Supplement to "Modeling Financial Contagion Using Mutually Exciting Jump Processes"*

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Abstract

This supplement to "Modeling Financial Contagion Using Mutually Exciting Jump Processes" contains tables and figures of (i) the Monte Carlo results, (ii) data on historical transmissions prior to or following major US stock market drops, and (iii) the robustness checks.

1. Monte Carlo Results

By means of extensive Monte Carlo simulations we have studied the degree of accuracy that our estimation method reaches. In this section, we present a selection of Monte Carlo results obtained in a setting that mimics that of our empirical data analysis. In particular, we impose similar parameter restrictions, use a similar sample period and use parameter values similar to the parameter estimates obtained from real data.

From the onset, one should be aware of the fact that one cannot expect degrees of accuracy similar or even close to what we are used to when estimating, e.g., a standard stochastic volatility model. By definition, extreme events occur infrequently and there is a positive probability that no jump occurs in any given finite time interval. When no jump occurs, there is no identification

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whatsoever, while when few jumps occur identification is expected to be weak. This is a consequence of the classical peso problem. Furthermore, we are effectively after subtle effects concerning the jumps of the process, not just whether they are present or not, but their finer structure: whether they self-excite, whether they cross-excite, etc. Finally, the estimation procedure can rely only on the time series of the log-returns of the various assets, since all other state variables are latent (volatility, jumps, jump intensities). In other words, we are asking for a lot out of the data. So one should not expect miracles from a time series of necessarily finite length. Nevertheless, what emerges out of the Monte Carlos is evidence that, using the estimation methodology outlined in our paper, the parameters of the data generating process can be recovered with sufficient degree of precision in a realistic context.

We first consider the univariate version of the Hawkes jump-diffusion model. Table 1.A reports quantiles of the small sample distribution of our estimators for the univariate Hawkes jump-diffusion process parameters obtained from 5,000 simulated sample paths. Specifically, the model is estimated for each of the 5,000 simulated sample paths and the reported values are the quantiles of the small sample distribution of the estimators obtained by estimating the parameters on each of the 5,000 sample paths. The true parameter values employed in the simulations' design are $\alpha = 20$, $\beta = 17$, $\lambda_\infty = 0.40$, $\sqrt{\theta} = 0.14$, $1/\gamma = 0.031$, $\mu = 0$ and $p = 1$. We use the interval-based expressions for the first moment and the autocovariances, derived in the appendix of our paper, and use the leading terms of the third and fourth moments, contained in the theorems. Because μ cannot be estimated consistently in finite time, we fix it in our GMM estimation and we further fix p.

In the bivariate case, which we are ultimately interested in, due to the cross-covariances (and under the restrictions that $\alpha_1 = \alpha_2 =: \alpha$ and $\lambda_{1,\infty} = \lambda_{2,\infty} =: \lambda_\infty$), we might expect even better identification than in the univariate case. Table 1.B reports quantiles of the small sample distribution of our estimators for the bivariate Hawkes jump-diffusion process parameters obtained from 5,000 simulated sample paths. The true parameter values are $\alpha = 20$, $\beta_{11} = 17$, $\beta_{12} = 1$, $\beta_{21} = 13$, $\beta_{22} = 7$, $\lambda_\infty = 0.40$, $\sqrt{\theta_1} = 0.14$, $\sqrt{\theta_2} = 0.17$, $\rho = 0.39$, $1/\gamma_1 = 0.031$, $1/\gamma_2 = 0.027$, $\mu_1 = 0.21$, $\mu_2 = 0.20$ and $p_1 = p_2 = 1$. Because on a fixed time horizon μ_1 and μ_2 can never be estimated consistently, we fix these parameters in our GMM estimation. Also, we fix $1/\gamma_1$, $1/\gamma_2$ and p_1, p_2 so that we need not consider the (approximated) third and fourth moments for identification, and rely solely on the interval-based moments derived in the appendix of our paper. Doing so, we can focus on the identification of the Hawkes process parameters, which are our prime interest in the paper.

The Monte Carlo results show that the population parameters of the Hawkes jump-diffusion model can be identified with sufficient degree of precision from data generated by the presupposed Hawkes jump-diffusion model. We have also analyzed the situation in which the data generating process is in fact a Poissonian jump-diffusion (with constant jump intensities) but is presupposed to be a Hawkes jump-diffusion (with stochastic jump intensities). We find that our estimation methodology is robust in this respect, finding parameter estimates for $\alpha, \beta_{11}, \beta_{12}, \beta_{21}$ and β_{22} that are very close to and statistically not significantly different from zero. With β's that are exactly zero the Hawkes jump-diffusion model reduces to a Poissonian jump-diffusion model.

Figure 1 plots the small sample distribution of our estimators for the bivariate Hawkes jump-diffusion process parameters obtained from 5,000 simulated sample paths. The true parameter values are $\alpha = n/a$, $\beta_{11} = 0$, $\beta_{12} = 0$, $\beta_{21} = 0$, $\beta_{22} = 0$, $\lambda_{\infty} = \lambda = 3.0$, $\sqrt{\theta_1} = 0.10$, $\sqrt{\theta_2} = 0.10$, $\rho = 0.15$, $1/\gamma_1 = 0.028$, $1/\gamma_2 = 0.028$, $\mu_1 = 0.25$, $\mu_2 = 0.25$ and $p_1 = p_2 = 1$. To focus attention on the identification of the jump process parameters, we fix again the parameters $\mu_1, \mu_2, 1/\gamma_1$, $1/\gamma_2$ and p in our GMM estimation. We conclude that our estimation methodology appropriately accounts for the possibility of having constant jump intensities.

2. Data on Historical Transmissions

To get a qualitative insight in the direction of jump transmissions, we sort daily US returns to find the most extreme declines (over 3.0% in a single day) in the US stock market in the period January 1, 1980 to May 31, 2012. If the inter-arrival time of these jumps was less than 6 weeks, we grouped the returns as being one event, or a related sequence of events. We end up with 86 declines and 28 groups.

We read the analysis in the press in the days following each event (statements such as "Tokyo opened lower *after* Wall Street closed down 3%" vs. "Wall Street opened lower *following* a rout in European markets") to confirm the sequencing: where and when the event started, and whether transmission (contagion) took place following one of these events. Tables 2, 3 and 4 summarize our findings.

3. Robustness Check I: Open-to-Close and Close-to-Open Returns

For a first robustness check, we use open and close international equity index data from finance.yahoo.com. We study two indices: S&P500 and Nikkei. Open-to-close and close-to-open data are available from January 4, 1984. Summary statistics are in Table 5. In Panel A, the opening of the US market marks the beginning of a new day, while in Panel B the opening of the Japanese market marks the beginning of a new day. Autocorrelograms and cross-correlograms are in Figure 2. GMM parameter estimates are in Table 6. The GMM parameter estimates are obtained under the same assumptions as in the analysis based on daily data.

4. Robustness Check II: Sample Split

As a second robustness check, we have estimated the Hawkes jump-diffusion model over two subsamples of the full sample. To assess the effect of sample changes on the parameter estimates, we have split the full sample into two subsamples: the first subsample covers January 1, 1980, to April 30, 2004, which corresponds to the first three quarters of the full sample ($[0, 3/4]$); the second subsample covers February 1, 1988, to May 31, 2012, which corresponds to the last three quarters of the full sample ($[1/4, 1]$). We focus attention on the pair (US,UK). The results are displayed in Table 7.

Panel A: Univariate	10%	25%	50%	75%	90%
α	5.68	12.55	18.61	22.49	42.81
β	1.84	7.57	12.94	16.70	20.04
λ_∞	0.10	0.27	0.45	0.63	1.02
$\sqrt{\theta}$	0.137	0.138	0.138	0.139	0.139
$1/\gamma$	0.0281	0.0342	0.0416	0.0525	0.0691

Panel B: Bivariate	10%	25%	50%	75%	90%
α	16.4	19.0	20.8	23.9	30.0
β_{11}	8.1	13.9	16.9	19.0	23.1
β_{12}	0.0	0.3	1.2	2.0	4.4
β_{21}	0.1	2.9	10.3	13.3	17.4
β_{22}	0.3	6.2	8.7	15.1	22.9
λ_∞	0.13	0.26	0.44	0.68	1.01
$\sqrt{\theta_1}$	0.138	0.139	0.139	0.140	0.140
$\sqrt{\theta_2}$	0.160	0.161	0.161	0.162	0.163
ρ	0.363	0.368	0.374	0.380	0.385

Table 1
Monte Carlo Results.

This table reports quantiles of the small sample distribution of our estimators for the Hawkes jump-diffusion model, obtained from 5,000 simulated sample paths. Each simulated sample path consists of 50 years of daily data. Panel A contains the results for the univariate case. The true parameter values are $\alpha = 20$, $\beta = 17$, $\lambda_\infty = 0.40$, $\sqrt{\theta} = 0.14$ and $1/\gamma = 0.031$. Panel B contains the results for the bivariate case. The true parameter values are $\alpha = 20$, $\beta_{11} = 17$, $\beta_{12} = 1$, $\beta_{21} = 13$, $\beta_{22} = 7$, $\lambda_\infty = 0.40$, $\sqrt{\theta_1} = 0.14$, $\sqrt{\theta_2} = 0.17$ and $\rho = 0.39$.

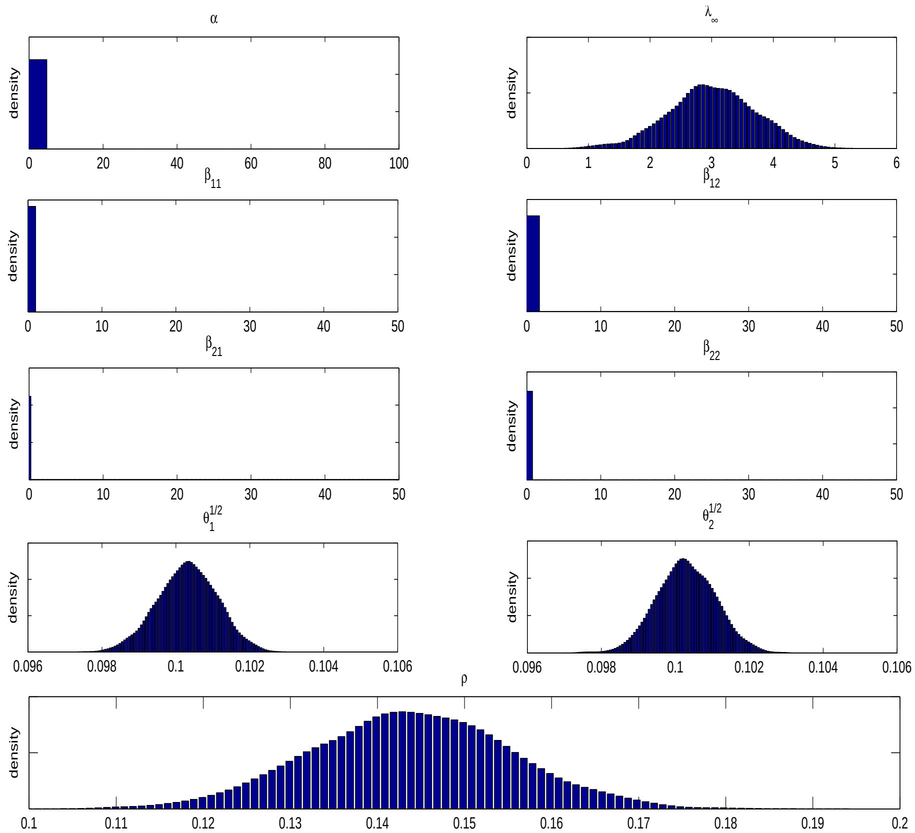

Figure 1
Monte Carlo Results: Hawkes vs. Poisson.

This figure plots the empirical distribution functions of the parameter estimators for the bivariate Hawkes jump-diffusion model, obtained from 5,000 simulated sample paths. The true parameter values are $\alpha = \text{n/a}$, $\lambda_\infty = \lambda = 3.0$, $\beta_{11} = 0$, $\beta_{12} = 0$, $\beta_{21} = 0$, $\beta_{22} = 0$, $\sqrt{\theta_1} = 0.10$, $\sqrt{\theta_2} = 0.10$, $\rho = 0.15$, $1/\gamma_1 = 0.028$, $1/\gamma_2 = 0.028$, $\mu_1 = 0.25$, $\mu_2 = 0.25$ and $p_1 = p_2 = 1$.

Group	Date	Event	Starts in	Transmits?
1	October 14, 1987	News on US trade deficit	US	Yes
	October 16, 1987	„	„	„
	October 19, 1987	Black Monday	Hong Kong	Yes
	October 22, 1987	„	„	„
	October 26, 1987	„	„	„
	November 30, 1987	Approaching US recession and accelerating inflation	US	Yes
	December 3, 1987	„	„	„
	January 8, 1988	„	„	„
2	September 9, 2008	Fannie Mae and Freddie Mac placed into conservatorship	US	No
	September 15, 2008	Bankruptcy of Lehman Brothers and sale of Merrill Lynch	US	Yes
	September 17, 2008	„	„	„
	September 22, 2008	Goldman Sachs and Morgan Stanley converted to bank holding companies	US	No
	September 29, 2008	Four bailouts in Europe and US House of Representatives voted against $ 700 billion rescue plan	Europe / US	Yes
	October 2, 2008	Prospects of the $700 billion US dollars bailout became reality after approval on October 1 (Senate) and October 3 (House of Representatives) "The financial markets, however, were not enthusiastic. ... weighed down by another round of bleak economic data, including a report showing that 159,000 jobs were lost in September..." (October 3, 2008; The New York Times)	US	Yes
	October 6, 2008	Major financial crisis in Iceland and several European governments guarantee bank deposits	Europe	Yes
	October 7, 2008	„	„	„
	October 9, 2008	Simultaneous cuts of interest rates by Central Banks	US	Yes
	October 15, 2008	Losses in Europe precipitated further losses in the US	Europe	Yes
	October 21, 2008	Stock markets continued to decline worldwide during the week of October 20, 2008.		
	October 22, 2008	„	„	„
	October 24, 2008	Losses in Europe precipitated further losses in the US. Stock markets declined sharply worldwide amidst growing fears among investors that a global recession is imminent if not already settled in. The panic was partly fueled by remarks by Alan Greenspan that the crisis is “a once-in-a-century credit tsunami”.	Europe / US	Yes
	October 27, 2008	„	US	Yes
	November 5, 2008	Bad news on economic activity	US	Yes
	November 6, 2008	„	„	„
	November 12, 2008	The prospects of a government rescue for US auto makers dwindled	US	No
	November 14, 2008	„	„	„
	November 19, 2008	Proposed federal bailouts of US auto makers failed	US	Yes
	November 20, 2008	„	„	„
	December 1, 2008	"The declines on Wall Street came after stocks fell in Europe..." (December 1, 2008; The Wall Street Journal) Announcement that US economy had officially entered recession in December 2007	Europe / US	Yes
	December 4, 2008	„	„	„

Table 2
Narrated Table of Major US Stock Market Drops.

Group	Date	Event	Starts in	Transmits?
2, cont.	January 7, 2009	Glum US economic and corporate news	US	No
	January 14, 2009	Glum US economic and corporate news and Greece downgrade	Europe / US	Yes
	January 20, 2009	Obama's inauguration, worries about global financial industry and Spain downgrade on the 19th	Europe / US	Yes
	January 29, 2009	Glum US economic news	US	Yes
	February 10, 2009	US bank rescue plan "Most Asian stock markets declined Wednesday after Wall Street gave the thumbs down to U.S. Treasury Secretary Timothy Geithner's revised proposal to shore up U.S. banks" (February 11, 2009; The Wall Street Journal)	US	Yes
	February 17, 2009	Continued worries about global financial industry	US	Yes
	February 23, 2009	„	„	„
	March 2, 2009	„	„	„
	March 5, 2009	Continued worries about global economy "The decline (Red. in Japan) came after major U.S. stock indexes plunged" (March 6, 2009; The Wall Street Journal)	US	Yes
	March 30, 2009	Worries about US corporate sector	US	Yes
	April 20, 2009	Continued worries about global financial industry	US	Yes
3	August 4, 1998	"Coming on a day with no significant bad news, the fall marks a shift in investors' mood." (August 5, 1998; The Wall Street Journal)	US	No
	August 27, 1998	"Global markets tumbled on speculation that Yeltsin may resign, along with an indefinite suspension of ruble trading and fear Russia may partially return to Soviet-style economics." (August 28, 1998; The Wall Street Journal)	Russia	Yes
	August 31, 1998	„	„	„
	October 1, 1998	"Greenspan painted a frightening picture of the damage that Long-Term Capital's failure could have inflicted on global economies, as he defended its rescue. He also suggested that more such threats may lurk in the markets." (October 2, 1998; The Wall Street Journal) "The Japanese economy, the world's second largest, appears to be on the brink of depression." (October 1, 1998; Financial Times)	Japan / US	Yes
4	August 4, 2011	Worries about US and European economy "Thursday's rout on Wall Street pushed investors (Red. globally) into safer assets" (August 5, 2011; The Wall Street Journal)	US	Yes
	August 8, 2011	US downgrade	US	Yes
	August 10, 2011	Worries about US and European economy	US	Yes
	August 18, 2011	„	„	„
	September 22, 2011	„	„	„
5	October 27, 1997	Asian flu	Thailand	Yes
6	October 13, 1989	"Friday's sell-off was triggered by the collapse of UAL's buy-out plan and a big rise in producer prices." (October 16, 1989; The Wall Street Journal)	US	Yes
7	April 14, 2000	US News	US	Yes
8	September 17, 2001	WTC terrorist attack	US	Yes
	September 20, 2001	„	„	„
9	September 11, 1986	Worries about higher interest rates and renewed inflation in US	US	Yes

Table 3
Narrated Table of Major US Stock Market Drops, continued.

Group	Date	Event	Starts in	Transmits?
10	March 12, 2001	"Europe's stock markets cracked under the pressure of the plunging Nasdaq Stock Market in the U.S. and other technology-industry anxieties." (March 13, 2001; The Wall Street Journal)	US	Yes
	April 3, 2001	"In Tokyo, the Nikkei Stock Average fell 4.1% as investors dumped shares following Friday's declines in the U.S. markets." (April 10, 2001; The Wall Street Journal)	US	Yes
11	April 4, 1988	Fed increases rates	US	Yes
12	July 10, 2002	"Stocks tumbled, hurt by waning confidence in the market and in corporate integrity. Stock markets plunged around the world following analyst downgrades and the S&P 500 shake-up." (July 11, 2002; The Wall Street Journal)	US	Yes
	July 19, 2002	Internet bubble burst	„	„
	July 22, 2002	Internet bubble burst	„	„
	August 1, 2002	Bearish economic data from the US; Internet bubble burst	US	Yes
	August 5, 2002	Internet bubble burst	„	„
	September 3, 2002	**"Foreign markets usually take their cue from the U.S. Yesterday, the opposite happened."** (September 4, 2002; The Wall Street Journal)	Japan	Yes
	September 19, 2002	Internet bubble burst	US	Yes
	September 27, 2002	"European markets began the day sharply lower in reaction to a late selloff in the U.S. on Friday." (October 1, 2002; The Wall Street Journal)	US	Yes
13	January 4, 2000	The prospect of rising US interest rates shook investor confidence in the technology shares	US	Yes
14	May 6, 2010	Worries about European sovereign debt "Thursday's plunge in U.S. stocks rattled investors around the globe" (May 7, 2010; The Wall Street Journal)	US	Yes
	May 20, 2010	„	„	„
	June 4, 2010	Worries about US jobs and European sovereign debt	US	Yes
	June 29, 2010	„	„	„
15	October 25, 1982	Decline of Nasdaq	US	Yes
16	November 15, 1991	US news (statements Bush)	US	Yes
17	November 9, 2011	Worries about European sovereign debt	Europe / US	Yes
18	March 24, 2003	"Waning optimism about quick Iraq war" (March 26, 2003; The Wall Street Journal)	US	Yes
19	December 20, 2000	US news (statements Fed)	US	Yes
20	February 27, 2007	See Introduction of the paper	China / US	Yes
21	August 6, 1990	Mideast conflict	US	Yes
22	February 5, 2008	US news on service sector	US	Yes
23	February 4, 2010	Worries about European sovereign debt	Europe / US	Yes
24	June 22, 2009	Worries about global economy	Europe / US	Yes
25	July 7, 1986	Worries about US economy	US	Yes
26	March 8, 1996	US jobs data	US	Yes
27	June 6, 2008	"Concerns about reliability of world (Red. oil) supplies; surge, ..., sends stock market plunging and raises fears that US economy could be in for period of inflation and slow growth" (June 7, 2008; The Wall Street Journal)	US	Yes
28	January 9, 1998	Indonesian crises	Indonesia	Yes

Table 4
Narrated Table of Major US Stock Market Drops, continued.

Panel A	S&P500	Nikkei
Mean	0.000241	-0.000153
St. Deviation	0.0114	0.0142
Skewness	-1.593	-0.514
Excess Kurtosis	34.61	8.50
Panel B	S&P500	Nikkei
Mean	0.000277	-0.000109
St. Deviation	0.0114	0.0134
Skewness	-1.582	0.092
Excess Kurtosis	35.47	6.00

Table 5
Descriptive Statistics Open-to-Close and Close-to-Open Data.

This table reports descriptive statistics for the log-returns derived from the open-to-close and close-to-open equity index data. Panel A: The opening of the US market marks the beginning of a new day. Panel B: The opening of the Japanese market marks the beginning of a new day. Sample period: January 4, 1984 to December 31, 2008.

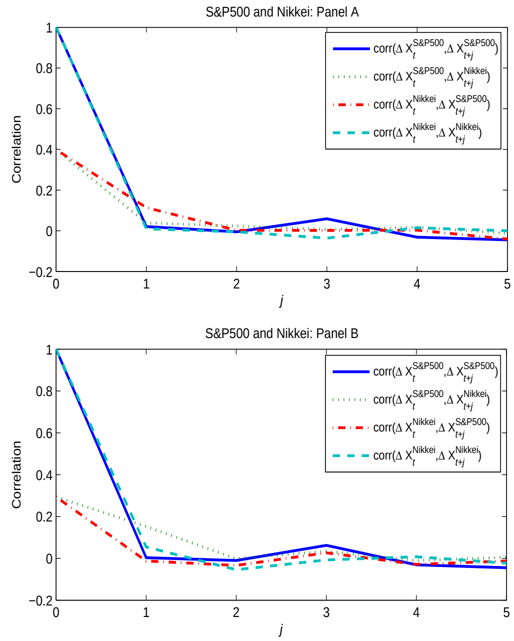

Figure 2
Autocorrelations and Cross-Correlations (Open-to-Close and Close-to-Open Data).

This figure plots autocorrelations and cross-correlations for the log-returns derived from the open-to-close and close-to-open equity index data of Table 5. In Panel A the opening of the US market marks the beginning of a new day, while in Panel B the opening of the Japanese market marks the beginning of a new day. The unit of the index j is days.

1	S&P500	1	S&P500
2	Nikkei	2	Nikkei
α	20.8***	$\sqrt{\theta_1}$	0.142***
	(3.7)		(0.0176)
β_{11}	17.6**	$\sqrt{\theta_2}$	0.195***
	(8.5)		(0.0064)
β_{12}	1.5	ρ	0.360***
	(4.6)		(0.0331)
β_{21}	10.8*	μ_1	0.206***
	(6.5)		(0.0452)
β_{22}	10.1*	μ_2	0.132***
	(5.8)		(0.0443)
λ_∞	0.38***	$1/\gamma_1$	0.0327***
	(0.070)		(0.00701)
λ_1	5.11	$1/\gamma_2$	0.0260***
λ_2	5.85		(0.00165)

Table 6
Parameter Estimates for the Bivariate Hawkes Jump-Diffusion Model: Open-to-Close and Close-to-Open Data.

This table reports the GMM parameter estimates for the 13 parameters of the bivariate Hawkes jump-diffusion model under exponential decay; standard errors are in parentheses. *, **, and *** indicate significance at the 95%, 97.5%, and 99.5% confidence levels, respectively. The underlying data are the log-returns derived from open-to-close and close-to-open equity index data. Sample period: January 4, 1984 to December 31, 2008.

	$[0, 3/4]$	$[1/4, 1]$
1	US	US
2	UK	UK
α	20.3***	20.4*
	(5.1)	(11.9)
β_{11}	16.2***	17.5**
	(3.3)	(8.3)
β_{12}	1.4	1.3
	(2.0)	(3.4)
β_{21}	12.9***	13.2**
	(2.2)	(6.1)
β_{22}	7.4***	7.2
	(2.6)	(6.7)
λ_∞	0.41***	0.46***
	(0.071)	(0.065)
λ_1	3.46	6.49
λ_2	4.10	7.20
$\sqrt{\theta_1}$	0.143***	0.159***
	(0.0074)	(0.0079)
$\sqrt{\theta_2}$	0.171***	0.177***
	(0.0032)	(0.0089)
ρ	0.347***	0.417***
	(0.0467)	(0.0267)
μ_1	0.208***	0.219***
	(0.0347)	(0.0341)
μ_2	0.190***	0.213***
	(0.0330)	(0.0416)
$1/\gamma_1$	0.0327***	0.0232***
	(0.00836)	(0.00169)
$1/\gamma_2$	0.0261***	0.0242***
	(0.00346)	(0.00240)

Table 7
Parameter Estimates for the Bivariate Hawkes Jump-Diffusion Model: Sample Split.

This table reports the GMM parameter estimates for the 13 parameters of the bivariate Hawkes jump-diffusion model under exponential decay; standard errors are in parentheses. We have split the full sample into two subsamples: the first subsample covers January 1, 1980, to April 30, 2004, which corresponds to the first three quarters of the full sample ($[0, 3/4]$); the second subsample covers February 1, 1988, to May 31, 2012, which corresponds to the last three quarters of the full sample ($[1/4, 1]$). *, **, and *** indicate significance at the 95%, 97.5%, and 99.5% confidence levels, respectively.