Rollover Risk

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Princeton Initiative
September 10, 2011
Overview

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  - Bankruptcy of Bear Stearns, Lehman Brothers
  - Debt crises of Greece and Spain, and concerns about European banks
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- Rollover risk depends on insolvency risk.
- Rollover risk also exacerbates conflicts among different stakeholders:
  - conflict between debt and equity holders
  - coordination problem between creditors
- Through these channels, rollover risk affects the borrower’s credit risk:
  - exacerbates its insolvency risk
  - exposes it to market liquidity risk
  - makes debt structure an important factor
Debt Structure

- Why do firms use debt? Frictions cause deviation from M-M Theorem.
  - Debt reduces cost of auditing the firms, e.g., Townsend (1979)
  - Short-term debt is a disciplinary device, e.g., Calomiris and Kahn (1991)
  - Short-term debt reduces adverse selection, e.g., Gorton and Pennacchi (1990)

Optimal leverage
- Tradeoff between tax shield and bankruptcy cost, e.g., Leland (1994)

Optimal debt maturity
- Debt overhang, e.g., Myers (1978) and He and Diamond (2010)

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  - The firm defaults when its asset value falls below its debt level, e.g., Merton (1973), Longstaff and Schwartz (1995).
- Second generation with endogenous default threshold:
  - The firm defaults when its equity value drops to zero, e.g., Leland (1994), Leland and Toft (1996).
- Rollover exposes the borrower to market liquidity risk, e.g., He and Xiong (2010).
- Third generation with endogenous default threshold:
  - The firm defaults when short-term creditors refuse to roll over, e.g., Morris and Shin (2004, 2010) and He and Xiong (2009).
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“Rollover Risk and Credit Risk” by He and Xiong (2010)

- Build on Leland (1994) and Leland and Toft (1996):
  - A firm has to constantly roll over its maturing debt by issuing new debt with the same maturity and face value at market price.
  - Equity holders of the firm bear the rollover gain/loss and endogenously default when the equity value drops to zero.
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Intrinsic conflict between debt and equity holders:
- at times of rollover losses, equity holders will inject capital to bail out maturing debt holders only to the extent the option value of keeping the firm alive is positive

Debt rollover exposes the firm to liquidity risk in bond markets.
- Deteriorating liquidity exacerbates default risk.
- Liquidity premium and default premium are entangled.
A firm repays maturing bonds by issuing new bonds at market prices.

- The rollover gain/loss is absorbed by equity holders;
- The firm defaults when equity value drops to zero.

The unlevered firm value follows a log-normal process under the $Q$-measure:

$$dV_t = (r \delta) dt + \sigma dZ_t.$$ 

Riskfree rate $r$, payout rate $\delta$.

In bankruptcy creditors recover $\alpha$ fraction of the firm value.
Model (1)

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- The unlevered firm value follows a log-normal process under the $Q$-measure:
  \[
  \frac{dV_t}{V_t} = (r - \delta) \, dt + \sigma dZ_t.
  \]
  - Riskfree rate $r$, payout rate $\delta$.

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Model (2): Debt Structure

- The firm commits to a stationary debt structure \((C, P, m)\):
  - aggregate face value \(P\) and annual coupon payment \(C\);
  - each bond has maturity \(m\);
  - debt expirations are uniformly spread across time, i.e., over \((t, t + dt)\), \(\frac{1}{m} dt\) fraction of the bonds matures.
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- The firm issues new bonds with the same face value, coupon rate and maturity to replace maturing bonds.
- Over \((t, t + dt)\), the net cash flow to equity holders is
  \[
  NC_t = \delta V_t - (1 - \pi) C + \frac{1}{m} \left[ \bar{d} (V_t, m) - P \right].
  \]
  - \(\bar{d} (V_t, m)\): market value of per unit newly issued bond;
  - When the bond price drops, equity holders face rollover losses.
  - Will show the loss is greater for short-term debt.
Model (3): Endogenous Default

- The firm defaults when $V_t$ drops to an endogenous threshold $V_B$.
  - At $V_B$, equity value $E(V_B) = 0$, i.e., the firm cannot raise any equity financing;
  - Optimality of $V_B$: smooth pasting $E'(V_B) = 0$. 
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  - Optimality of $V_B$: smooth pasting $E'(V_B) = 0$.
- Intrinsic conflict of interest between debt and equity holders:
  - When the bond price falls (for either fundamental or liquidity reasons), equity holders bear the rollover loss while the maturing debt holders get paid in full.
  - Equity holders face a tradeoff: rollover loss vs option value of keeping the firm alive.
Model (4): The Secondary Bond Markets

- The secondary markets of corporate bonds are highly illiquid.
  - Large bid-ask spreads and price impact.
  - Edwards, Harris, and Piwowar (2007): bid/ask spread on corporate bonds ranges from 4 to 75 bps.
  - Bao, Pan, and Wang (2009): trading cost (bid/ask spread & price impact) ranges from 74 to 221 bps; and the cost is higher for long-term bonds.
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- When a bond holder sells a bond, he only recovers a fraction \((1 - k)\) of the value.
  - \(k\) represents the liquidity discount (trading cost, info problem,...)
- Each bond investor is subject to Poisson liquidity shocks with intensity \(\xi\), a la Amihud and Mendelson (1986).
  - Upon the arrival of a liquidity shock, he has to sell his bond holdings.
- We assume no cost for trading equity and issuing new bonds.
Solving the Equilibrium

- For a given $V_B$, PDE for the debt value $d(V_t, \tau; V_B)$:

\[ r + \xi k \mid \text{liquidity premium} \mid d(V_t, \tau) = c \frac{\partial d(V_t, \tau)}{\partial \tau} + (r - \delta) V_t \frac{\partial d(V_t, \tau)}{\partial V_t} + \frac{1}{2} \sigma^2 V_t^2 \frac{\partial^2 d(V_t, \tau)}{\partial V_t^2}. \]

- At the bankruptcy, $d(V_B, \tau; V_B) = \alpha V_B m$, for all $\tau \in [0, m]$.

- At maturity, $d(V_t, 0; V_B) = p$, for all $V_t > V_B$.

- ODE for equity value $E(V_t)$:

\[ r E(V_t) = (r - \delta) V_t E(V_t) + \frac{1}{2} \sigma^2 V_t^2 E(V_t) + \delta V_t (1 - \pi) C + d(V_t, m) p. \]

- With boundary condition $E(V_B) = 0$.

- Closed-form solution for $E(V_t)$ using Laplace transformation.

- Smooth pasting $E_0(V_B) = 0$: closed-form solution for $V_B$. 
Solving the Equilibrium

- For a given $V_B$, PDE for the debt value $d(V_t, \tau; V_B)$:

$$ \left( r + \xi k \right) d(V_t, \tau) = c - \frac{\partial d(V_t, \tau)}{\partial \tau} + (r - \delta) V_t \frac{\partial d(V_t, \tau)}{\partial V} + \frac{1}{2} \sigma^2 V_t \frac{\partial^2 d(V_t, \tau)}{\partial V^2} . $$

  - At the bankruptcy, $d(V_B, \tau; V_B) = \frac{\alpha V_B}{m}$, for all $\tau \in [0, m]$.
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ODE for equity value $E (V)$:

$$r E = (r - \delta) V_t E_V + \frac{1}{2} \sigma^2 V_t^2 E_{VV} + \delta V_t - (1 - \pi) C + d (V_t, m) - p.$$ 

with boundary condition $E (V_B) = 0$:

- Closed-form solution for $E (V)$ using Laplace tranformation.
Solving the Equilibrium

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liquidity premium
Key Channels of Liquidity Effects

- Consider an unanticipated liquidity shock which increases $\xi$ or $k$.
  - e.g., increased redemption risk, margin risk, or market illiquidity.
Baseline Model Parameters for Illustration

- Risk-free rate: $r = 8\%$.
- Tax rate: $\pi = 27\%$.
- Asset volatility $\sigma = 23\%$; payout rate $\delta = 2\%$.
- Trading cost $k = 1\%$; Intensity of liquidity shocks $\xi = 1$.
  - Consistent with Bao, Pan, and Wang (2009) who focus on a relatively liquid sample.
- Liquidation recovery rate: $\alpha = 0.5$.
- Debt maturities $m = 1$; total principal $P = 61.68$; total coupons $C = 6.39$.
- Current asset value: $V_t = 100$. 
Market Liquidity and Endogenous Default

- Two channels of liquidity effects: liquidity premium and endogenous default risk.
**Amplification by Short-term Debt**

- Shorter maturity forces equity holders to quickly realize rollover loss.
  - Rollover loss per unit of time: \( \frac{d (V_t, m) - P}{m} \).
  - More severe conflict b/w debt- and equity-holders.
- Short-term maturity makes an individual bond safer, but a firm with more short-term debt is riskier.

![Panel A: Rollover Loss](image1)

![Panel B: Bankruptcy Boundary](image2)

![Panel C: Bond Spread](image3)
Implications: Predicting Defaults

- Our model predicts market liquidity as a new factor for predicting bond defaults, in addition to
  - Distance to default: leverage, asset volatility
  - Firms’ liquidity holdings: cash, credit lines

- The existing structural credit risk models have mixed successes:
  - Leland (2004): Leland model does a good job in capturing average default probabilities of bonds with different ratings.
  - Bharath and Shumway (2008): distance-to-default variable constructed from Merton model is not a sufficient statistic for default probability.
  - Davydenko (2007): distance to default cannot capture the cross section of bond spreads;

- Collin-Dufresne, Goldstein, and Martin (2001): standard variables cannot explain the changes of credit spreads.

- Das, Duffie, Kapadia, and Saita (2007): distance-to-default variables cannot fully capture default correlation observed in the data.
Implications: Decomposing Credit Spreads

- Both academics and policy makers have recognized the important effect of market liquidity on credit spreads, but tend to treat it as independent from default risk.

- Several studies, e.g., Longstaff, Mithal, and Neis (2005), Beber, Brandt, and Kavajecz (2008), and Schwarz (2009), decompose credit spreads to assess contributions of liquidity premium and default risk:

\[
CreditSpread_{i,t} = \alpha + \beta \cdot CDS\_Spread_{i,t} + \delta \cdot LIQ_{i,t} + \epsilon_{i,t}
\]

- Default risk explains a majority part of the cross-sectional variation, although the liquidity effect is also significant.

- But these two effects are correlated through endogenous default.
  - How to classify the correlated part?
  - In the empirical analysis, the more precise measure of default risk (via traded prices) could have favored the default risk effect.
Several recent studies examine the impact of TAF on LIBOR-OIS spread.

They tend to control for default risk using certain credit spread, such as CDS spread or LIBOR-REPO spread.
  - Example: Taylor and Williams (2009)

\[ (LIBOR - OIS)_t = a \cdot (LIBOR - REPO)_t + b \cdot TAF_t + \epsilon_t \]

- The control variables can also absorb liquidity effects and thus leading to an under-estimation.
Implications: Maturity Risk

- Our model implies that firms’ debt maturity structure is an important determinant of credit risk.
- Evidence on non-financial firms with more maturing long-term debt during the recent credit crisis period had to cut down more investment and had greater credit spread increases.
- Evidence on credit ratings had ignored maturity risk.
  - Gopalan, Song, and Yerramilli (2009).
Debt Runs

- In practice, short-term debt tends to be the marginal financing.
  - Short-term debt holders’ rollover decision is crucial.
- Runs by short-term debt holders on financial institutions was one of the main causes of the credit crisis of 2007-2008.
- Similar concerns looming over European countries and banks.
Co ordination between Debt Holders

- The classic Diamond-Dybvig model on bank runs:
  - The simultaneous coordination problem among depositors leads to a self-fulfilling bank-run equilibrium.

- Global-games models of bank runs:
  - Signal noise leads to strategic uncertainty and prevents multiple equilibria, e.g., Carlsson and van Damme (1993) and Morris and Shin (2003).

- Dynamic coordination between creditors:
  - In a multiple-period setting, each maturing creditor is concerned by rollover decisions of future maturing creditors, e.g., He and Xiong (2009).

- Fundamental volatility plays a key role.

- Debt maturity, credit lines, and liquidity also matter.
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“Dynamic Debt Runs” by He and Xiong (2009)

- A firm holds a long-term risky asset by rolling over short-term debt.
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The environment of illiquid/imperfect capital markets:
- The firm cannot find a single creditor (with deep pockets) to finance all the debt, and has to rely on a continuum of small creditors.
- When some creditors choose to run, the firm needs to draw on unreliable credit lines.
- The firm asset is illiquid, i.e., the firm can only recover a fraction of its fundamental value in a premature liquidation.
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- Two key assumptions:
  - The asset fundamental is time-varying and publicly observable.
  - A staggered debt structure.
The firm holds a long-term asset:

- The asset generates constant cash flow $r dt$ over a period $dt$.
- At a Poisson arrival time $r \phi$, the asset matures with a final payoff equal to $r \phi$ value of a publicly observable process:

$$\frac{dy_t}{y_t} = \mu dt + \sigma dZ_t.$$
The firm holds a long-term asset:

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\[
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Risk-neutral agents with discount rate $\rho$. Asset fundamental value:

\[
F(y_t) = E_t \left[ \int_t^{\tau_\phi} e^{-\rho(s-t)} r ds + e^{-\rho(\tau_\phi-t)} y_{\tau_\phi} \right] = \frac{r}{\rho + \phi} + \frac{\phi}{\rho + \phi - \mu} y_t
\]

- $y_t$ is the firm fundamental.
- Our model ignores complications from private information.
Staggered Debt Financing

- We assume a unit measure of short-term creditors (discount rate $\rho$).
  - Interest payment $rdt$.
  - $r > \rho$. 

Threshold strategy $y$: roll over if and only if $y$. 

A staggered debt structure: Each contract matures with a probability of $\delta dt$, a la Calvo (1983).

In aggregate, $\delta dt$ fraction of debt matures over $(t, t + dt)$. 

This fraction is small and thus avoiding the Diamond-Dybvig type simultaneous coordination problem.

Rollover risk: during a contract period, other creditors might run.

At $\tau\delta$, an individual creditor decides to run or roll over. 

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- At $\tau_\delta$, an individual creditor decides to run or roll over.
  - Threshold strategy $y_*$: roll over if and only if $y \geq y_*$. 
Debt Run and Liquidation

- Over \((t, t + dt)\), \(\delta dt\) fraction of contracts matures.
- If they choose to run, the firm needs to draw on its credit lines.
  - With prob \(\theta \delta dt\), the credit lines fail, causing the firm to fail.
    - \(\theta\): unreliability of credit lines.
    - Can also be interpreted as imperfect government bailout.
  - With prob \(1 - \theta \delta dt\), the firm raises new fund and pays the creditors.
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  - With prob \(1 - \theta \delta dt\), the firm raises new fund and pays the creditors.
- Early liquidation recovers \(\alpha \in (0, 1)\) of the fundamental value:
  \[
  \tilde{L}(y_t) = \alpha F(y_t).
  \]
- Liquidation decision is irreversible, no partial liquidation.
- The firm’s liquidation value, \(\tilde{L}(y)\), is equally divided among all creditors, including the running ones.
- Because the probability of firm failing by one’s own run is tiny, the expected payoff from running is 1.
Three Possible Paths for An Individual Creditor

- A creditor receives $r$ until a random time $\tau = \min(\tau_\phi, \tau_\delta, \tau_\theta)$;
- Other creditors’ rollover threshold $y_*$: rollover when $y > y_*$, run otherwise.
An Individual Creditor’s Problem

- Given other creditors’ threshold \( y_\ast \), his value function is

\[
V (y_t; y_\ast) = E_t \left\{ \int_t^\tau e^{-\rho(s-t)} rds \right. \\
+ \left. e^{-\rho(\tau-t)} \min (1, y_\tau) 1\{\tau=\tau_\phi\} \right. \\
+ \left. e^{-\rho(\tau-t)} \max \{1, V (y_\tau; y_\ast)\} 1\{\tau=\tau_\delta\} \right. \\
+ \left. e^{-\rho(\tau-t)} \min (1, L + ly_\tau) 1\{\tau=\tau_\theta\} \right\}. 
\]

- Top path, the asset matures and pays off
- Middle path, make the rollover decision when contract expires
- Bottom path, the firm fails due to other creditors’ run

- Debt run externality: each creditor’s run imposes an externality on the other creditors who are locked in.
The Unique Monotone Equilibrium

- There exists a **unique** equilibrium threshold $y^*$ s.t. $V(y^*; y^*) = 1$.
  - Equilibrium uniquely defined in upper and lower dominance regions.
  - Knowing future maturing creditors will not run in dominance regions, backward induction uniquely determines equilibrium in the middle.

- Strategic uncertainty originates from time-varying fundamental, e.g., Frankel and Pauzner (2000).
- In contrast to Carlsson and van Damme (1993) and Morris and Shin (1998), strategic uncertainty arises from noise in private signals.
- Requires a well spread-out fundamental process.
- Continuous time not essential.
- Does not rely on specific information structure and immune from information revealed by market prices, e.g., Angeletos and Werning (2006) and Hellwig, Mukherji and Tsyvinski (2006).
There exists a unique equilibrium threshold $y^*$ s.t. $V(y^*, y^*) = 1$.

- Equilibrium uniquely defined in upper and lower dominance regions.
- Knowing future maturing creditors will not run in dominance regions, backward induction uniquely determines equilibrium in the middle.

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Rat Race after a Drop in Liquidation Value

- Consider an unexpected drop in liquidation recovery rate $\alpha$.
- A creditor’s optimal response $y'$ to other creditors’ threshold $y_\star$.

![Graph showing the best responses $y'(y_\star)$ for different values of $\alpha$.](image)
Calibrating Model Parameters

- We use a set of parameters for illustration.
  - Discount rate $\rho = 1.5\%$.
  - Asset cashflow $r = 7\%$; asset duration $1/\phi = 13$.
  - Asset’s liquidation recovery rate $\alpha = 55\%$.
  - Asset’s volatility $\sigma = 20\%$, growth rate $\mu = 1.5\%$, and current fundamental $y_0 = 1.4$.
  - Debt rollover frequency $\delta = 10$.
  - Unreliability of credit lines $\theta = 5$. 
Effects of Liquidation Value

- Illiquidity exacerbates runs.
  - Similar to Rochet and Vives (2004).
- Threshold $y_*$ sensitive to $\alpha$.
  - Amplification effect by the rat race.
Effects of Fundamental Volatility

- Volatility affects each creditor in three channels:
  - Insolvency risk, causing \( y_* \) to increase with \( \sigma \);
  - Rollover risk (strategic uncertainty), causing \( y_* \) to increase with \( \sigma \);
  - Embedded option, causing \( y_* \) to decrease with \( \sigma \).
Effects of Credit Lines

- Credit lines can temporarily sustain a firm under runs.
  - Common intuition: stronger credit lines should deter runs.
- When volatility is sufficiently large, credit lines exacerbate runs because fundamental can deteriorates during the period the firm lives on credit lines.
  - Uncertain government bailouts can be counter productive.

Panel A: Equilibrium Rollover Threshold

Panel B: Firm Value
Effects of Debt Maturity

- Common intuition: longer debt maturities mitigate runs.
- Two offsetting effects of longer maturities:
  - 1) the firm faces less frequent rollover with other creditors and thus less likely to fail under runs.
  - 2) internally, longer lock-in effect for each creditor, which motivates runs, especially severe when volatility is high.
- Longer maturities exacerbate runs when volatility is sufficiently high.
  - consistent with experience of runs on ABCP, e.g., Covitz, Liang, and Suarez (2009).

Panel A: Equilibrium Rollover Threshold

Panel B: Firm Value
Further Discussion

- **Synchronous vs Asynchronous Debt Structure**
  - It is common for firms to spread out debt expirations.
  - The synchronous structure leads to more severe runs than the static-rollover benchmark when volatility is sufficiently high.
  - Which structure is optimal?

- **Optimal Debt Maturity**
  - Cheng and Milbradt (2010) extend the model to allow the firm switching b/w two projects: one with high growth and low volatility, the other with low growth and high volatility.
  - The optimal debt maturity trades off discipline on risk shifting and debt run risk.

- **Spillover and Systemic Risk**
  - When firms hold similar assets and face a downward sloping curve, runs on one firm can spill over to other firms.
  - Each firm’s optimal debt structure and debt maturity depend on its own characteristics (fundamental volatility and asset illiqidity) and peer characteristics.
Rollover risk is a key determinant of the health of the credit markets.

Rollover risk exacerbates conflicts among different stakeholders:
- conflict between debt and equity holders
- coordination problem between creditors

Through these channels, rollover risk affects the borrower’s credit risk:
- exacerbates its insolvency risk
- exposes it to market liquidity risk
- makes debt structure an important factor