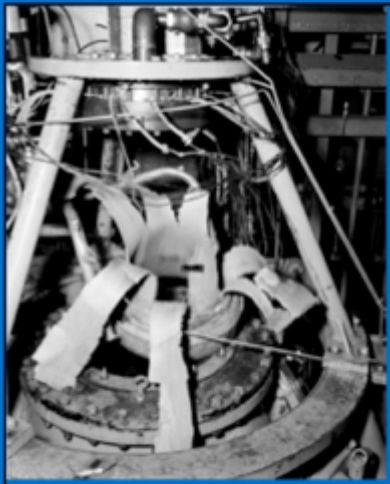
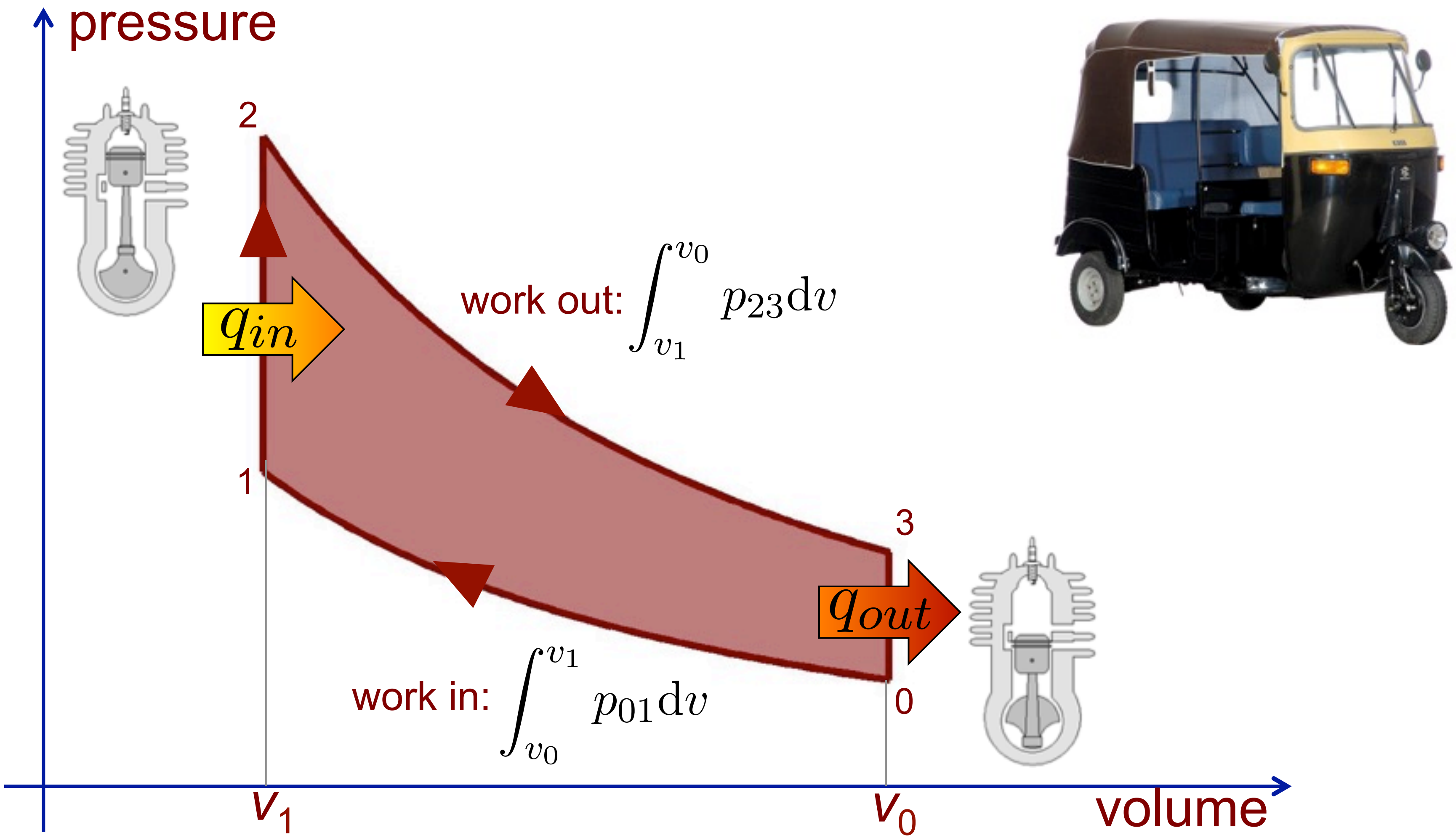
UNIVERSITY OF CAMBRIDGE
Department of Engineering

Sensitivity analysis of combustion instability
Position Lecture



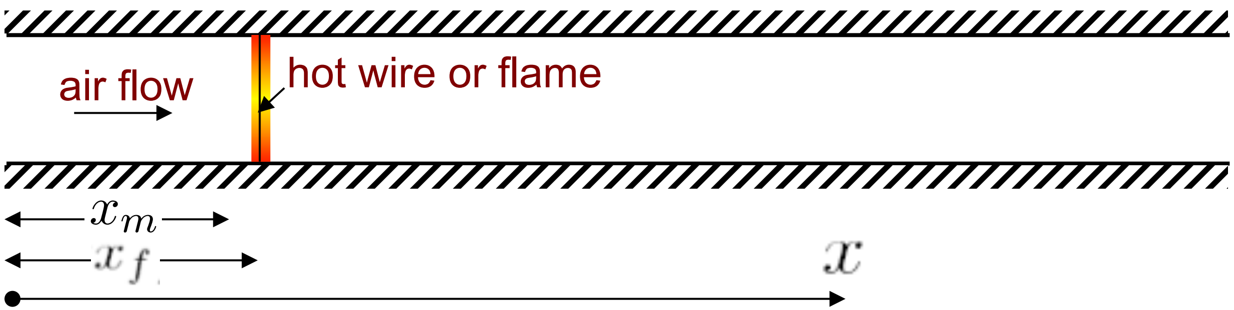
Matthew Juniper, Hans Yu, Nicholas Jamieson, José Aguilar

If more heat is injected at times of high pressure and less heat at times of low pressure, then heat is converted into work



A simple thermoacoustic model contains the acoustic momentum and energy equations, with heat release applied at a point in space

Diagram of the Rijke tube



Non-dimensional governing equations

acoustics

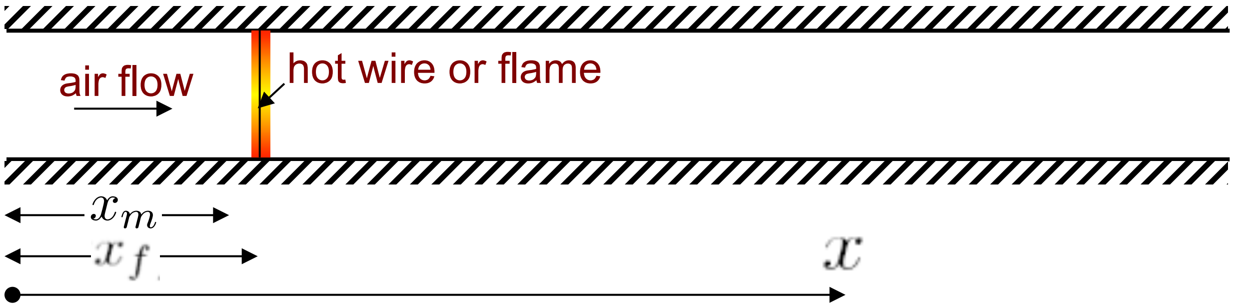
acoustic momentum: $$\gamma \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0$$

acoustic energy: $$\frac{\partial p}{\partial t} + \gamma \frac{\partial u}{\partial x} = (\gamma - 1) q \delta_D (x - x_f)$$

dilatation due to heat release

In the simplest possible model, the heat release is linearly proportional to the velocity at a measurement point, with a time delay.

Diagram of the Rijke tube



Non-dimensional governing equations

acoustics

dilatation due to heat release

acoustic momentum

$$\gamma \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0$$

acoustic energy

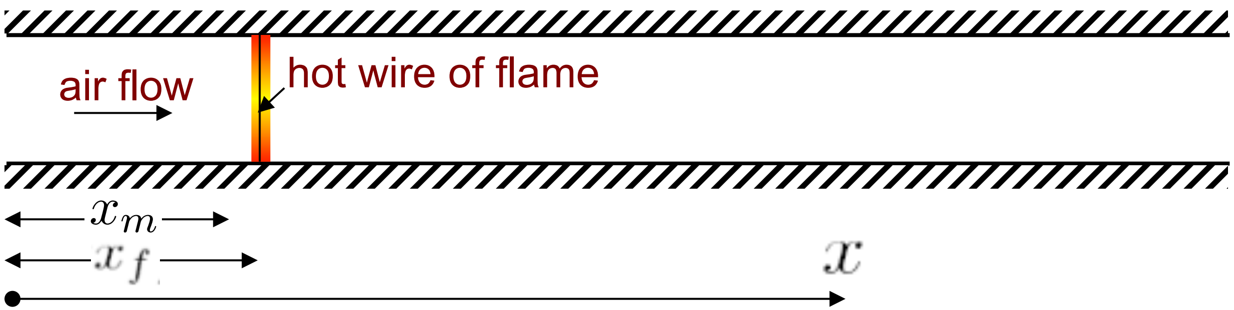
$$\frac{\partial p}{\partial t} + \gamma \frac{\partial u}{\partial x} = (\gamma - 1) q \delta_D (x - x_f)$$

n-tau heat release model

$$q(t) = n u(x_m, t - \tau)$$

The coefficients of the heat release model are found from models, experiments, or numerical simulations. The behaviour is very sensitive to errors in these coefficients.

Diagram of the Rijke tube



Non-dimensional governing equations

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acoustic momentum
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dilatation due to heat release

n-tau heat release model
$$q(t) = n u(x_m, t - \tau)$$

n and tau are found from:

- models
- experiments
- numerical simulations

acoustics

dilatation due to heat release

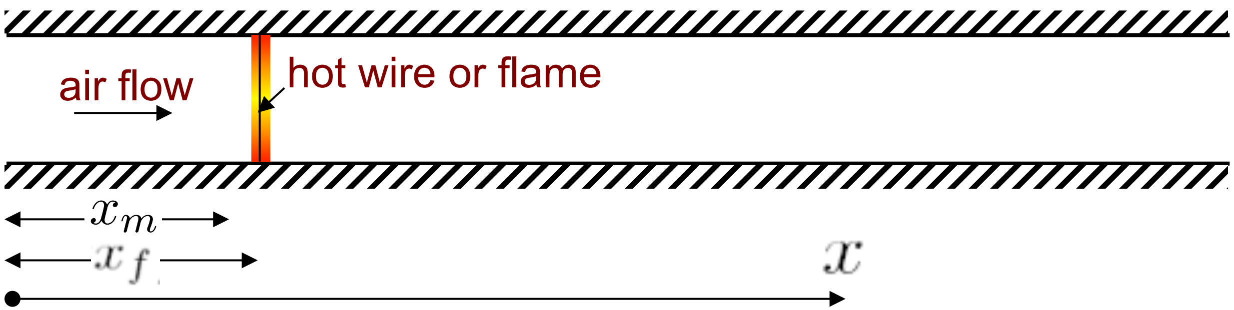
$$\gamma \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0$$

$$\frac{\partial p}{\partial t} + \gamma \frac{\partial u}{\partial x} = (\gamma - 1) q \delta_D (x - x_f)$$

The thermoacoustic mechanism is simple and can be easily modelled

To explain why this thermoacoustic model is so sensitive, we reduce it to a single equation for the eigenvalue, s, by projecting onto the natural acoustic modes.

Diagram of the Rijke tube



Non-dimensional governing equations;

acoustics

acoustic momentum

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dilatation due to heat release

n-tau heat release model

$$q(t) = n u(x_m, t - \tau)$$

single mode Galerkin model

project onto an acoustic mode, k = 1…N

$$u(x,t) = u_k(t)\cos(k\pi x)$$
$$p(x,t) = p_k(t)\sin(k\pi x)$$

perform a Laplace transform

$$u_k(t) = U_k \mathrm{e}^{st}$$
$$p_k(t) = P_k \mathrm{e}^{st}$$

$$s^2 + n_k \mathrm{e}^{-s\tau} + (k\pi)^2 = 0$$

Even for small heat release, n, we see that small changes in time delay, tau, can cause the most unstable mode to switch.

Laplace transform

$$u_k(t) = U_k \mathrm{e}^{st}$$
$$p_k(t) = P_k \mathrm{e}^{st}$$

Single mode Galerkin model; analysis in the limit of small heat release, n

$$s^2 + n_k \mathrm{e}^{-s\tau} + (k\pi)^2 = 0$$

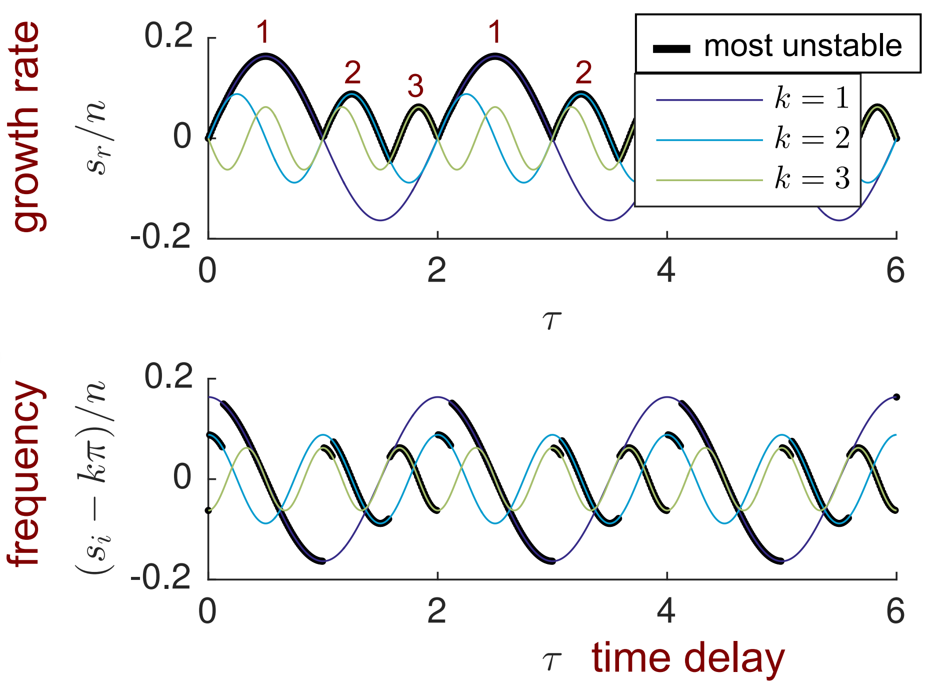
impose small heat release

$$n_k \rightarrow \epsilon n_k$$
$$s = s_0 + \epsilon s_1$$

find solutions at ε^0 and ε^1

$$s_0 = \pm k\pi \mathrm{i}$$
$$s_1 = -(n_k/2s_0)\mathrm{e}^{-s_0\tau}$$

$$Real(s) = \frac{n_k}{2k\pi}\sin(k\pi\tau)$$



- extreme sensitivity to time delay, tau
- strong sensitivity to geometric parameters

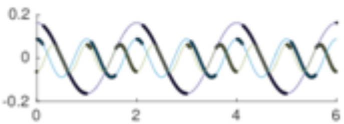
acoustics

$$\gamma \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0$$

dilatation due to heat release

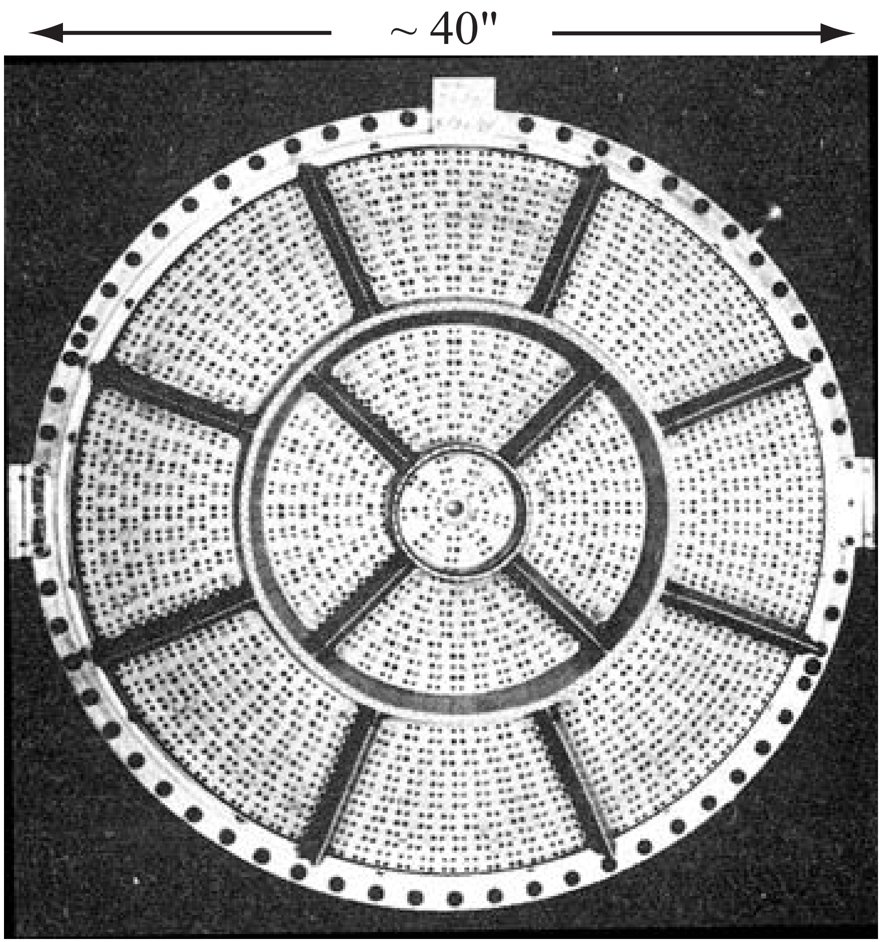
$$\frac{\partial p}{\partial t} + \gamma \frac{\partial u}{\partial x} = (\gamma - 1) q \delta_D (x - x_f)$$

The thermoacoustic mechanism is simple and can be easily modelled



But thermoacoustic models are very sensitive to parameters

For the F1 engine of the Saturn V, thermoacoustic instability was eliminated with baffles on the injector plate. But finding this design required 2000 full scale tests.



Despair:

- Practical thermo-acoustic systems are extremely sensitive to geometric parameters and to the heat release time delay, tau

- Some of these parameters, e.g. tau, are difficult to model or simulate accurately.

Despair:

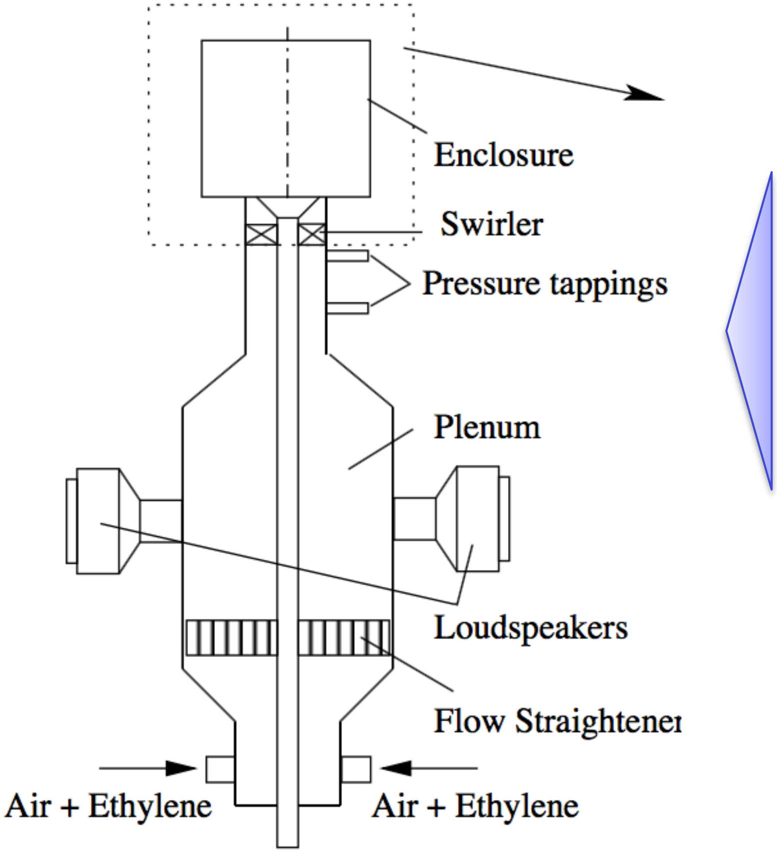
- Practical thermo-acoustic systems are extremely sensitive to geometric parameters and to the heat release time delay, tau
- Some of these parameters, e.g. tau, are difficult to model or simulate accurately.

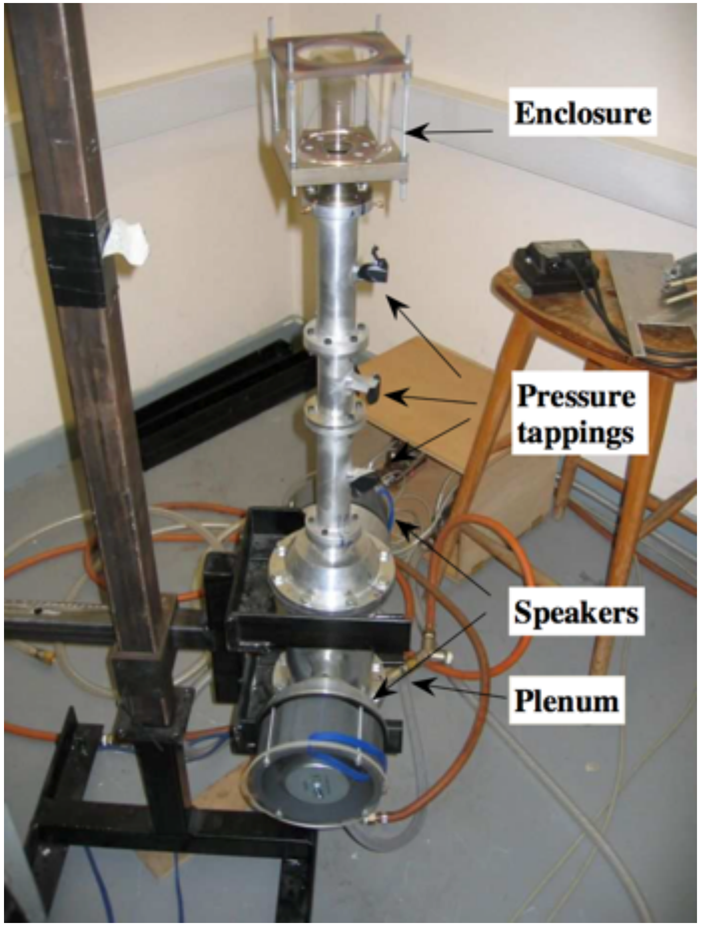
Hope:

- Usually (due to damping) only a few eigenvalues are unstable
- Many parameters can be changed (including flame parameters)
- To find stable operating points, linear stability analysis is sufficient
- Experience shows that most systems can be stabilized

We model a thermoacoustic system as an acoustic network and exploit the features described earlier to predict how to stabilize it:

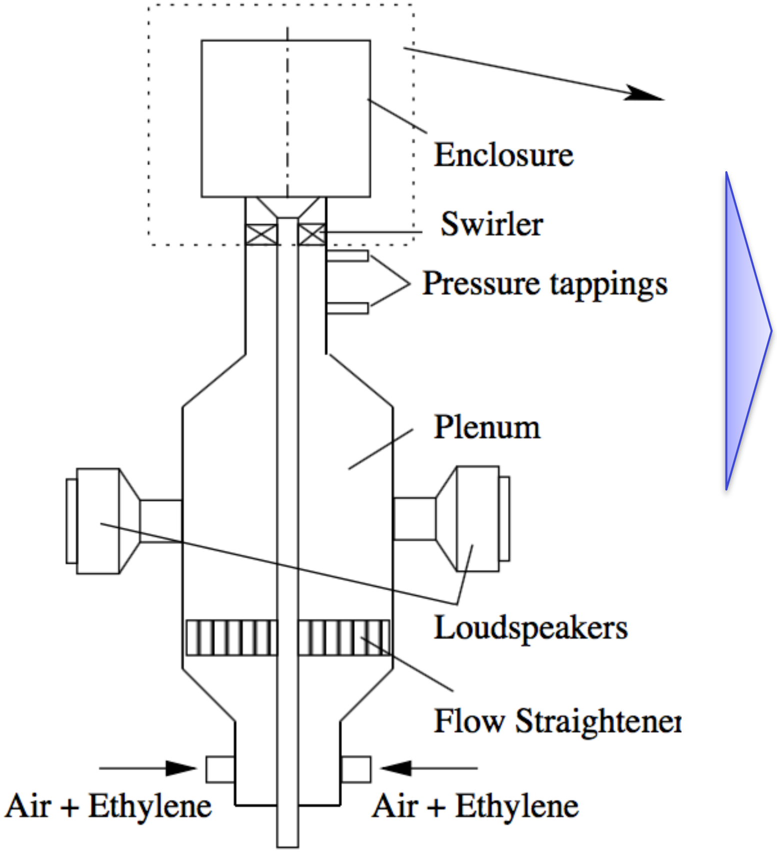
Thermoacoustic model (flame + acoustics)



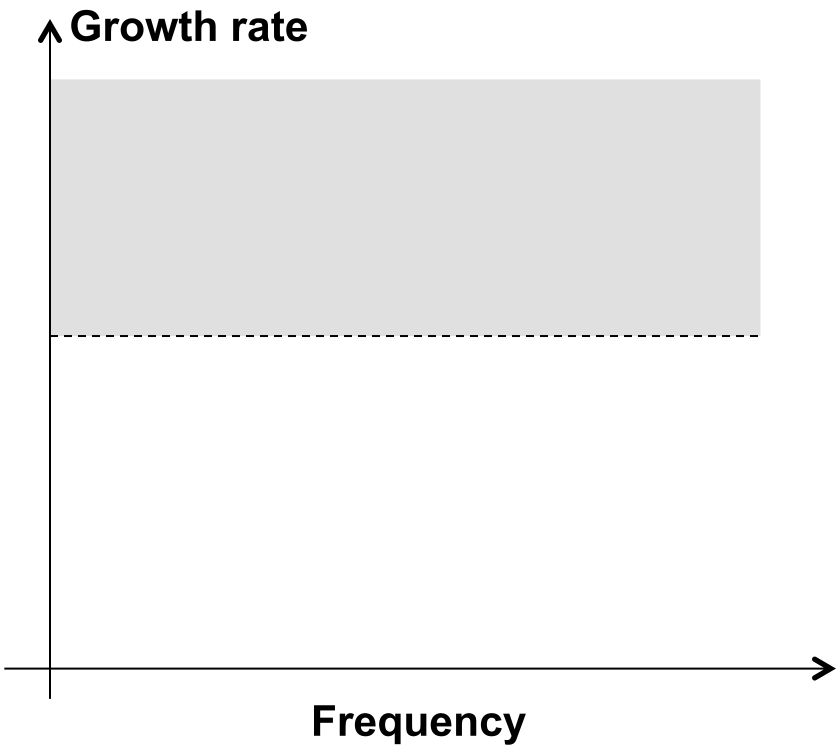


A linear analysis is sufficient.

Thermoacoustic model (flame + acoustics)

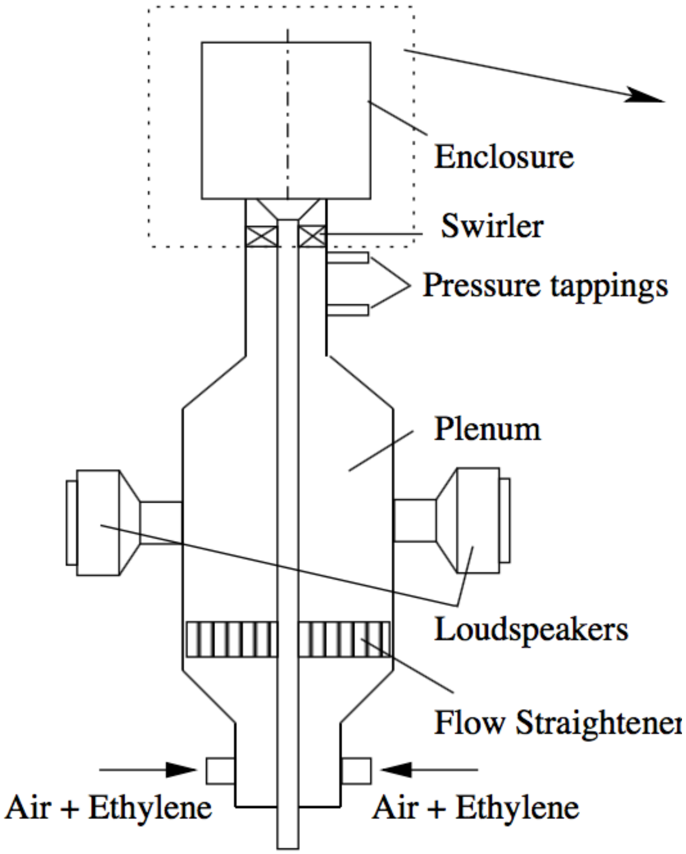


Linear modes of the model



A linear analysis is sufficient. Only a few modes are unstable

Thermoacoustic model (flame + acoustics)



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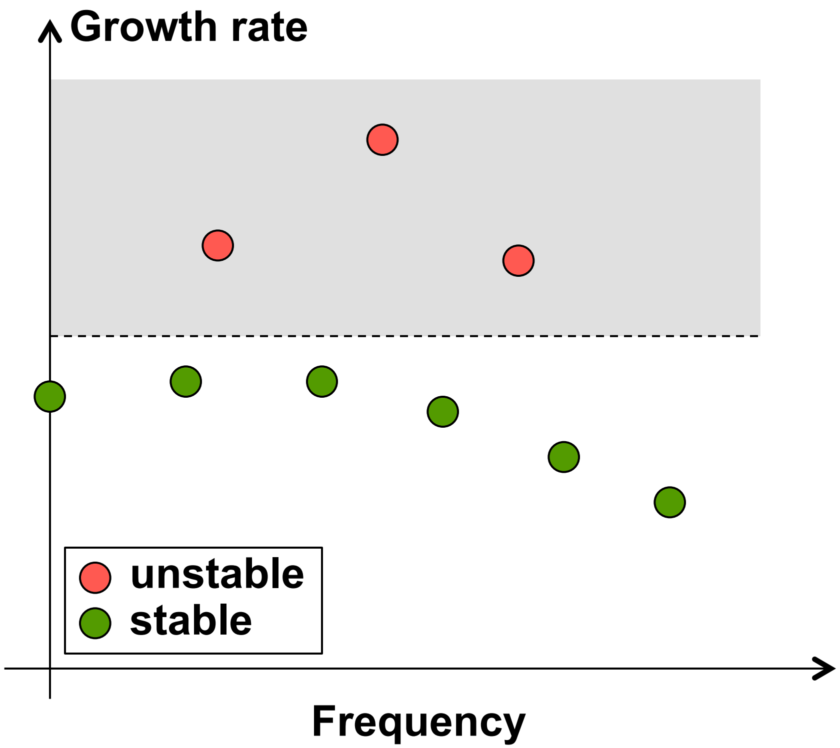
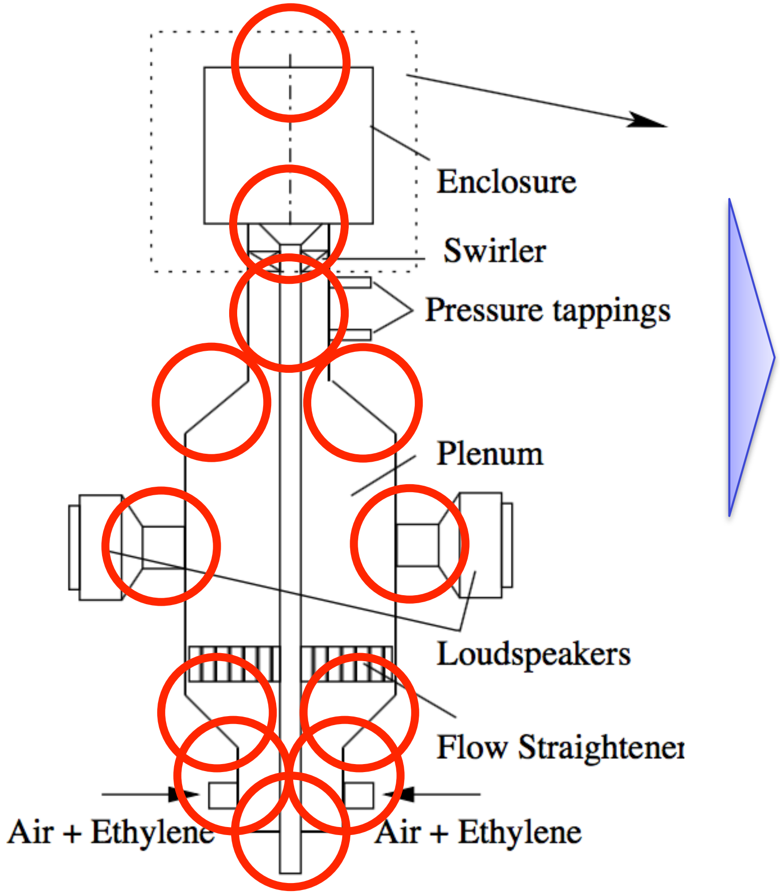
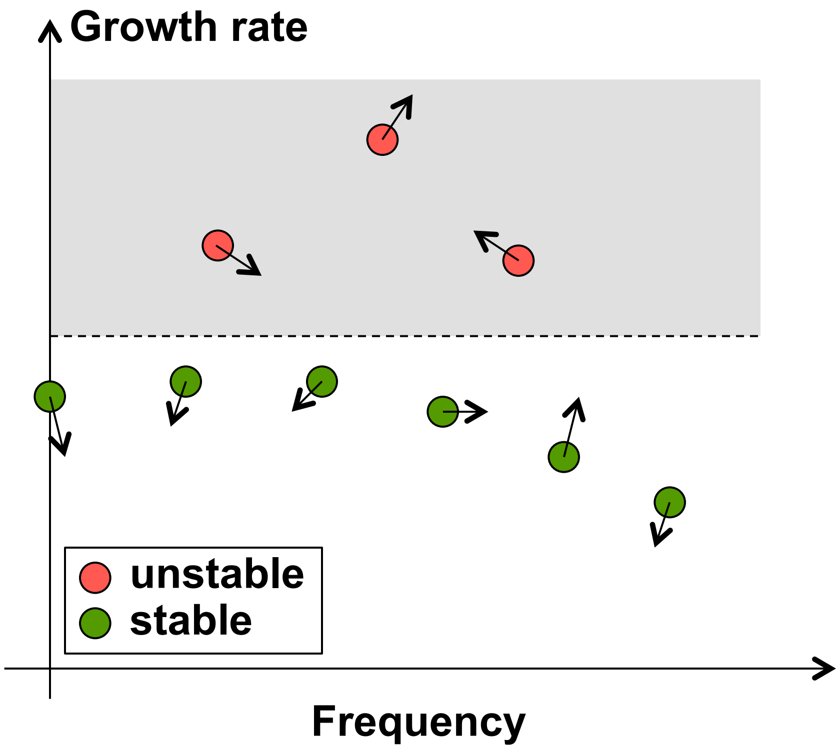


A linear analysis is sufficient. Only a few modes are unstable. Many parameters can be changed.

Thermoacoustic model (flame + acoustics)

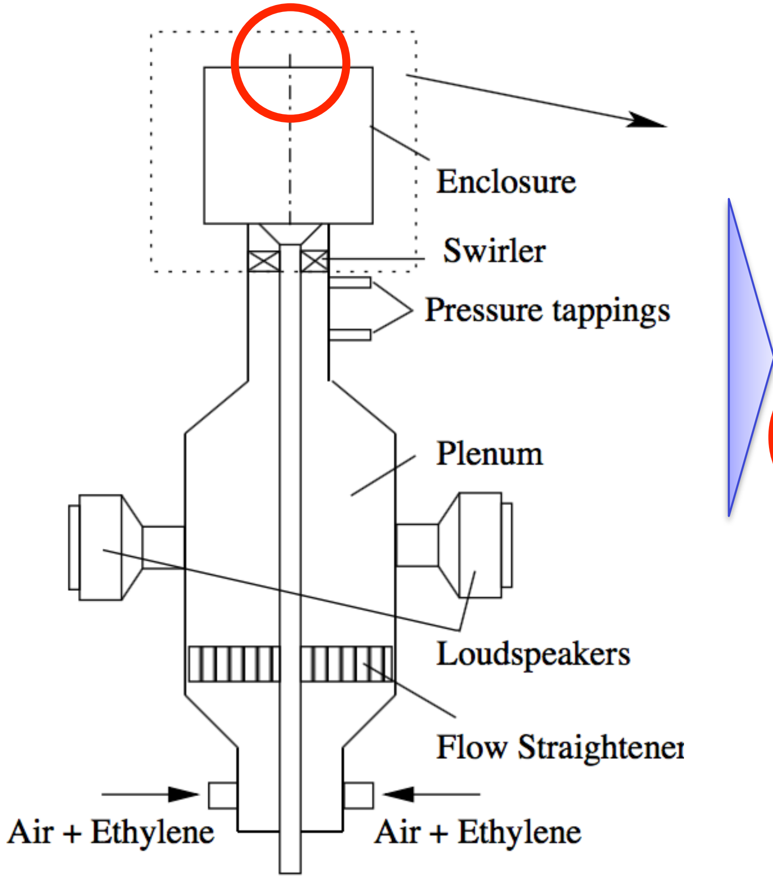


Linear modes of the model

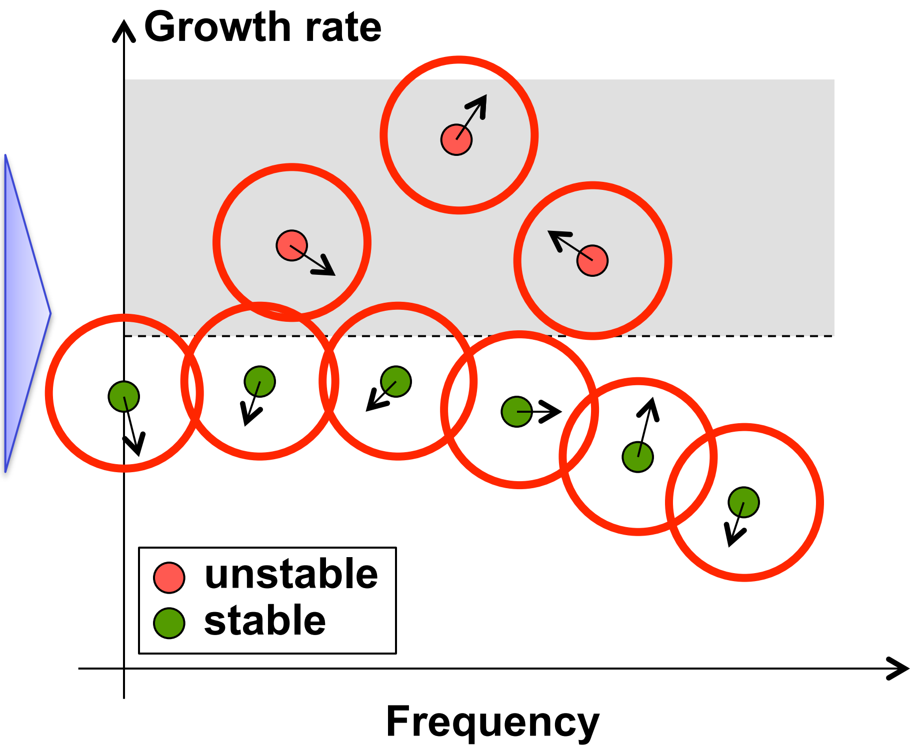


The 'brute force' approach would be to change each parameter in turn and examine its influence on the eigenvalues.

Thermoacoustic model (flame + acoustics)

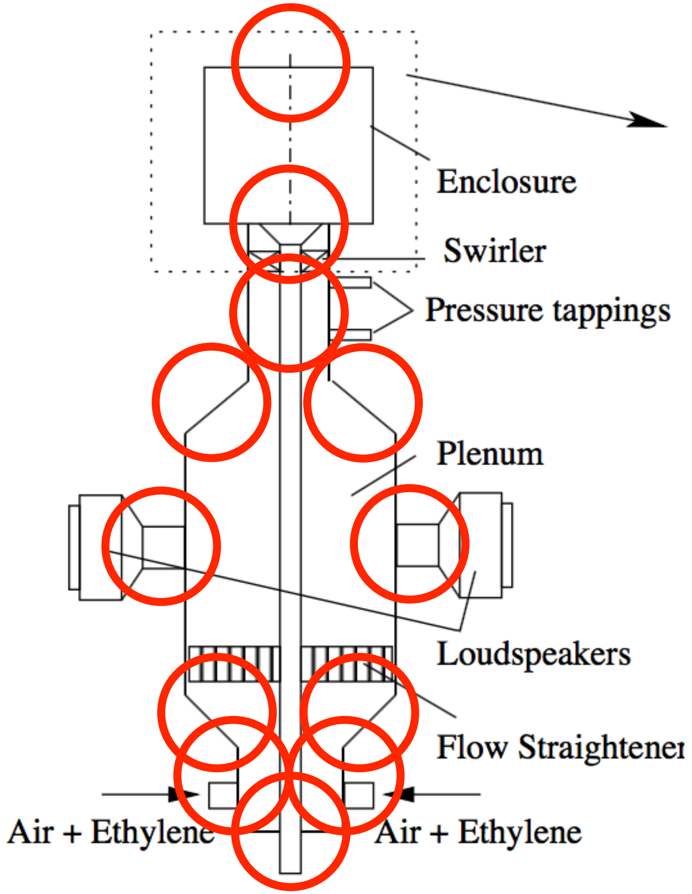


Linear modes of the model

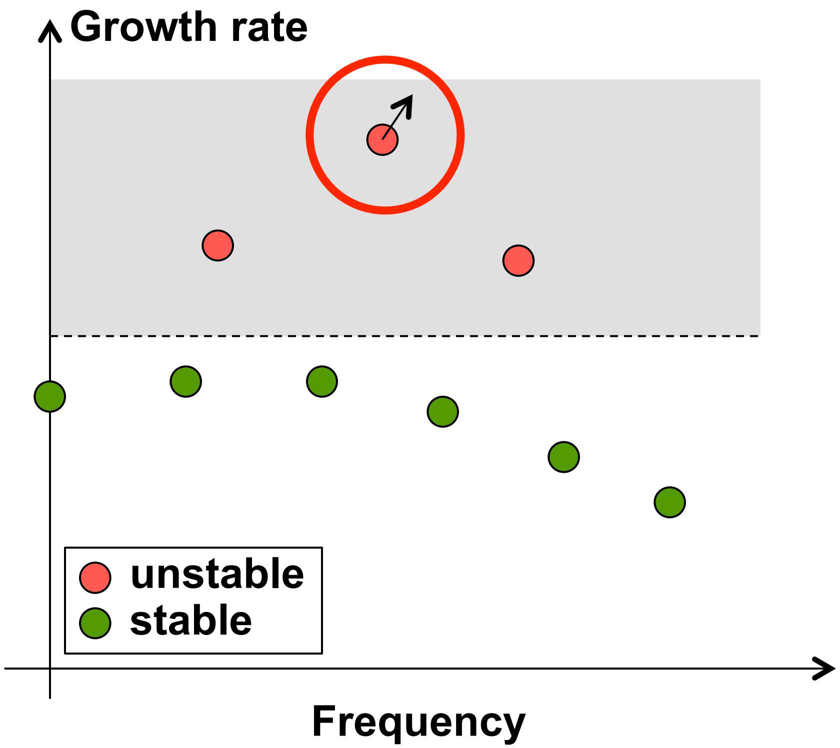


But with a single adjoint calculation, one obtains the sensitivity of a single eigenvalue to every parameter.

Thermoacoustic model (flame + acoustics)

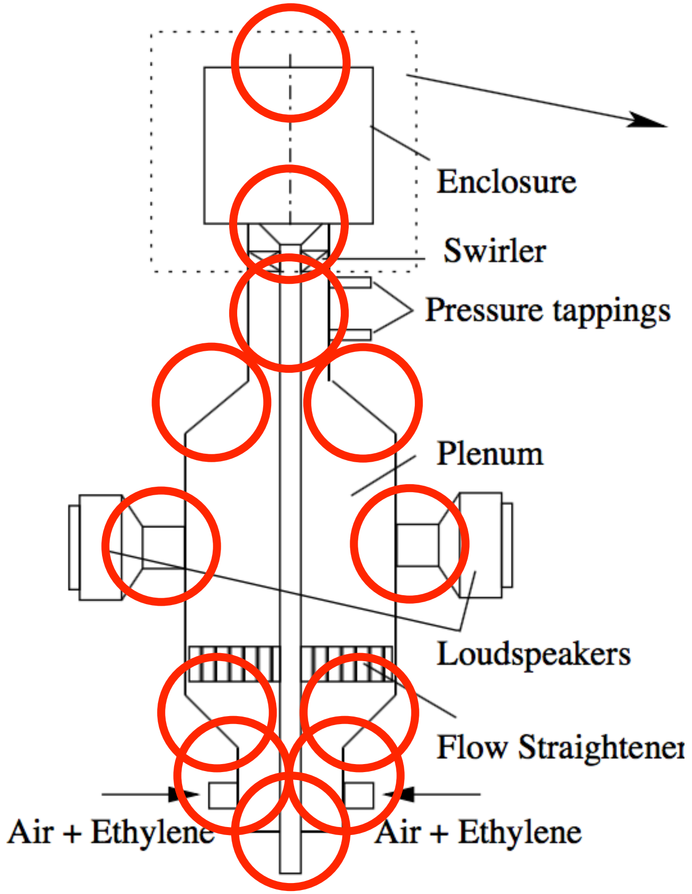


Linear modes of the model

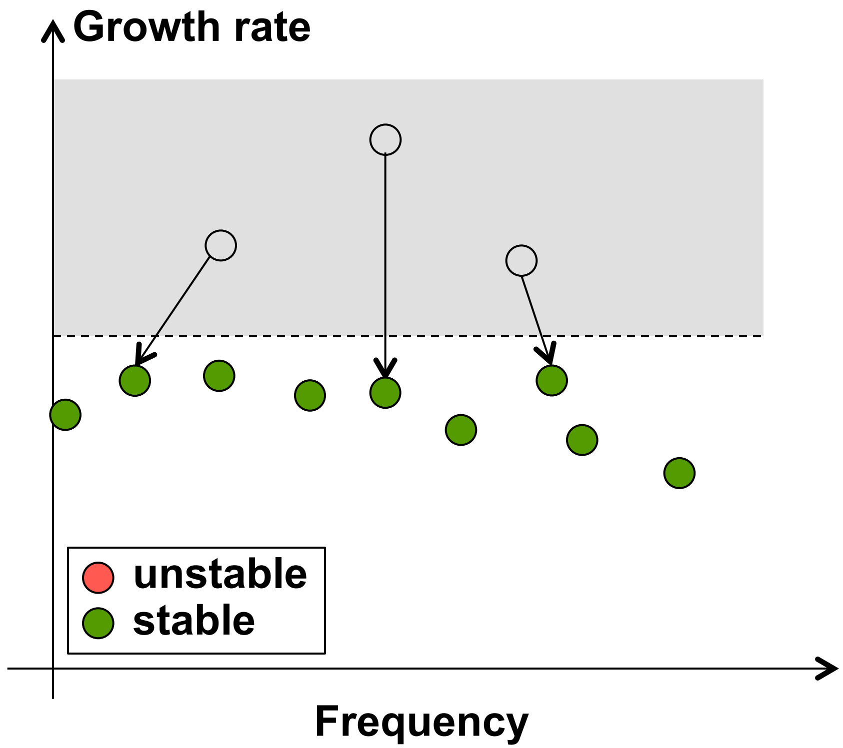


Conceivably, one could use this gradient information to stabilize all eigenvalues at a reasonable cost

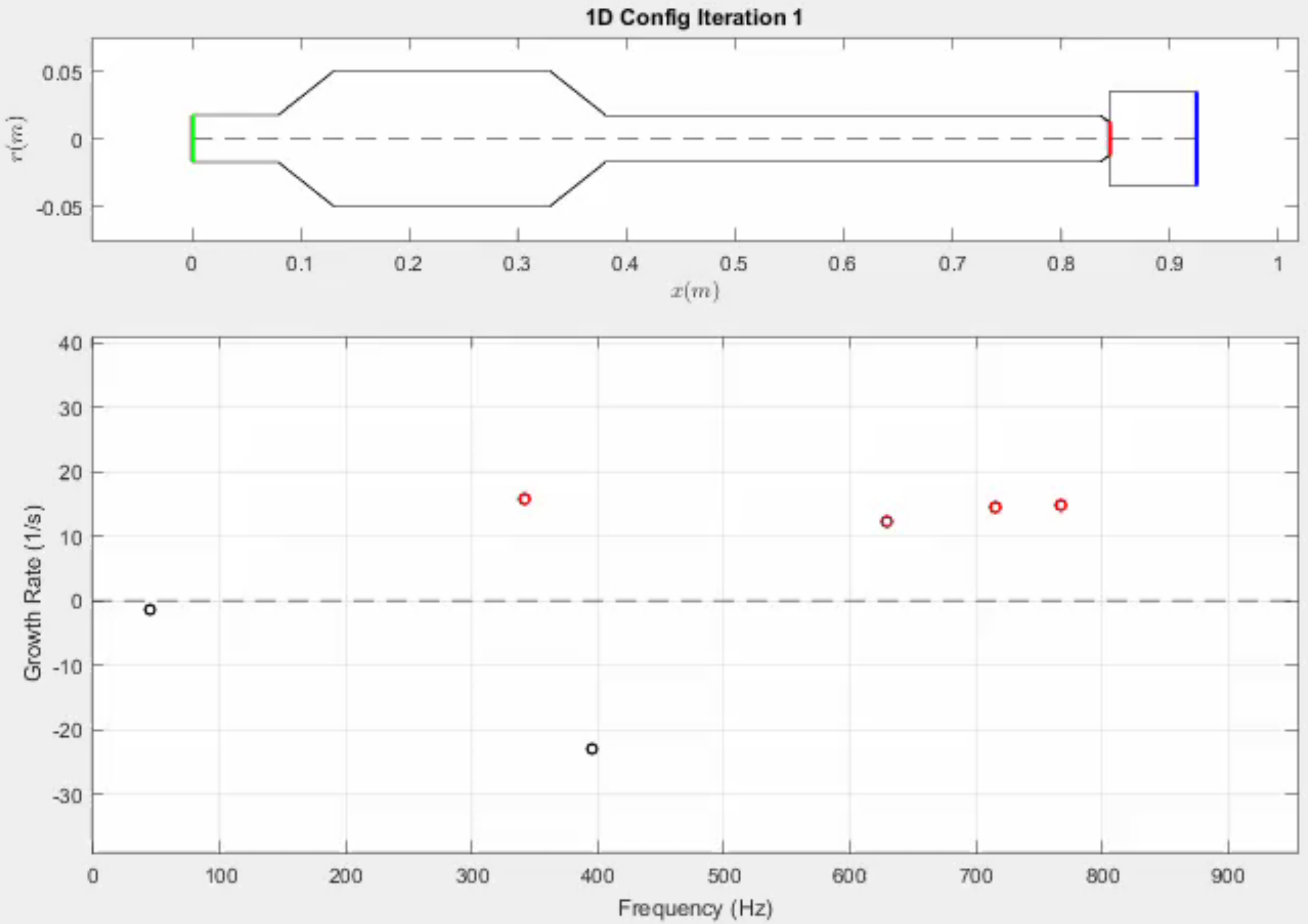
Thermoacoustic model (flame + acoustics)

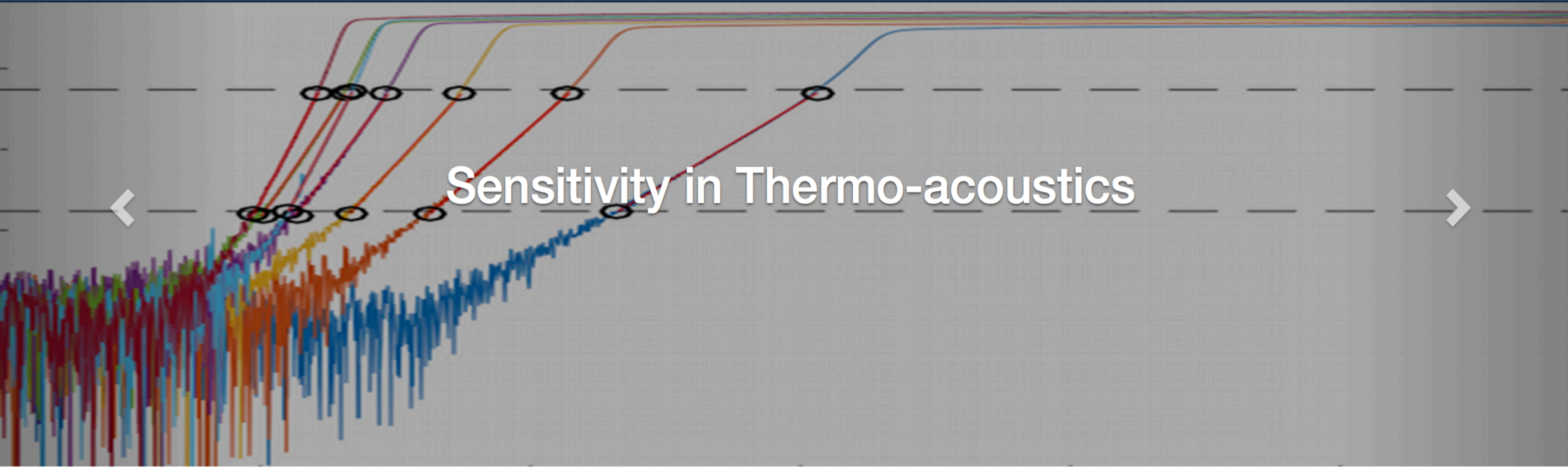


Linear modes of the model



We have applied adjoint-based sensitivity analysis in thermoacoustics to stabilize a model of a practical burner.





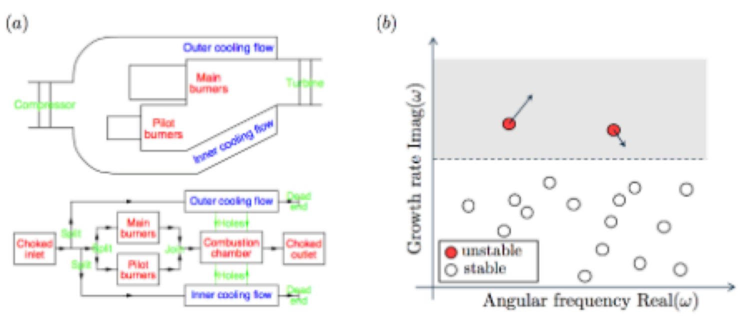
Thermoacoustic instability is a persistent problem in rocket engines and gas turbines because oscillations are so sensitive to many design parameters. This, however, presents an excellent opportunity for the application of adjoint methods and the combination of experiments and modelling.

Background

The first scientific reports of thermoacoustic oscillations appeared over two centuries ago. Their practical consequences have been evident since liquid rocket engine development in the 1930's: they cause thrust oscillations, structural damage, increased heat transfer, and component or payload failure. Despite decades of research by Germany from the 1930's, by the USA and USSR during the cold war, and recently by the gas turbine industry, these oscillations remain a severe problem today.

Adjoint methods applied to thermoacoustic models

There are many ways to model a thermoacoustic system. Acoustic network models are widely used in industry. Typically there will be many stable modes and just a handful of unstable modes:



A typical thermoacoustic network model and corresponding natural modes

We wish to stabilize the handful of unstable modes by altering the design parameters. There may be several thousand of these and it is impractical to work out the influence of each one by varying each in turn. With adjoint methods, however, we can evaluate the influence of every design parameter on a given eigenvalue with a single calculation. Therefore we need only as many calculations as there are unstable eigenvalues.

Our first application of this technique was to a simple model of a hot wire in a tube and the second was to a Burke-Schumann flame in a tube:

Sensitivity analysis of a time-delayed thermoacoustic system via an adjoint-based approach
L. Magri and M. P. Juniper
Journal of Fluid Mechanics **719**, 183--202, (2013), doi:10.1017/jfm.2012.639

Global modes, receptivity, and sensitivity analysis of diffusion flames coupled with duct acoustics
L. Magri, M. P. Juniper
Journal of Fluid Mechanics **752**, 237--265, (2014), doi:10.1017/jfm.2014.328

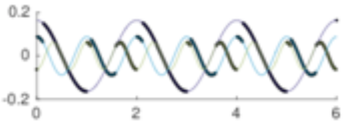
acoustics

dilatation due to heat release

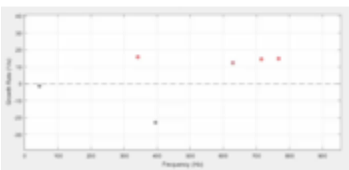
$$\gamma \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0$$

$$\frac{\partial p}{\partial t} + \gamma \frac{\partial u}{\partial x} = (\gamma - 1) q \delta_D (x - x_f)$$

The thermoacoustic mechanism is simple and can be easily modelled

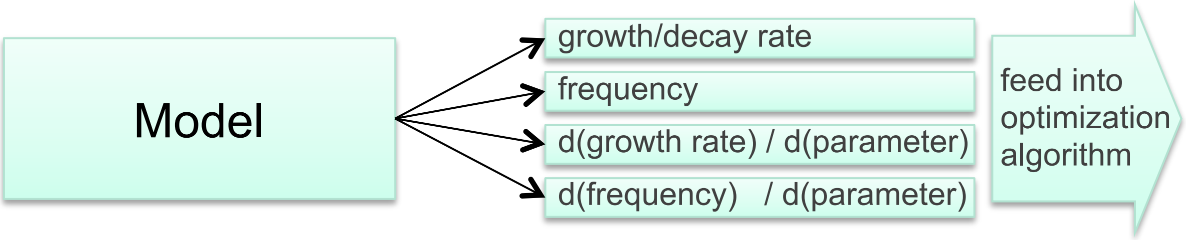


But thermoacoustic models are very sensitive to parameters



This sensitivity can be exploited to stabilize a model

We want to use the model in an optimization algorithm. If the growth rate and frequency are sensitive to the model parameters, their gradients will be severely prone to error.



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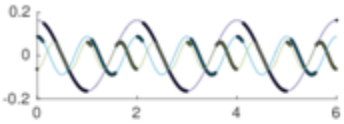

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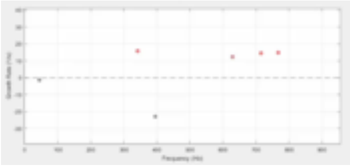

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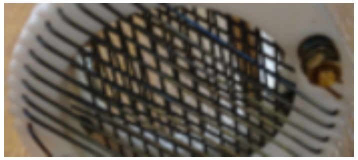
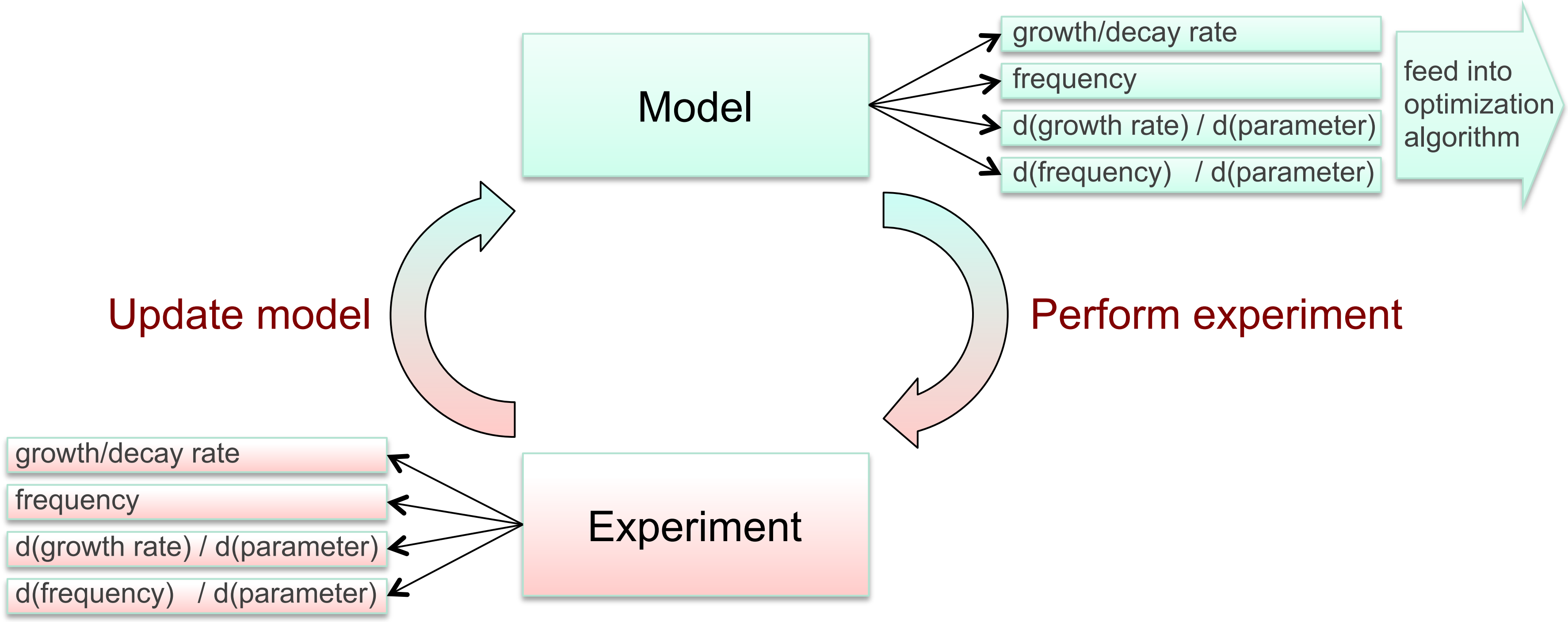
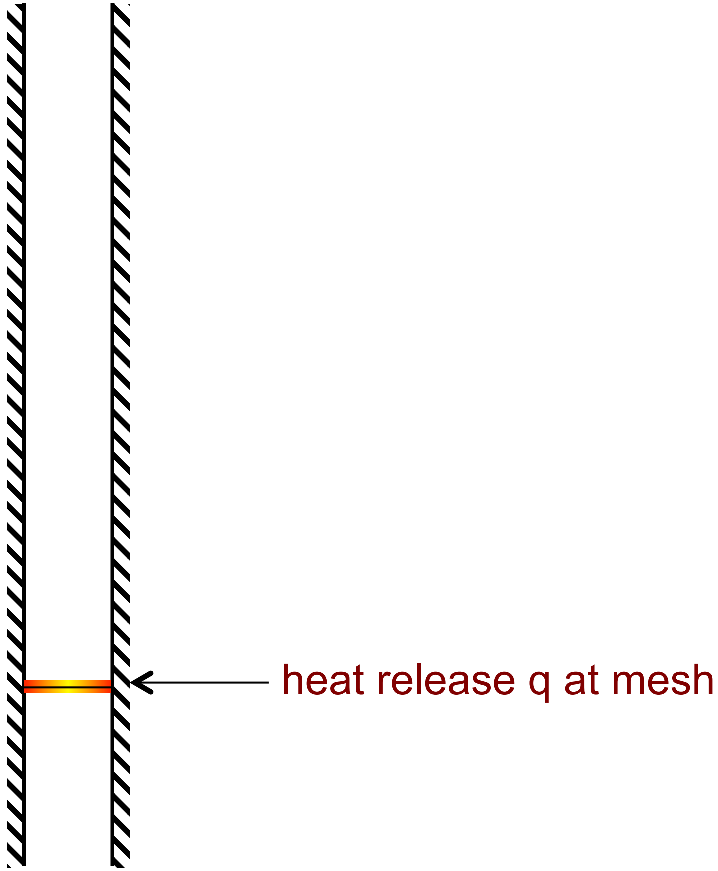


But models need to be accurate and the sensitivity to parameters introduces large systematic error

Ideally, therefore, we would measure growth/decay rates, frequencies, and their sensitivity to experimental parameters.



We carefully characterize a laboratory-scale thermoacoustic system



side view of heater

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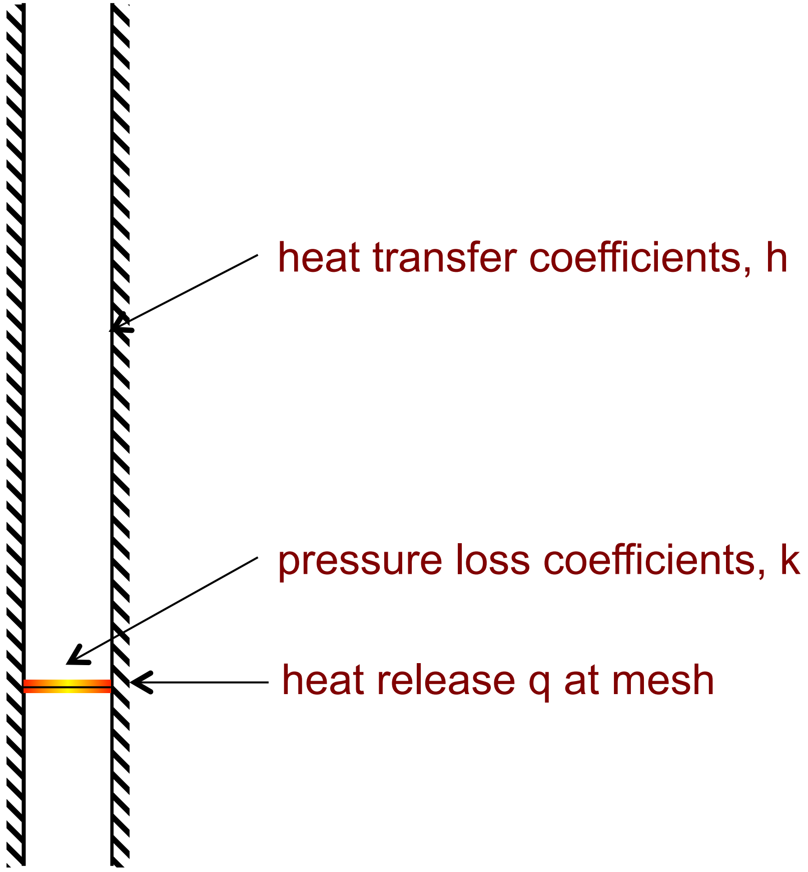

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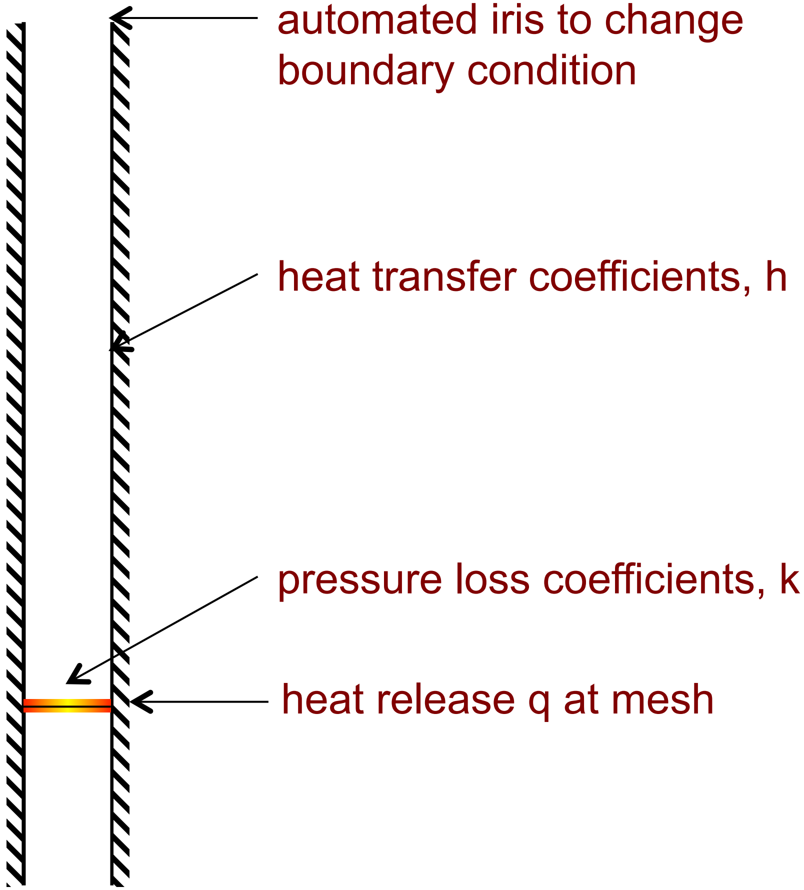

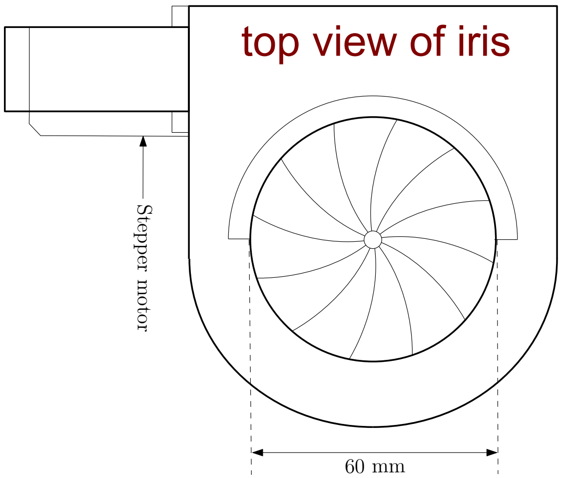

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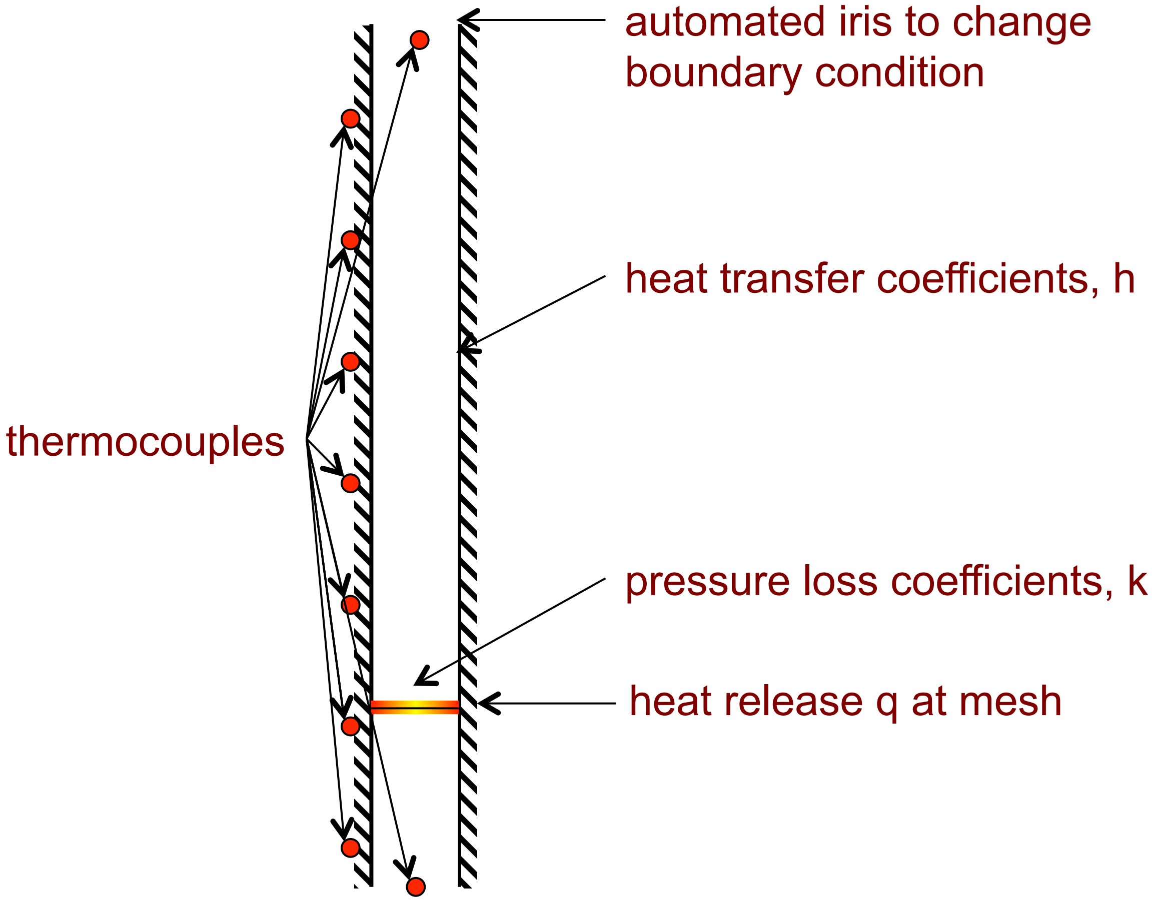
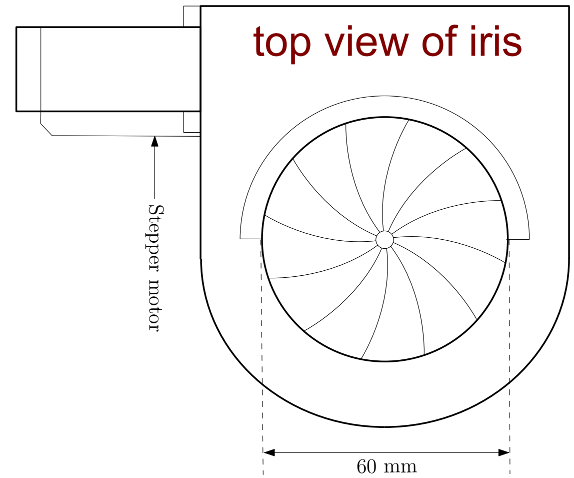
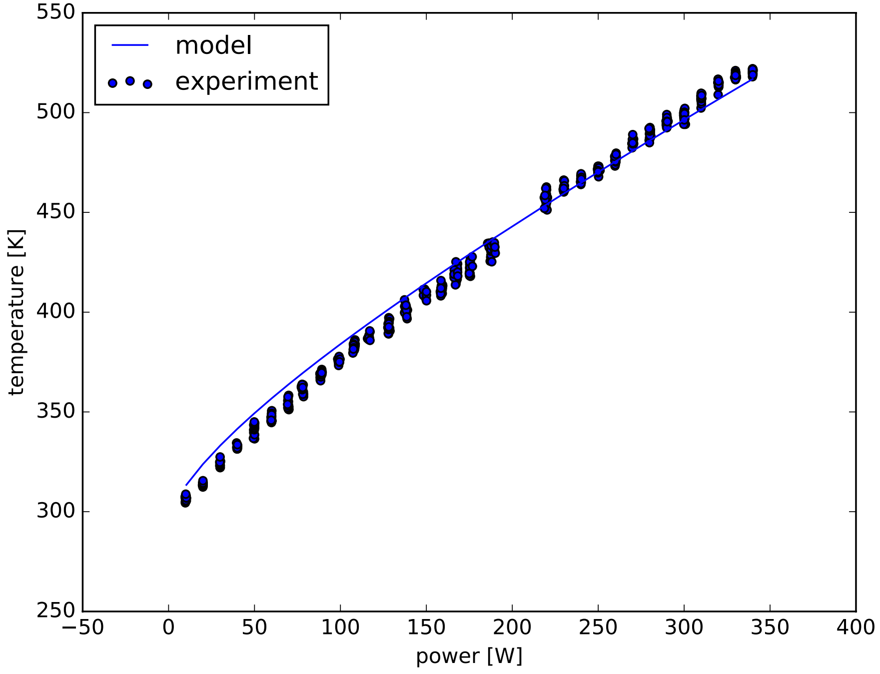




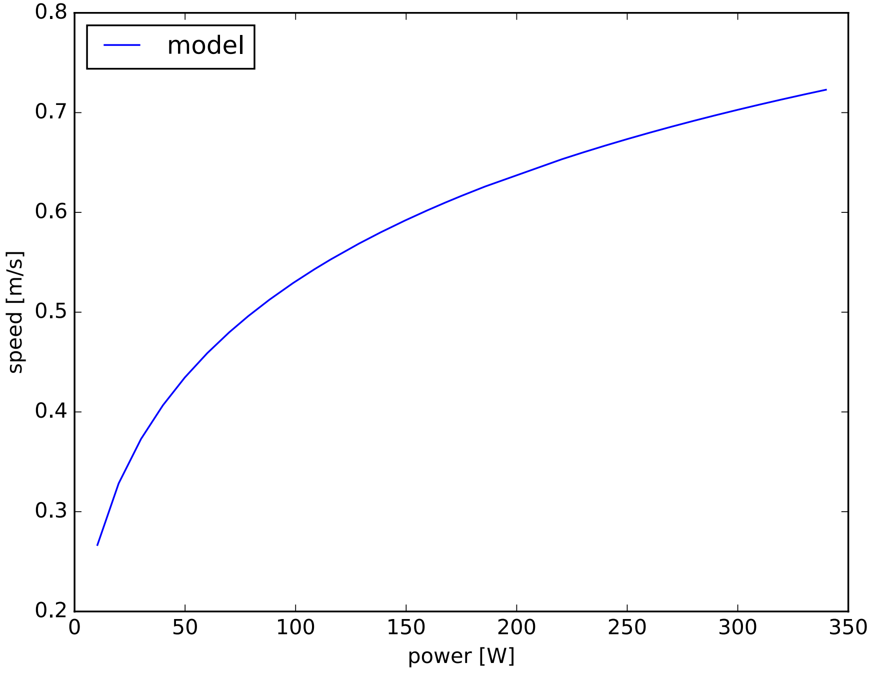
side view of heater

We use nonlinear regression to find the values of heat transfer coefficients and pressure loss coefficients that best fit the experimental data

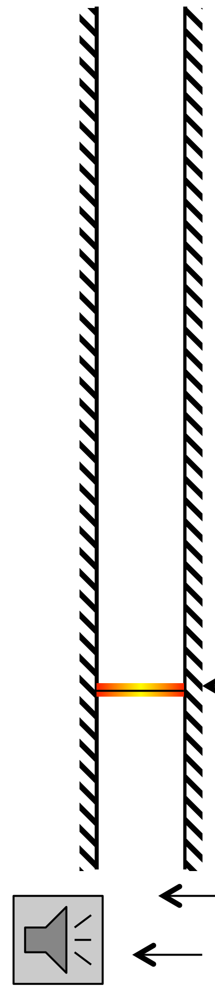
Exit temperature (K)



Entrance speed of air (m/s)



Having characterized the base state, we measure growth/decay rates and frequencies of thermoacoustic oscillations in order to infer n and tau



$$n = k_n \overline{q}$$

heat release at mesh $q(t) = n u(x_m, t - \tau)$

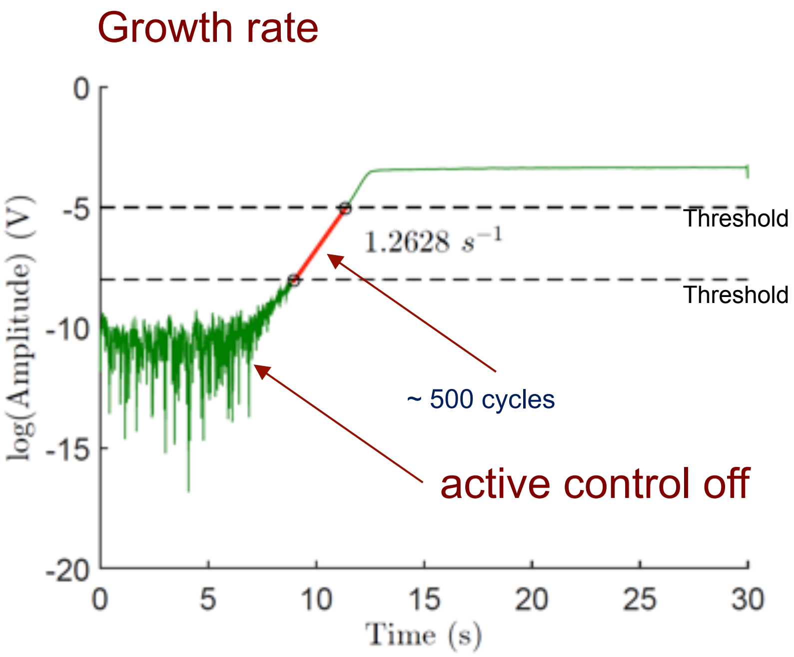
$$\tau = k_\tau \times 0.2 \frac{d_{wire}}{U_{inlet}}$$

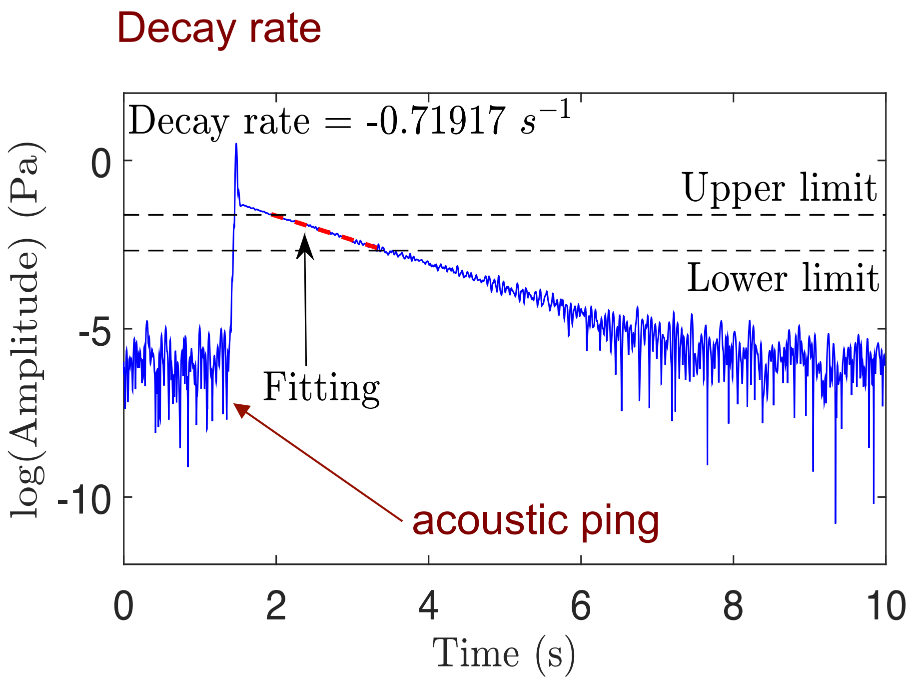
two microphones

loudspeaker for active control or pinging

Jamieson, N. P., Rigas, G., and Juniper, M. P.

We use a linear fit to obtain the linear growth rates between pre-determined thresholds. Typically (depending on the growth/decay rate) there are around 500 cycles in the linear regime.





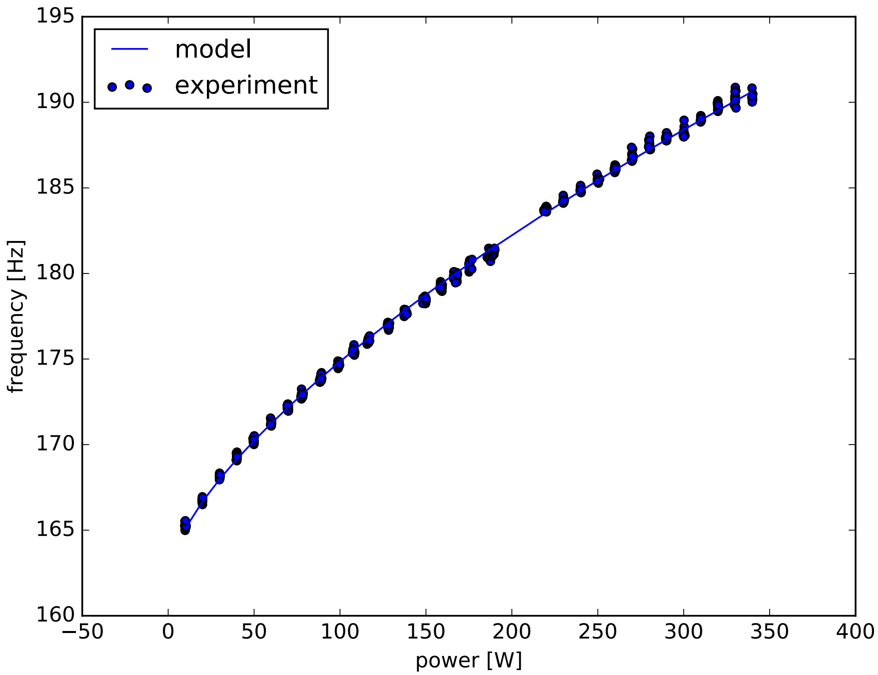
Jamieson, N. P., Rigas, G., and Juniper, M. P.

With nonlinear regression on many thousand measurements, we infer models for n and tau.

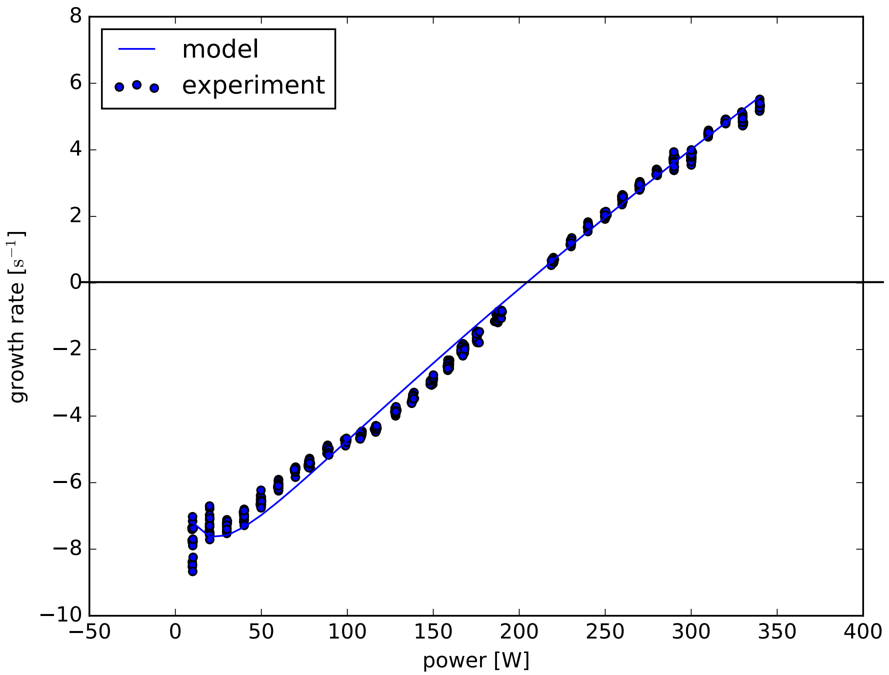
$$n = 103.8\overline{q}$$

$$\tau = 14.7 \times 0.2 \frac{d_{wire}}{U_{inlet}}$$

Frequencies as function of heater power



Growth / decay rates as fn of heater power



acoustics

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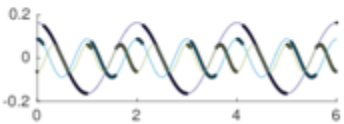

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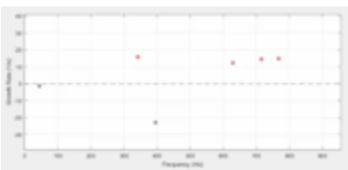
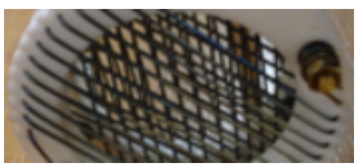
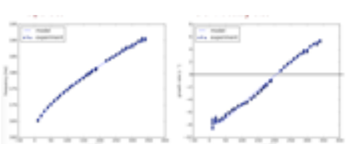


This sensitivity can be exploited to stabilize a model

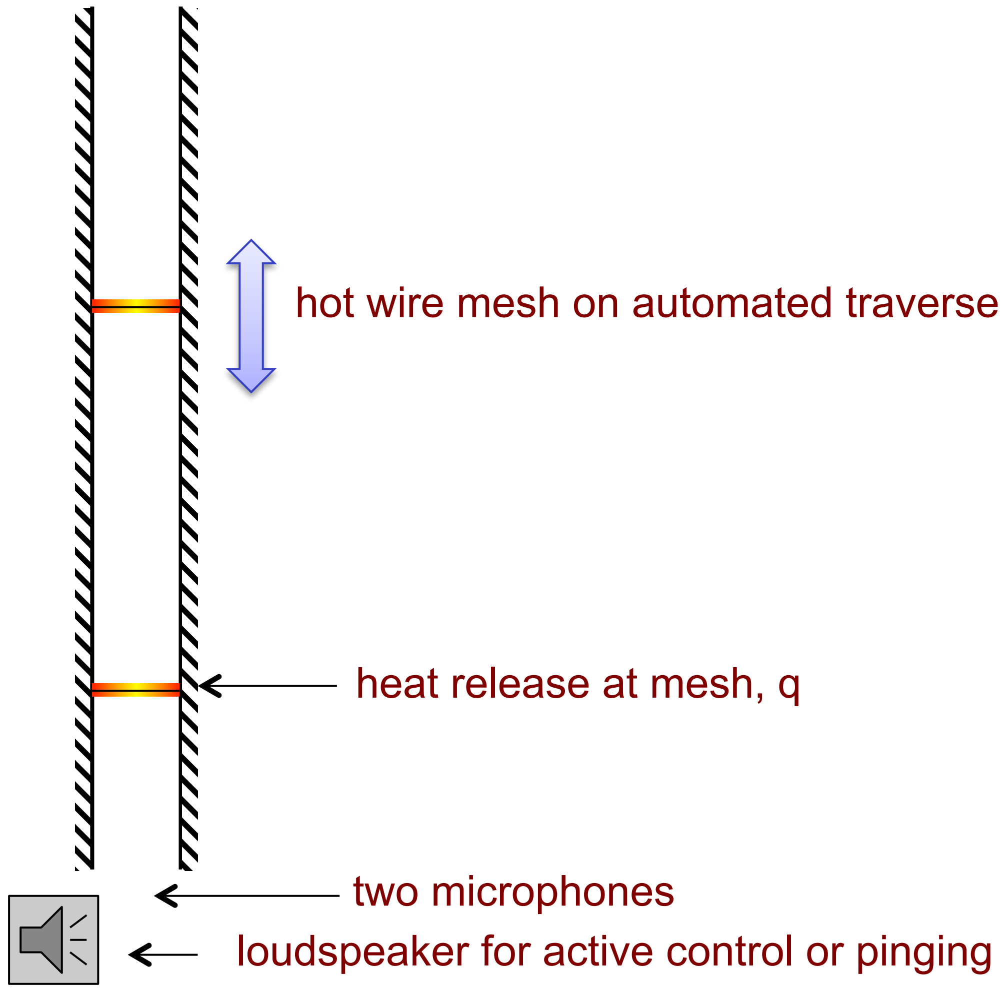


But models need to be accurate and the sensitivity to parameters introduces large systematic error



One solution is to infer parameters from experiments

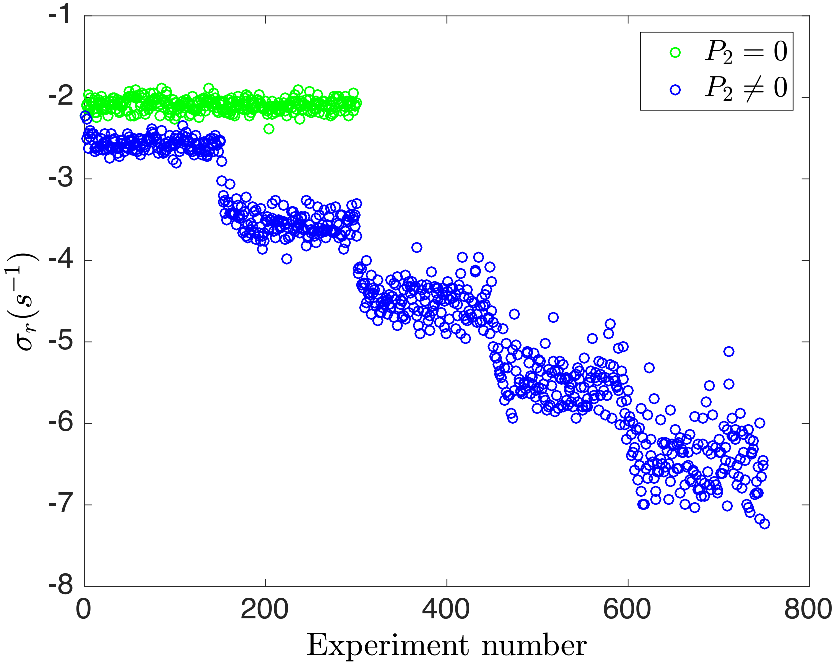
We then use these to predict the behaviour of a new experiment



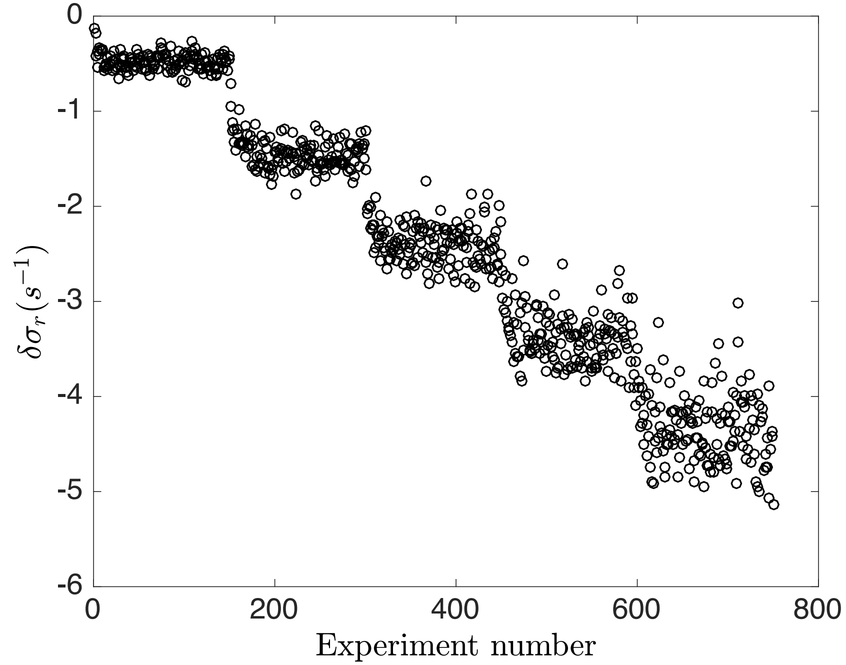
We operate in the stable regime and obtain a decay rate every 6 seconds. We obtain 150 datapoints at each operating point and step to a new operating point automatically.

Measured decay rates as the secondary heater power, P2, changes

Decay rate



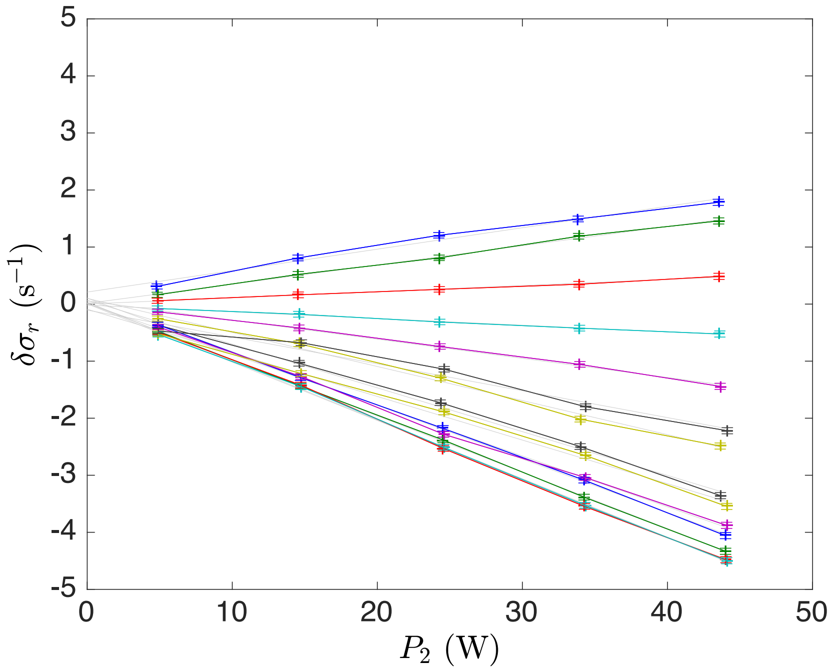
Shift in decay rate



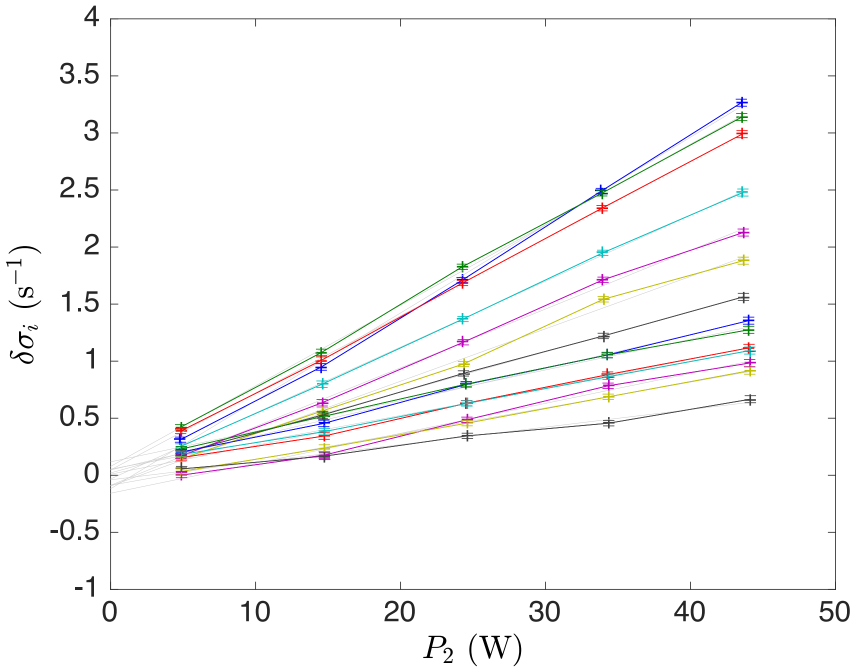
This provides growth/decay rates, frequencies, and their sensitivities with respect to the model parameters.

Shift in decay rate and frequency as the secondary heater power changes

Shift in decay rate

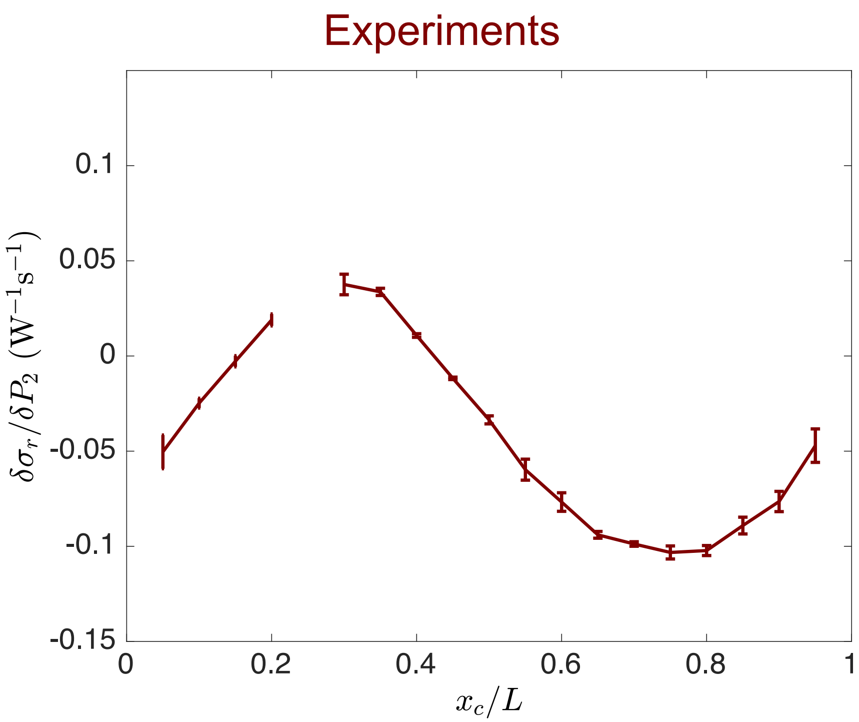


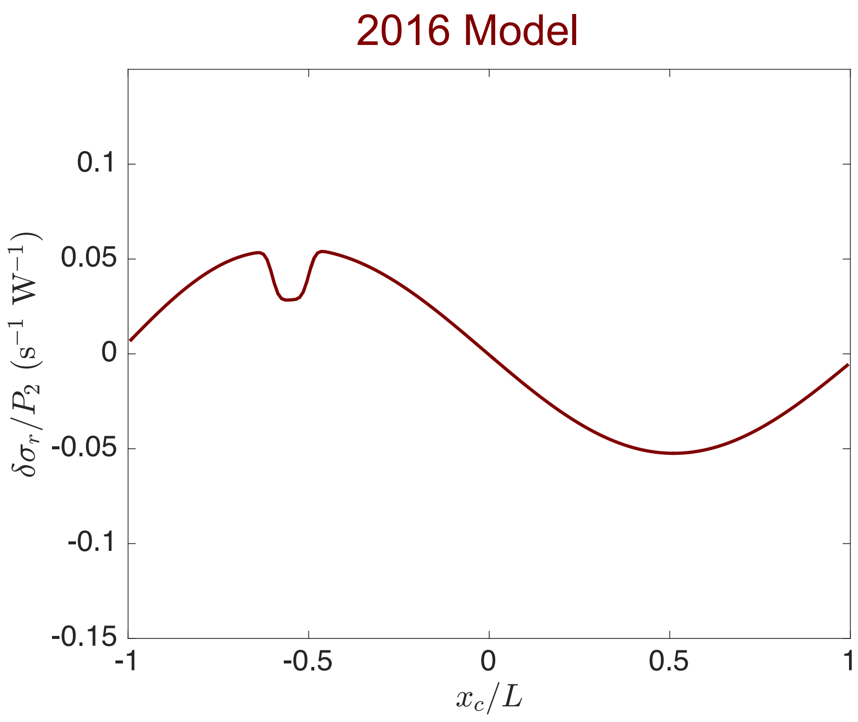
Shift in frequency



We characterize the model with a single heater, add a secondary heater, and compare predicted and measured growth rate sensitivities to the secondary heater power. This forces us to improve our model

d(growth rate) / d(secondary heater power)





Jamieson, N. P., Rigas, G., and Juniper, M. P.

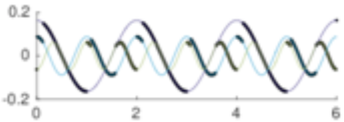
acoustics

$$\gamma \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0$$

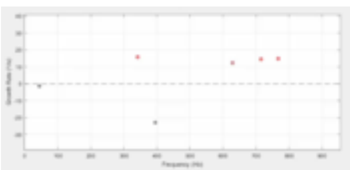
dilatation due to heat release

$$\frac{\partial p}{\partial t} + \gamma \frac{\partial u}{\partial x} = (\gamma - 1) q \delta_D (x - x_f)$$

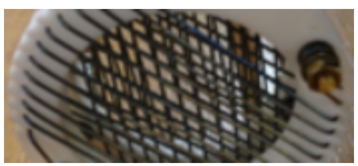
The thermoacoustic mechanism is simple and can be easily modelled



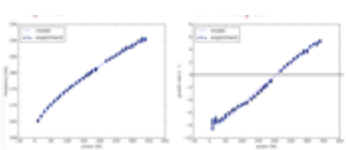
But thermoacoustic models are very sensitive to parameters



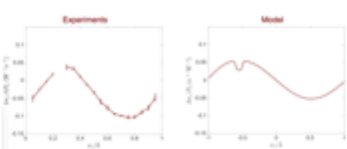
This sensitivity can be exploited to stabilize a model



But models need to be accurate and the sensitivity to parameters introduces large systematic error



One solution is to infer parameters from experiments



This leads to accurate models, which can predict stabilization strategies