Fluid Mechanics General Examination with Prof. Luigi Martinelli

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1 Words of Advice

My advisor, Prof. Richard Miles, provided me with some very helpful advice. During the exam, be sure to listen closely to the question and request clarification if you do not fully understand what is being asked. Do not babble, but be mindful that you can and should interact with your examiners. Most faculty want to hear a well-focused dialog on the topic, so refrain from jumping to memorized answers. Your examiners will often give helpful hints or suggestions; therefore, it is crucial that you listen to everything they have to say.

Lastly, be aware that this summary was compiled quite a while after the exam, so it may be incomplete and/or contain errors. Use the solutions with caution and a bit of skepticism.

2 Classes Covered

1. MAE 527 Physics of Gases (Prof. Edgar Choueiri)
2. MAE 551 Fluid Mechanics (Prof. Alexander Smits)
3. MAE 553 Turbulent Flows (Prof. Marcus Hultmark)
4. MAE 555 Non-Equilibrium Gas Dynamics (Prof. Yiguang Ju)

3 Questions

3.1 Shock Tube

Prof. Martinelli began by asking me what I know about shock tubes and how they work. I drew the standard diagram (cf. Figure 1) and explained the salient features such as the shock wave, contact surface and expansion wave. Some of the questions he asked included

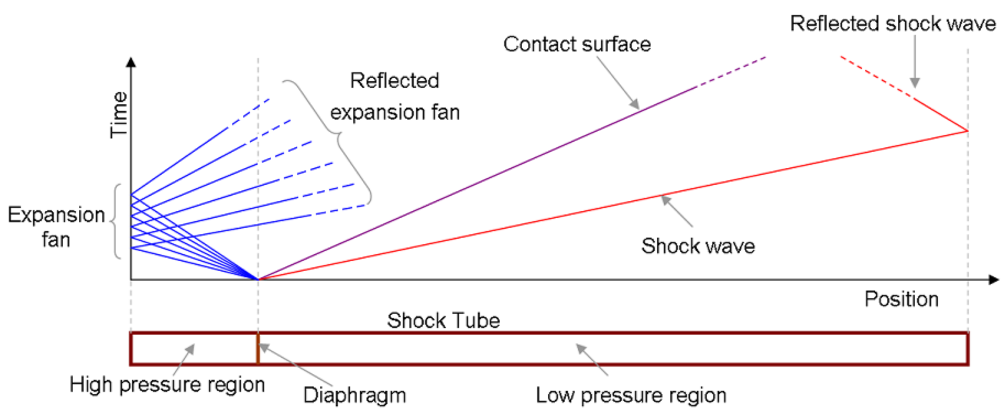

Figure 1: XT-Diagram for a Shock Tube [1].

What gases do you use? Like or unlike gases can be used to fill the driver and driven sections. Often, a light gas (e.g., He) at high temperature is chosen for the driver and a heavier gas (e.g., Ar) for the driven in order to maximize the shock wave's Mach number. The driven section will be at very low pressure.

When does the shock wave form? According to 1D inviscid theory, a shock wave will eventually form for any initial pressure ratio if the tube is long enough to allow characteristics of like type to intersect.

What is a contact surface? This surface separates two portions of fluid that share values of pressure and velocity, but potentially nothing else (e.g., different temperatures, speeds of sound, entropies).

What is the speed of the shock wave, u_{wave}?

The last question regarding u_{wave} initiated a discussion about steady and unsteady frames of reference since the common solution method for u_{wave} involves converting the unsteady moving shock into a steady problem. For moving shock waves, a control volume (CV) stationary w.r.t. the laboratory frame results in an unsteady problem whereas a CV moving w.r.t. the laboratory (at u_{wave}) yields a steady problem. Eventually, Prof. Martinelli asked

Why can you simply use relative velocities?

I began the proof to show the equivalence, but thankfully Prof. Martinelli did not require me to complete the (quite tedious) proof and reminded me that although most textbooks take this fact for granted, the proof is not trivial.

We then discussed the idea of a thermodynamic system (system in equilibrium), a control mass (no mass enters/exits) and ultimately, the related concept of the Reynolds transport theorem.

3.2 Vortices

Prof. Martinelli asked me how I would find the pressure distribution in a tornado. I drew Figure 2 on the board and stated that a tornado can be roughly modeled as a vortex with solid body rotation in the core and irrotational motion surrounding the core (i.e., Rankine vortex). For the equations

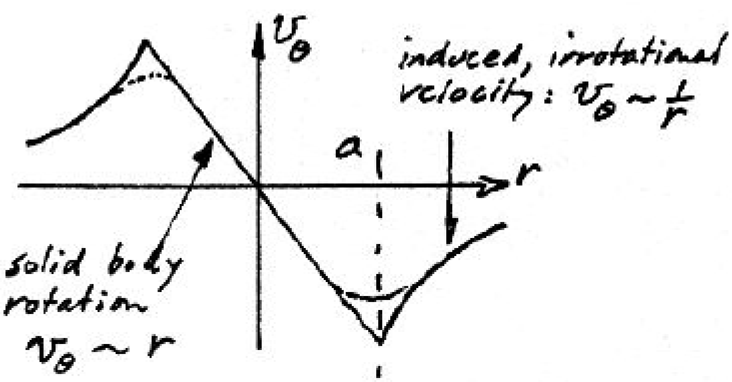

Figure 2: Tangential Velocity Distribution for a Rankine Vortex [2, p. 240].

specifying velocity, I wrote

$$v_\theta = \frac{\Gamma}{2\pi}\frac{r}{a^2} \text{ for } r < a, \tag{1}$$

$$v_\theta = \frac{\Gamma}{2\pi r} \text{ for } r > a. \tag{2}$$

Pressure is found from the velocity distribution by using the Bernoulli equation in the irrotational region and a radial momentum balance in the rotational core. The pressures (and velocities) are equal at $r = a$. Then, Prof. Martinelli asked how I knew that the tangential velocity in a potential vortex is given by $v_\theta = \frac{\Gamma}{2\pi r}$. I responded by saying it is found from analyzing an infinitely long vortex filament with the Biot-Savart law. He followed up by asking me if I knew or had seen the derivation for the Biot-Savart law. I said no and the questioning ended there. Note that the Biot-Savart law is not purely empirical and can be derived from principles of vector calculus.

3.3 Turbulence Modeling

Prof. Martinelli directed me to write the following partial differential equation on the board

$$u_t + uu_x + u_{xx} + \nu u_{xxxx} = 0 \text{ where } u = u(x,t), \tag{3}$$

and said that this equation can be used to model turbulence. He then proceeded to ask

How does (3) compare with the 1D momentum equation? The dissipation term is of opposite sign and is a fourth-order instead of a second-order derivative.

Prof. Martinelli suggested I use Fourier transform methods and see if I could glean any more information (and practice the skills I had learned while studying for the mathematics exam). Neglecting the non-linear term for the sake of simplicity, the Fourier transform of (3) w.r.t. x becomes

$$\hat{u}_t - k^2\hat{u} + \nu k^4 \hat{u} = 0 \text{ where } \mathcal{F}[u(x,t)] = \hat{u}(k,t), \tag{4}$$

and k represents wavenumber. Solving (4), which is an ordinary differential equation in t with k as a parameter,

$$\hat{u}(k,t) = e^{(k^2-\nu k^4)t}. \tag{5}$$

Why is (3) a good model for turbulence? According to (5), as $k \to 0$ (i.e., large length scales), the exponent is positive and as $k \to \infty$ (i.e., small length scales), the exponent is negative. These trends agree with the turbulence energy cascade, where the large length scales contain energy production and the small length scales contain energy dissipation.

References

[1] "Shock tube." [Online]. Available: `http://en.wikipedia.org/wiki/Shock_tube`, 2013. [Accessed 10 February 2014].

[2] A. J. Smits, "Lectures in fluid mechanics: Viscous flows and turbulence." Princeton University, Department of Mechanical and Aerospace Engineering, 2010.