U^{-1} is convexity and increasing, we obtain the intended result.

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