**Dictionary-based Classification**

- **RLS problem:** \( \arg \min_{w_i \in \mathbb{R}^n} \frac{1}{2} \| x - \sum_{i=1}^{c} D_i w_i \|_2^2 + \lambda \sum_{i=1}^{c} f(w_i) \)

- **SRC (Sparse Representation-based Classification)**
  \[ f(w_i) = \| w_i \|_1 \quad \tilde{w} = h(x) \text{ is sparse.} \]
  \[ \text{Decision Rule: } \arg \min_{i} \kappa_i(x) = \| x - D_i \tilde{w}_i \|_2^2 \]

- **CRC-RLS (Collaborative Representation-based Classification Using l1-Regularized Least Squares)**
  \[ f(w_i) = \| w_i \|_2^2 \quad \tilde{w} = (D^T D + \lambda I)^{-1} D^T x \]
  \[ \text{Decision Rule: } \arg \min_{i} \kappa_i(x) = \| x - D_i \tilde{w}_i \|_2^2 / \| \tilde{w}_i \|_2^2 \]

- **"Uncollaborative Problem"**
  \[ \arg \min_{w_i \in \mathbb{R}^n} \sum_{i=1}^{c} \frac{1}{2} \| x - D_i w_i \|_2^2 + \lambda f(w_i) \]
  \( \text{subspace-SRC (S-SRC), subspace-RLS (S-RLS)} \)

**Collaboration, Sparsity, Nonlinearity**

- **Linear Coding Scheme**
  \[ \bar{w}_j = P_j x \quad \bar{x}_j = D_j P_j x = L_j x \quad r_j(x) = (I - L_j)x \]
  \[ s_j(x) = \| r_j(x) \|_2^2 = x^T Q_j x, \text{ where } Q_j = (I - L_j)^T (I - L_j) \]
  Boundary between classes j and k:
  \[ d_{jk}(x) = x^T (Q_j - Q_k) x \]
  \[ Q_k = Q_j - Q_k = V \Sigma V^T, \quad \Sigma = \text{diag}(\sigma) \in \mathbb{R}^{m \times m} \]

- Write \( x = V \tilde{b} \), then \( d_{jk}(x) = b^T V^T (V \Sigma V^T) b = \sum_{i=1}^{n} \sigma_i b_i^2 \)

  The set of boundaries is (modulo V):
  \[ B = \{ b \mid \sum_{j=1}^{n} \sigma_j b_j^2 = 0, \quad \sum_{j=1}^{n} b_j^2 = 1 \} \]

- **Quadratic kernel embedding:** \( d_{jk}(x) = x^T Q_{jk} x = q_{jk}(\phi(x)) \)
  \[ \phi(x) = (x_1^2, \ldots, x_n^2, x_1 x_2, \ldots, x_{n-1} x_n)^T \in \mathbb{R}^{2n}, \quad m = (n+1)/2 \]

**Experimental Results**

- **Datasets:**
  - **MNIST:** 60,000 digit images, dimension 28x28=784. Randomly selected 2,500 training images and 500 testing images.
  - **GTZAN:** 100 music clips (30s, sampled at 22,050 Hz), 10 genres of music. Clips are divided into 3-sec, 50% overlap texture windows (TW), represented by 1st-order scattering coefficients. Randomly selected 18,000 TWs for training and 2,000 TWs for testing.

**Main Observations**

- Multi-class Q-SVM is a viable alternative to CRC-RLS.
- Collaborative representation is not always necessary.
- SRC & multi-class SVM are the "right" benchmarks.
- Sparsity induced nonlinearity is key to SRC.