Rollover Risk

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- Rollover risk depends on insolvency risk.
- Rollover risk also exacerbates conflicts among different stakeholders:
  - conflict between debt and equity holders
  - coordination problem between creditors
- Through these channels, rollover risk affects the borrower's credit risk:
  - exacerbates its insolvency risk
  - exposes it to market liquidity risk
  - makes debt structure an important factor

Debt Structure

- Why do firms use debt? Frictions cause deviation from M-M Theorem.
  - Debt reduces cost of auditing the firms, e.g., Townsend (1979)
  - Short-term debt is a disciplinary device, e.g., Calomiris and Kahn (1991)
  - Short-term debt reduces adverse selection, e.g., Gorton and Pennacchi (1990)

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  - Short-term debt reduces adverse selection, e.g., Gorton and Pennacchi (1990)
- Optimal leverage
  - Tradeoff between tax shield and bankruptcy cost, e.g., Leland (1994)
- Optimal debt maturity
  - Debt overhang, e.g., Myers (1978) and He and Diamond (2010)
  - Tradeoff between lower cost of short-term debt financing and greater rollover risk, e.g., He and Xiong (2010)

Credit Risk

- Standard credit risk models focus on fundamental insolvency risk.
  - key determinant: distance to default (volatility adjusted leverage).
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  - Rollover exposes the borrower to market liquidity risk, e.g., He and Xiong (2010).
- Third generation with endogenous threshold:
  - The firm defaults when short-term creditors refuse to roll over, e.g., Morris and Shin (2004, 2010) and He and Xiong (2009).

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- Build on Leland (1994) and Leland and Toft (1996):
  - A firm has to constantly roll over its maturing debt by issuing new debt with the same maturity and face value at market price.
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  - Equity holders of the firm bear the rollover gain/loss and endogenously default when the equity value drops to zero.
- Intrinsic conflict between debt and equity holders:
  - at times of rollover losses, equity holders will inject capital to bail out maturing debt holders only to the extent the option value of keeping the firm alive is positive
- Debt rollover exposes the firm to liquidity risk in bond markets.
  - Deteriorating liquidity exacerbates default risk.
  - Liquidity premium and default premium are entangled.

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  - The rollover gain/loss is absorbed by equity holders;
  - The firm defaults when equity value drops to zero.
- The unlevered firm value follows a log-normal process under the Q-measure:

$$\frac{dV_t}{V_t} = (r - \delta)\, dt + \sigma dZ_t.$$

  - Riskfree rate r, payout rate δ.
- In bankruptcy creditors recover α fraction of the firm value.

Model (2): Debt Structure

- The firm commits to a stationary debt structure (C, P, m):
  - aggregate face value P and annual coupon payment C;
  - each bond has maturity m;
  - debt expirations are uniformly spread across time, i.e., over $(t, t + dt)$, $\frac{1}{m}dt$ fraction of the bonds matures.

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- The firm issues new bonds with the same face value, coupon rate and maturity to replace maturing bonds.
- Over $(t, t+dt)$, the net cash flow to equity holders is

$$NC_t = \delta V_t - (1-\pi)\,C + \frac{1}{m}\left[\overline{d}\,(V_t, m) - P\right].$$

  - $\overline{d}\,(V_t, m)$: market value of per unit newly issued bond;
  - When the bond price drops, equity holders face rollover losses.
  - Will show the loss is greater for short-term debt.

Model (3): Endogenous Default

- The firm defaults when V_t drops to an endogenous threshold V_B.
  - At V_B, equity value $E(V_B) = 0$, i.e., the firm cannot raise any equity financing;
  - Optimality of V_B: smooth pasting $E'(V_B) = 0$.

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  - Optimality of V_B: smooth pasting $E'\left(V_B\right) = 0$.
- Intrinsic conflict of interest between debt and equity holders:
  - When the bond price falls (for either fundamental or liquidity reasons), equity holders bear the rollover loss while the maturing debt holders get paid in full.
  - Equity holders face a tradeoff: rollover loss vs option value of keeping the firm alive.

Model (4): The Secondary Bond Markets

- The secondary markets of corporate bonds are highly illiquid.
  - Large bid-ask spreads and price impact.
  - Edwards, Harris, and Piwowar (2007): bid/ask spread on corporate bonds ranges from 4 to 75 bps.
  - Bao, Pan, and Wang (2009): trading cost (bid/ask spread & price impact) ranges from 74 to 221 bps; and the cost is higher for long-term bonds.

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- When a bond holder sells a bond, he only recovers a fraction $(1-k)$ of the value.
  - k represents the liquidity discount (trading cost, info problem,...)
- Each bond investor is subject to Poisson liquidity shocks with intensity ξ, a la Amihud and Mendelson (1986).
  - Upon the arrival of a liquidity shock, he has to sell his bond holdings.
- We assume no cost for trading equity and issuing new bonds.

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  - At the bankruptcy, $d\left(V_B,\tau;V_B\right)=\frac{\alpha V_B}{m}$, for all $\tau\in\left[0,m\right]$.
  - At maturity, $d\left(V_t,0;V_B\right)=p$, for all $V_t>V_B$.

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- ODE for equity value $E(V)$:

$$rE = (r - \delta) V_t E_V + \frac{1}{2}\sigma^2 V_t^2 E_{VV} + \delta V_t - (1 - \pi) C + d(V_t, m) - p.$$

  with boundary condition $E(V_B) = 0$:

  - Closed-form solution for $E(V)$ using Laplace tranformation.

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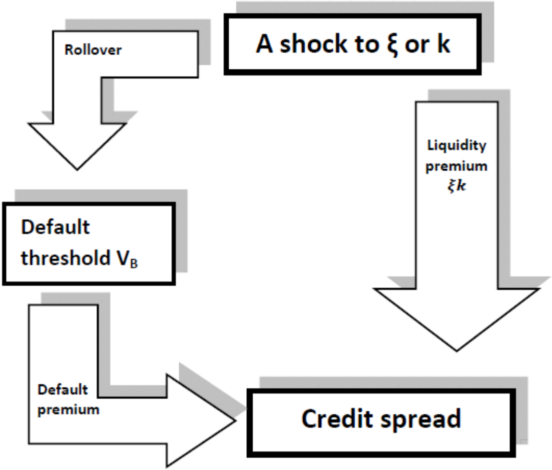
- Smooth pasting $E'(V_B) = 0$: closed-form solution for V_B.

Key Channels of Liquidity Effects

- Consider an unanticipated liquidity shock which increases ξ or k.
  - e.g., increased redemption risk, margin risk, or market illiquidity.

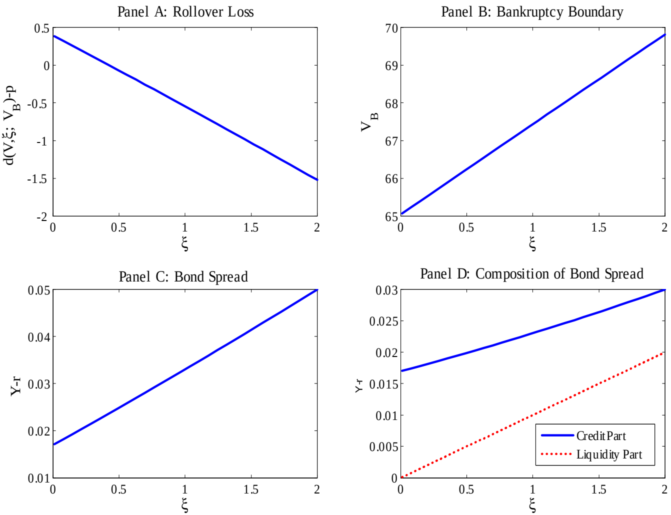


Baseline Model Parameters for Illustration

- Risk-free rate: $r = 8\%$.
- Tax rate: $\pi = 27\%$.
- Asset volatility $\sigma = 23\%$; payout rate $\delta = 2\%$.
- Trading cost $k = 1\%$; Intensity of liquidity shocks $\xi = 1$.
  - Consistent with Bao, Pan, and Wang (2009) who focus on a relatively liquid sample.
- Liquidation recovery rate: $\alpha = 0.5$.
- Debt maturities $m = 1$; total principal $P = 61.68$; total coupons $C = 6.39$.
- Current asset value: $V_t = 100$.

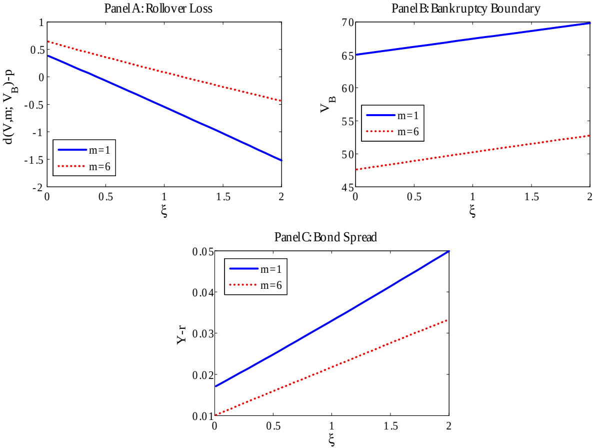
Market Liquidity and Endogenous Default

- Two channels of liquidity effects: liquidity premium and endogenous default risk.



Amplification by Short-term Debt

- Shorter maturity forces equity holders to quickly realize rollover loss.
  - Rollover loss per unit of time: $\left[\overline{d}\left(V_t, m\right) - P\right] / m$.
  - More severe conflict b/w debt- and equity-holders.
- Short-term maturity makes an individual bond safer, but a firm with more short-term debt is riskier.



Implications: Predicting Defaults

- Our model predicts market liquidity as a new factor for predicting bond defaults, in addition to
  - Distance to default: leverage, asset volatility
  - Firms' liquidity holdings: cash, credit lines
- The existing structural credit risk models have mixed successes:
  - Leland (2004): Leland model does a good job in capturing average default probabilities of bonds with different ratings.
  - Bharath and Shumway (2008): distance-to-default variable constructed from Merton model is not a sufficient statistic for default probability.
  - Davydenko (2007): distance to default cannot capture the cross section of bond spreads;
- Collin-Dufresne, Goldstein, and Martin (2001): standard variables cannot explain the changes of credit spreads.
- Das, Duffie, Kapadia, and Saita (2007): distance-to-default variables cannot fully capture default correlation observed in the data.

Implications: Decomposing Credit Spreads

- Both academics and policy makers have recognized the important effect of market liquidity on credit spreads, but tend to treat it as independent from default risk.
- Several studies, e.g., Longstaff, Mithal, and Neis (2005), Beber, Brandt, and Kavajecz (2008), and Schwarz (2009), decompose credit spreads to assess contributions of liquidity premium and default risk:

$$CreditSpread_{i,t} = \alpha + \beta \cdot CDS\_Spread_{i,t} + \delta \cdot LIQ_{i,t} + \epsilon_{i,t}$$

  - Default risk explains a majority part of the cross-sectional variation, although the liquidity effect is also significant.
- But these two effects are correlated through endogenous default.
  - How to classify the correlated part?
  - In the empirical analysis, the more precise measure of default risk (via traded prices) could have favored the default risk effect.

Implications: Measuring Liquidity Effects

- Several recent studies examine the impact of TAF on LIBOR-OIS spread.
  - e.g., Taylor and Williams (2009), McAndrews, Sarkar, and Wang (2008), Wu (2008).
- They tend to control for default risk using certain credit spread, such as CDS spread or LIBOR-REPO spread.
  - Example: Taylor and Williams (2009)

$$(LIBOR - OIS)_t = a \cdot (LIBOR - REPO)_t + b \cdot TAF_t + \epsilon_t$$

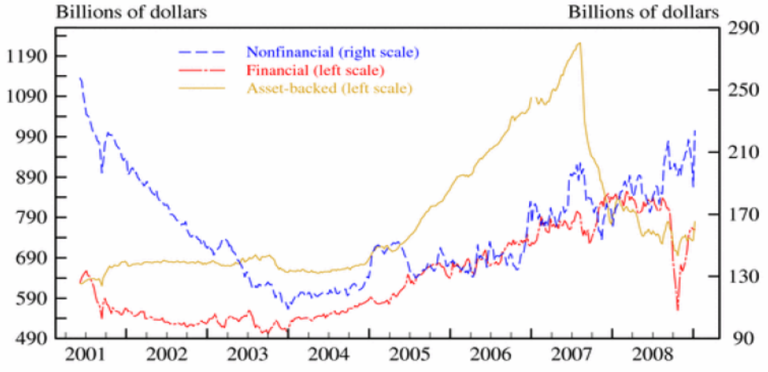
  - The control variables can also absorb liquidity effects and thus leading to an under-estimation.

Implications: Maturity Risk

- Our model implies that firms' debt maturity structure is an important determinant of credit risk.
- Evidence on non-financial firms with more maturing long-term debt during the recent credit crisis period had to cut down more investment and had greater credit spread increases.
  - Almeida, et al. (2009), Hu (2010).
- Evidence on credit ratings had ignored maturity risk.
  - Gopalan, Song, and Yerramilli (2009).

Debt Runs

- In practice, short-term debt tends to be the marginal financing.
  - Short-term debt holders' rollover decision is crucial.
- Runs by short-term debt holders on financial institutions was one of the main causes of the credit crisis of 2007-2008.
- Similar concerns looming over European countries and banks.



Coordination between Debt Holders

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- Global-games models of bank runs:
  - Rochet and Vives (2004) and Goldstein and Pauzner (2005) allow depositors holding noisy private signals about bank fundamental.
  - Signal noise leads to strategic uncertainty and prevents multiple equilibria, e.g., Carlsson and van Damme (1993) and Morris and Shin (2003).

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  - Signal noise leads to strategic uncertainty and prevents multiple equilibria, e.g., Carlsson and van Damme (1993) and Morris and Shin (2003).
- Dynamic coordination between creditors:
  - In a multiple-period setting, each maturing creditor is concerned by rollover decisions of future maturing creditors, e.g., He and Xiong (2009).
  - Fundamental volatility plays a key role
  - Debt maturity, credit lines, and liquidity also matter

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- The environment of illiquid/imperfect capital markets:
  - The firm cannot find a single creditor (with deep pockets) to finance all the debt, and has to rely on a continuum of small creditors.
  - When some creditors choose to run, the firm needs to draw on unreliable credit lines.
  - The firm asset is illiquid, i.e., the firm can only recover a fraction of its fundamental value in a premature liquidation.

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  - The firm asset is illiquid, i.e., the firm can only recover a fraction of its fundamental value in a premature liquidation.
- Two key assumptions:
  - The asset fundamental is time-varying and publicly observable.
  - A staggered debt structure.

Long-Term Asset

- The firm holds a long-term asset:
  - The asset generates constant cash flow rdt over a period dt.
  - At a Poisson arrival time τ_ϕ, the asset matures with a final payoff equal to τ_ϕ value of a publicly observable process:

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- Risk-neutral agents with discount rate ρ. Asset fundamental value:

$$F(y_t) = E_t\left[\int_t^{\tau_\phi} e^{-\rho(s-t)} rds + e^{-\rho\left(\tau_\phi - t\right)} y_{\tau_\phi}\right] = \frac{r}{\rho+\phi} + \frac{\phi}{\rho+\phi-\mu} y_t$$

  - y_t is the firm fundamental.
  - Our model ignores complications from private information.

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    - Each contract matures with a probability of δdt, a la Calvo (1983).
    - In aggregate, δdt fraction of debt matures over $(t, t + dt)$.
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- At τ_δ, an individual creditor decides to run or roll over.
  - Threshold strategy y_*: roll over if and only if $y \geq y_*$.

Debt Run and Liquidation

- Over $(t, t + dt)$, δdt fraction of contracts matures.
- If they choose to run, the firm needs to draw on its credit lines.
  - With prob $\theta\delta dt$, the credit lines fail, causing the firm to fail.
    - θ: unreliability of credit lines.
    - Can also be interpreted as imperfect government bailout.
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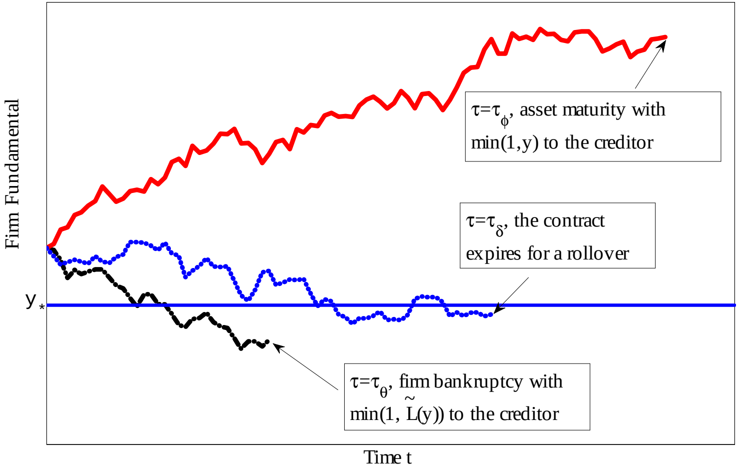
  - With prob $1 - \theta\delta dt$, the firm raises new fund and pays the creditors.
- Early liquidation recovers $\alpha \in (0, 1)$ of the fundamental value:

$$\widetilde{L}(y_t) = \alpha F(y_t).$$

  - Liquidation decision is irreversible, no partial liquidation.
- The firm's liquidation value, $\widetilde{L}(y)$, is equally divided among all creditors, including the running ones.
- Because the probability of firm failing by one's own run is tiny, the expected payoff from running is 1.

Three Possible Paths for An Individual Creditor

- A creditor receives r until a random time $\tau = \min\left(\tau_\phi, \tau_\delta, \tau_\theta\right)$;
- Other creditors' rollover threshold y_*: rollover when $y > y_*$, run otherwise.



An Individual Creditor's Problem

- Given other creditors' threshold y_*, his value function is

$$
\begin{aligned}
V\left(y_t; y_*\right) \quad = \quad & E_t \left\{ \int_t^{\tau} e^{-\rho(s-t)} r ds \right. \\
& + \underbrace{e^{-\rho(\tau-t)} \min\left(1, y_\tau\right) \mathbf{1}_{\{\tau=\tau_\phi\}}}_{\text{Top path, the asset matures and pays off}} \\
& + \underbrace{e^{-\rho(\tau-t)} \max_{\text{rollover or run}} \left\{1, V\left(y_\tau; y_*\right)\right\} \mathbf{1}_{\{\tau=\tau_\delta\}}}_{\text{Middle path, make the rollover decision when contract expires}} \\
& \left. + \underbrace{e^{-\rho(\tau-t)} \min\left(1, L + l y_\tau\right) \mathbf{1}_{\{\tau=\tau_\theta\}}}_{\text{Bottom path, the firm fails due to other creditors' run}} \right\}.
\end{aligned}
$$

- Debt run externality: each creditor's run imposes an externality on the other creditors who are locked in.

The Unique Monotone Equilibrium

- There exists a **unique** equilibrium threshold y_* s.t. $V(y_*; y_*) = 1$.
  - Equilibrium uniquely defined in upper and lower dominance regions.
  - Knowing future maturing creditors will not run in dominance regions, backward induction uniquely determines equilibrium in the middle.

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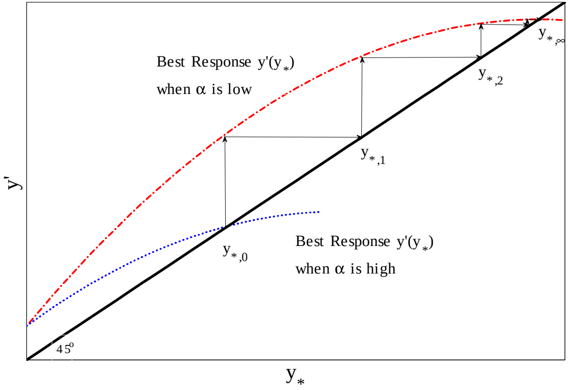
- Strategic uncertainty originates from time-varying fundamental.
  - e.g., Frankel and Pauzner (2000).
  - In contrast to Carlsson and van Damme (1993) and Morris and Shin (1998), strategic uncertainty arises from noise in private signals.
- Requires a well spread-out fundamental process.
  - Continuous time not essential.
  - Does not rely on specific information structure and immune from information revealed by market prices, e.g., Angeletos and Werning (2006) and Hellwig, Mukherji and Tsyvinski (2006).

Rat Race after a Drop in Liquidation Value

- Consider an unexpected drop in liquidation recovery rate α.
- A creditor's optimal response y' to other creditors' threshold y_*.

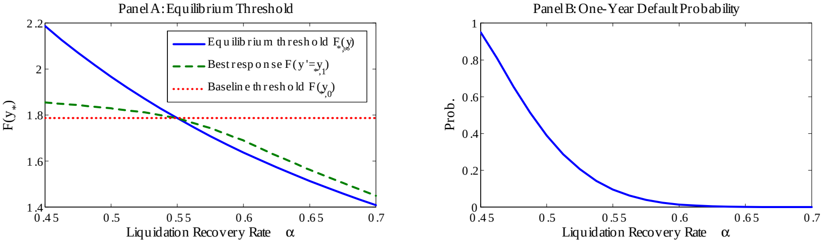


Calibrating Model Parameters

- We use a set of parameters for illustration.
  - Discount rate $\rho = 1.5\%$.
  - Asset cashflow $r = 7\%$; asset duration $1/\phi = 13$.
  - Asset's liquidation recovery rate $\alpha = 55\%$.
  - Asset's volatility $\sigma = 20\%$, growth rate $\mu = 1.5\%$, and current fundamental $y_0 = 1.4$.
  - Debt rollover frequency $\delta = 10$.
  - Unreliability of credit lines $\theta = 5$.

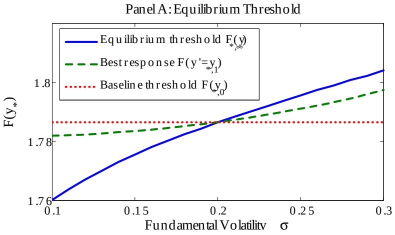
Effects of Liquidation Value

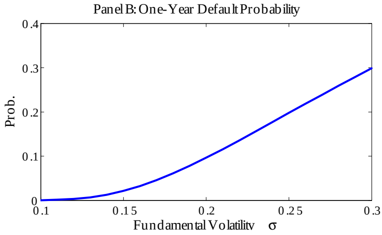
- Illiquidity exacerbates runs.
  - Similar to Rochet and Vives (2004).
- Threshold y_* sensitive to α.
  - Amplification effect by the rat race.



Effects of Fundamental Volatility

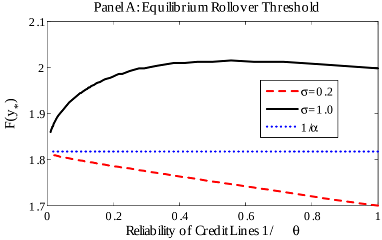
- Volatility affects each creditor in three channels:
  - Insolvency risk, causing y_* to increase with σ;
  - Rollover risk (strategic uncertainty), causing y_* to increase with σ;
  - Embedded option, causing y_* to decrease with σ.

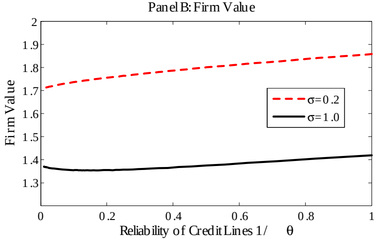




Effects of Credit Lines

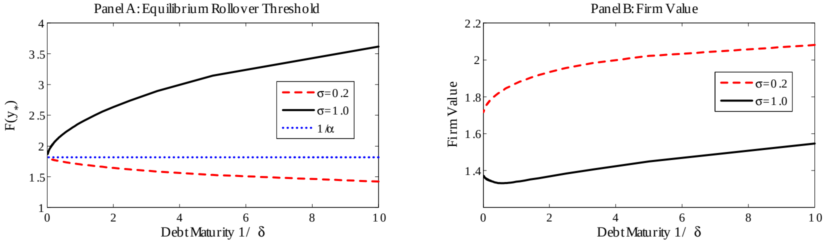
- Credit lines can temporarily sustain a firm under runs.
  - Common intuition: stronger credit lines should deter runs.
- When volatility is sufficiently large, credit lines exacerbate runs because fundamental can deteriorates during the period the firm lives on credit lines.
  - Uncertain government bailouts can be counter productive.





Effects of Debt Maturity

- Common intuition: longer debt maturities mitigate runs.
- Two offsetting effects of longer maturities:
  - 1) the firm faces less frequent rollover with other creditors and thus less likely to fail under runs.
  - 2) internally, longer lock-in effect for each creditor, which motivates runs, especially severe when voaltility is high.
- Longer maturities exacerbate runs when volatility is sufficiently high.
  - consistent with experience of runs on ABCP, e.g., Covitz, Liang, and Suarez (2009).



Further Discussion

- Synchronous vs Asynchronous Debt Structure
  - It is common for firms to spread out debt expirations.
  - The synchronous structure leads to more severe runs than the static-rollover benchmark when volatility is sufficiently high.
  - Which structure is optimal?
- Optimal Debt Maturity
  - Cheng and Milbradt (2010) extend the model to allow the firm switching b/w two projects: one with high growth and low volatility, the other with low growth and high volatility.
  - The optimal debt maturity trades off discipline on risk shifting and debt run risk.
- Spillover and Systemic Risk
  - When firms hold similar assets and face a downward sloping curve, runs on one firm can spill over to other firms.
  - Each firm's optimal debt structure and debt maturity depend on its own characteristics (fundamental volatility and asset illiqidity) and peer characteristics.

Summary

- Rollover risk is a key determinant of the health of the credit markets.
- Rollover risk exacerbates conflicts among different stakeholders:
  - conflict between debt and equity holders
  - coordination problem between creditors
- Through these channels, rollover risk affects the borrower's credit risk:
  - exacerbates its insolvency risk
  - exposes it to market liquidity risk
  - makes debt structure an important factor