Bunsen burner flames at elevated pressures: Lessons from simple chemistry Direct Numerical Simulation analysis

Nilanjan Chakraborty
School of Mechanical and Systems Engineering, Newcastle University, Newcastle Upon Tyne, UK

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Outline of the presentation

- Motivation
- Computational challenges
- Objectives
- Mathematical Background
- Numerical implementation & DNS database
- Results & Discussion
- Conclusions
Motivation

- In many engineering applications (e.g. spark ignition engines and industrial gas turbines) turbulent premixed combustion takes places at elevated pressures.

- With the advent of high performance computing CFD plays an increasingly important role for the purpose of optimising combustion processes in internal combustion engines and gas turbines.

- Direct Numerical Simulations (DNS), which resolve all the length and time scales of fluid turbulence without any recourse to physical approximations, are not feasible for the configurations relevant to aforementioned engineering applications. Most industrial calculations are done using Reynolds-Averaged Navier Stokes (RANS) and Large Eddy Simulations (LES) which need combustion models.

- Most combustion models are derived and validated only for atmospheric pressure. Therefore it is crucial to assess if these models are valid for elevated pressure conditions.
Challenges for high pressure simulations (1)

- The laminar burning velocity for hydrocarbon-air (e.g. methane-air) mixtures often scales as: \( S_L \sim S_L^0 (p/p_0)^{-0.5} \).

- This suggests that for a given rms velocity fluctuation the velocity ratio \( u'/S_L \) scales as: \( u'/S_L \sim (p/p_0)^{0.5} \).

Taken from: “An introduction to combustion, Concepts and Applications” by S.R. Turns
For isobaric combustion of ideal gases: $\rho \sim \rho$ and thus the kinematic viscosity $\nu$ scales as: $\nu \sim \rho^{-1}$.

Thus the flame thickness scales as: $\delta_{th} \sim \nu/S_L \sim \rho^{-1/2}$. Thus integral length scale to flame thickness ratio $l/\delta_{th}$ scales as: $l/\delta_{th} \sim \rho^{0.5}$.

As a result, the turbulent Reynolds number for a given burner with a given set of values of rms turbulent velocity fluctuation $u'$ and integral length scale $l$ scales as:

$$Re_t = \frac{u'l}{\nu} \sim \frac{u'l}{S_L\delta_{th}} \sim \rho$$
Objectives

- To analyse the scalar gradient (or flame thickness) and curvature statistics with an increase in pressure for Bunsen burner configuration
- To assess if a high pressure flame behaves in the same manner (in terms of above statistics) as that of a flame with same turbulent Reynolds number but at a lower pressure
- It is recognised that a change in pressure for a given flow condition for a burner means the flames are at different locations on the combustion regime diagram.
- The present analysis focuses only on fluid-dynamical aspects of pressure variation.
Mathematical Background

The generic chemical reaction for premixed flames can be written as:

\[ \text{Reactants} \rightarrow \text{Products} \]

It is possible to define a reaction progress variable \( c \) which increases monotonically from 0 to 1 from fresh reactants to completely burned products:

\[
c = \frac{Y_{R0} - Y_R}{Y_{R0} - Y_{R\infty}}; \quad \text{Subscript 0: unburned gas value}
\]

\[
\text{Subscript } \infty: \text{burned gas value}
\]

Transport equation of reaction progress variable (\( c \)):

\[
\rho \frac{\partial c}{\partial t} + \rho u_k \frac{\partial c}{\partial x_k} = \frac{\partial}{\partial x_k} \left( \rho D \frac{\partial c}{\partial x_k} \right) + \dot{\omega}
\]

The above equation can be written in kinematic form as:

\[
\frac{\partial c}{\partial t} + u_j \frac{\partial c}{\partial x_j} = (S_d) \left| \nabla c \right| \quad \text{Displacement speed}
\]

\[
S_d = \left[ \frac{\dot{\omega} + \nabla \cdot (\rho D \nabla c)}{\rho \left| \nabla c \right|} \right]
\]
Deformation & stretching of premixed flames

\[ c(x, t) = \Gamma + d\Gamma \]

\[ c(x, t) = \Gamma \]

\[ \frac{d\vec{r}}{dt} = \vec{r} \cdot \nabla \vec{w} \]

\[ \frac{dr_i}{dt} = r_j \frac{\partial w_i}{\partial x_j} \]

Here

\[ \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{w} \cdot \nabla \]
The transport equation of $c$ takes the following form:

$$\frac{\partial (\rho c)}{\partial t} + \frac{\partial (\rho u_j c)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \rho D \frac{\partial c}{\partial x_j} \right) + \dot{w}$$

(1)

Reaction rate

Molecular diffusion rate

For a given $c$ –isosurface the above equation can be recast in kinematic form in the following manner:

$$\frac{dc}{dt} = \frac{\partial c}{\partial t} + \dot{w} \cdot \nabla c = \frac{\partial c}{\partial t} + w_j \frac{\partial c}{\partial x_j} = 0$$

(2)

Here $\vec{w}$ is the propagation velocity and a comparison between Eqs. (1) and (2) suggest that

$$w_j = u_j + \left[ \frac{\dot{w} + \nabla \cdot (\rho D \nabla c)}{\rho |\nabla c|} \right] N_j$$

Deformation & stretching (1)
Deformation & stretching (2)

A surface $A(t)$ is considered to be moving in space with an arbitrary velocity $\vec{w}$. The surface is bounded by a circuit $C(t)$ and its normal vector is $\vec{N}$. The velocity field $\vec{w}$ is assumed to be prescribed over a domain swept by the moving surface $A(t)$. Under these assumptions the rate of change of flux of a vector across the surface $A(t)$ is given by:

$$\frac{d}{dt} \int_{A(t)} \vec{G} \cdot \vec{N} dA = \int_{A(t)} \left[ \frac{\partial \vec{G}}{\partial t} + \vec{w} \cdot \nabla \vec{G} - \vec{G} \cdot \nabla \vec{w} + \nabla \vec{w} \cdot \vec{G} \right] \cdot \vec{N} dA = \int_{A(t)} \left[ \frac{\partial G_i}{\partial t} + w_j \frac{\partial G_i}{\partial x_j} - G_j \frac{\partial w_i}{\partial x_j} + G_i \frac{\partial w_j}{\partial x_j} \right] N_i dA$$

$\vec{G} = \vec{N}$

**Stretch rate:**

$$\frac{1}{\partial A} \frac{d(\partial A)}{dt} = (\delta_{ij} - N_i N_j) \frac{\partial w_i}{\partial x_j} = (\delta_{ij} - N_i N_j) \frac{\partial u_i}{\partial x_j} + S_d \frac{\partial N_j}{\partial x_j} = a_T + 2S_d \kappa_m$$

**Tangential strain rate:**

$$a_T = (\delta_{ij} - N_i N_j) \frac{\partial u_i}{\partial x_j}$$

**Stretch/Effective tangential strain rate:**

$$a_T^{\text{eff}} = (\delta_{ij} - N_i N_j) \frac{\partial w_i}{\partial x_j} = a_T + 2S_d \kappa_m$$

**Mean curvature (i.e. mean of principal curvatures):**

$$\kappa_m = (\kappa_1 + \kappa_2) / 2 = 0.5 \nabla \cdot \vec{N}$$

Pope (1988); Dopazo et al. (2015)
Deformation & stretching (3)

- The kinematic equation of reaction progress variable can be recast as:

\[
\frac{\partial c}{\partial t} + w_j \frac{\partial c}{\partial x_j} = 0 \quad \Rightarrow \quad \frac{\partial c}{\partial t} + u_j \frac{\partial c}{\partial x_j} + S_d N_j \frac{\partial c}{\partial x_j} = 0 \quad \Rightarrow \quad \frac{\partial c}{\partial t} + u_j \frac{\partial c}{\partial x_j} = S_d |\nabla c|
\]

- Taking gradient of the final equation as:

\[
\frac{d|\nabla c|}{dt} = \left[ \frac{\partial |\nabla c|}{\partial t} + w_j \frac{\partial |\nabla c|}{\partial x_j} \right] = -\left[ N_j N_k \frac{\partial u_j}{\partial x_k} + N_k \frac{\partial S_d}{\partial x_k} \right] |\nabla c| = -N_j N_k \frac{\partial w_j}{\partial x_k} |\nabla c|
\]

**Effective normal strain rate:**

\[a^{\text{eff}}_N = N_j N_k \frac{\partial w_j}{\partial x_k} = N_j N_k \frac{\partial u_j}{\partial x_k} + N_k \frac{\partial S_d}{\partial x_k}\]

**Normal strain rate:** \[a_N = N_j N_k \frac{\partial u_j}{\partial x_k}\]

**Additional strain rate due to propagation:** \[N_k \frac{\partial S_d}{\partial x_k}\]

- Let us now consider these relations:

\[
\frac{1}{\Delta x_N} \frac{d\Delta x_N}{dt} = \left[ N_j N_k \frac{\partial u_j}{\partial x_k} + N_k \frac{\partial S_d}{\partial x_k} \right] = a^{\text{eff}}_N
\]

\[
\frac{1}{|\nabla c|} \frac{d|\nabla c|}{dt} = -\left[ N_j N_k \frac{\partial u_j}{\partial x_k} + N_k \frac{\partial S_d}{\partial x_k} \right] = -a^{\text{eff}}_N
\]

Pope (1988); Dopazo et al. (2015)
Numerical Implementation

- 3D compressible DNS with single step irreversible Arrhenius type chemistry carried out using a well-known code SENGA (Jenkins & Cant, 1999).
- Outflow boundary conditions are specified using NSCBC technique (Poinsot & Lele, 1992).
- High order finite difference (10th order central difference scheme at the internal grid points and gradually dropping to 2nd order one-sided scheme at non-periodic boundaries and 3rd order explicit Runge-Kutta scheme have been used spatial discretisation and explicit time advancement respectively.
- Standard values are chosen for Prandtl number = 0.7 and Zel’dovich number = 6.0. Lewis number is taken to be unity. These values are representative of stoichiometric methane-air combustion.
- The flame is initialised as:

  - Unburned gas \( (c = 0) \)
  - Burned gas \( (c = 1) \)

  \[ f(\vec{r}) = \]
  - Unstrained laminar flame solution as a function of radial vector
  - \( U_{inlet} \)
  - \( D \)
Inflow Boundary condition specification (1)

- Synthetic inlet turbulence is generated by digital filtering random data following Klein et al. (JCP, 186, 2003)

- For a 1D example:
  - $r_n$ be random numbers with $\langle r_n \rangle = 0, \langle r_n^2 \rangle = 1$
  - $b_k, k = -N \ldots N$ be the filter coefficients: $b_k = \overline{b_k}/B$, $B$ is a normalization factor and $\overline{b_k} = \exp(-\pi k^2/2n^2)$
  - $u_m = \sum_{n=-N}^{N} b_n \cdot r_{m+n}$ is a correlated signal with length scale $N \cdot \Delta x$

- Besides the length scale, first and second order one point statistics can be prescribed
In this method the filter size is proportional to the desired turbulent length (time) scale divided by $\Delta x$ ($\Delta t$).

If the filter has a 1D size of $N$, generation of inlet data for one cell in the inlet plane is an $O(N^3)$ operation.

This method has been applied successfully to a large number of flow problems.

However, direct application to large scale problems can be inefficient.
The digital filter method can be tuned for high parallel performance in 4 steps

1. The Gaussian filter in time direction has been replaced by an autoregressive AR1 process to avoid excessive filter length (due to small $\Delta t$ in the compressible solver)

2. The 2D filter in the inlet plane can be replaced by the tensor product of two 1D filters. This replaces the $O(N^2)$ by an $O(N)$ operation
Inflow Boundary condition specification (4)

3. The filtering makes use of all available processors not only those located at the inflow plane.

4. Transferring random data before filtering can be avoided by assigning a local unique random seed at start of simulation in overlapping regions.

For more details refer to Klein et al. (JCP, 186, 2003) and Kempf et al. (C&F, 60, 2012), Nachtigal (2017).
**Simulation and flow parameters (1)**

<table>
<thead>
<tr>
<th>Case</th>
<th>$P/P_0$</th>
<th>$U_{inlet}/S_L$</th>
<th>$u'_{inlet}/S_L$</th>
<th>$L_{11}/D$</th>
<th>$L_{domain}/D$</th>
<th>$N_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0</td>
<td>6.0</td>
<td>1.0</td>
<td>0.2</td>
<td>2.0</td>
<td>250</td>
</tr>
<tr>
<td>B</td>
<td>5.0</td>
<td>6.0</td>
<td>1.0</td>
<td>0.2</td>
<td>2.0</td>
<td>560</td>
</tr>
<tr>
<td>C</td>
<td>10.0</td>
<td>6.0</td>
<td>1.0</td>
<td>0.2</td>
<td>2.0</td>
<td>795</td>
</tr>
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</table>

- $P_0 = 1.0$ bar and statistics are taken after at least 2-throughpass times

<table>
<thead>
<tr>
<th>Case</th>
<th>$P/P_0$</th>
<th>$Re_D$</th>
<th>$Re_t$</th>
<th>$Da$</th>
<th>$Ka$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0</td>
<td>399.0</td>
<td>13.3</td>
<td>5.00</td>
<td>0.45</td>
<td>4.5</td>
</tr>
<tr>
<td>B</td>
<td>5.0</td>
<td>892.0</td>
<td>29.26</td>
<td>11.36</td>
<td>0.30</td>
<td>4.5</td>
</tr>
<tr>
<td>C</td>
<td>10.0</td>
<td>1262.0</td>
<td>41.22</td>
<td>16.13</td>
<td>0.25</td>
<td>4.5</td>
</tr>
</tbody>
</table>

- $Re_D = \frac{\rho_0 U_{inlet} D}{\mu_0}$; $Re_t = \frac{\rho_0 u'_{inlet} L_{11}}{\mu_0}$; $Da = \frac{S_L L_{11}}{u'_{inlet} \delta_{th}}$; $Ka = \left(\frac{u'_{inlet}}{S_L}\right)^{1.5} \left(\frac{L_{11}}{\delta_{th}}\right)^{-0.5}$; $\delta_{th} = \frac{(T_{ad} - T_0)}{\max|\nabla T|_L}$ & $\tau = \frac{(T_{ad} - T_0)}{T_0}$
Flame morphology (1)

Case A
(p = p₀)

Case B
(p = 5p₀)

Case C
(p = 10p₀)
$P_0 = 1.0$ bar and statistics are taken after at least 2-throughpass times.

$$\begin{align*}
\Delta T &= (T_{ad} - T_0) / \max |\nabla T|_L \quad \& \quad \tau = (T_{ad} - T_0) / T_0
\end{align*}$$
Flame morphology (2)

Case C
\[(u'_{inlet})/S_L = 1\]
\[L_{11}/\delta_{th} = 16.13\]
\[Re_t = 41.22\]
\[Re_D = 1262.0\]
\[p = 10p_0\]

Case D
\[(u'_{inlet})/S_L = 3.1\]
\[L_{11}/\delta_{th} = 5.20\]
\[Re_t = 41.22\]
\[Re_D = 399.0\]
\[p = p_0\]

Case E
\[(u'_{inlet})/S_L = 1\]
\[L_{11}/\delta_{th} = 16.13\]
\[Re_t = 41.22\]
\[Re_D = 399.0\]
\[p = p_0\]
Regime diagram locations

\[ \frac{u'}{S_L} \]

\[ \frac{L_{11}}{\delta_{th}} \]
The quantity $|\nabla c|$ scales as: $|\nabla c| \sim 1/\delta$ where $\delta$ is the flame thickness and thus the quantity $|\nabla c| \times \delta_{th} \sim \delta_{th}/\delta$ can be taken to be a measure of the ratio of laminar flame thickness to turbulent flame thickness. Here we can see a small amount of flame thinning under turbulence.

- The standard deviations are shown as bars. Clearly the uncertainly in the difference in the mean value is not significant to draw any meaningful conclusion.

- No major pressure dependence on the normalised flame thickness.
The standard deviations are shown as bars. Clearly the uncertainly in the difference in the mean value is not significant to draw any meaningful conclusion. Standard deviation increases with increasing $u'/S_L$.

No major $u'/S_L$ and $L_{11}/\delta_{th}$ dependences on the mean normalised flame thickness within this parameter range.
No major pressure dependence on the mean normalised fluid-dynamic normal strain rate.

No major $u'/S_L$ and $L_{11}/\delta_{th}$ dependences on the mean normalised fluid-dynamic normal strain rate within this parameter range.
Case A shows higher mean dilatation rate in comparison to Cases B and C.

Case E shows higher mean dilatation rate in comparison to Cases C and D.

Difference in mean dilatation rate warrant further explanation

Statistics of $\nabla \cdot \mathbf{u} \times \delta_{th}/S_L$: normalised dilatation rate
The mean behaviour of normalised fluid-dynamic tangential strain rate shows some difference. The difference in mean tangential strain rate (i.e. $a_T = \nabla \cdot \vec{u} - a_N$) behaviour originates due to the difference in mean values of dilatation rate and normal strain rate.
Curvature PDFs (1)

Curvature PDFs for cases A-C. Mean curvature is normalised with thermal flame thickness.

Curvature PDF for case C shows large extent of negative skewness. This negative skewness is there also for cases A and B.
Curvature PDFs (2)

- Creta et al. (Phys. Rev. E 2016) suggested that negative skewness of curvature PDF is a marker for DL (Darrieus-Landau) instability.
- The DL instability of a thermo-diffusively neutral flame is most effective in an intermediate range of length scales.

Lower limit: Critical wavelength $\lambda_c$

Upper limit: Nozzle diameter

Matalon & Matkowsky (1982 JFM)
Curvature PDFs (3)

- In fact DL instability is promoted for high pressure flames.

<table>
<thead>
<tr>
<th>(\lambda_c/D)</th>
<th>(P_0)</th>
<th>(5P_0)</th>
<th>(10P_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creta &amp; Matalon (JFM 2011)</td>
<td>0.96</td>
<td>0.42</td>
<td>0.3</td>
</tr>
</tbody>
</table>

- Nevertheless DL instability is unlikely to explain the negative skewness for the \(P_0\) flames.

- Huygens principle can be responsible for negative curvature PDF skewness in absence of DL instability.

- Solution of G-equation
  \[
  \frac{\partial G}{\partial t} = |\nabla G|
  \]
  Time evolves from red to blue to green.
The standard deviations are shown as bars. Clearly the uncertainty in the difference in the mean value is not significant to draw any meaningful conclusion.

No major pressure dependence on the mean normalised displacement speed.
Statistics of $S_d/S_L$: normalised displacement speed

- The standard deviations are shown as bars. Clearly the uncertainty in the difference in the mean value is not significant to draw any meaningful conclusion. Standard deviation increases with increasing $u'/S_L$.
- No major $u'/S_L$ and $L_{11}/\delta_{th}$ dependences on the mean normalised displacement speed within this parameter range.
The mean reaction component of displacement speed $S_r/S_L$ assumes positive values across $c$ and increasingly positive toward burned gas.

The mean normal diffusion component of displacement speed $S_n/S_L$ assumes positive toward unburned gas side but becomes increasingly negative toward burned gas side.

The normalised tangential diffusion component $S_t/S_L$ does not contribute significantly to the mean displacement speed but it affects the local variation.
The contribution of mean $N_j \frac{\partial S_r}{\partial x_j}$ is majorly negative values through the flame, due to $S_r / S_L$ being positive across the flame opposing the flame normal.

The contribution of mean $N_j \frac{\partial S_n}{\partial x_j}$ exhibits small negative values on the unburned side and increasingly positive toward burned side.

The mean contribution of $N_j \frac{\partial S_t}{\partial x_j}$ is negligible in comparison to $N_j \frac{\partial S_r}{\partial x_j}$ and $N_j \frac{\partial S_n}{\partial x_j}$.

The difference in the mean value $[N_j \frac{\partial S_{\alpha}}{\partial x_j} (\alpha = r, n & t)]$ is not significant to draw any meaningful conclusion. **No major pressure dependence on the mean normalised added normal strain rate components.**
Statistics of additional normal strain rate $\partial S_d / \partial x_N$

- The mean $N_j \partial S_d / \partial x_j$ remains negative throughout flame.

- The difference in the mean value $N_j \partial S_d / \partial x_j$ is not significant to draw any meaningful conclusion. **No major pressure dependence on the mean added normal strain rate.**
For the corrugated flamelets regime cases A-C and E the mean behaviour of $2S_d\kappa_m$ is governed by the mean contribution of $2(S_r + S_n)\kappa_m$ where the mean contribution of $2S_t\kappa_m$ remain small.

- The mean contributions of $2(S_r + S_n)\kappa_m$ and $2S_t\kappa_m = -4D\kappa_m^2$ remain comparable for case D representing the thin reactions zone regime of premixed turbulent combustion.

- The magnitude of the mean value of $2S_d\kappa_m$ is the highest for case D and the differences in the mean values of $2S_d\kappa_m$ originate due to the difference in curvature distributions.
Statistics of curvature stretch rate $2S_d\kappa_m$

- The mean $2S_d\kappa_m$ remains negative throughout flame.
- The difference in the mean value of $2S_d\kappa_m$ arises due to the difference in curvature distribution and also due to the change in combustion regime.
\( a_N^{\text{eff}} \times \delta_{th}/S_L \): effective normalised normal strain rate (1)

- The magnitude of \( a_N^{\text{eff}} \) remains small in comparison to the magnitudes of \( a_N \) and \( N_j \partial S_d / \partial x_j \).

- The mean contributions of \( a_N \) and \( N_j \partial S_d / \partial x_j \) almost cancel each other.

- The above behaviour does not seem to be affected by the pressure variation.
The standard deviations are shown as bars. Clearly the uncertainty in the difference in the mean value is not significant to draw any meaningful conclusion.

No difference in effective normal strain rate is reflected in no difference in $|\nabla c| \times \delta_{th}$.

No major pressure dependence on the mean normalised effective normal strain rate.

$a_N^{\text{eff}} \times \delta_{th}/S_L$: effective normalised normal strain rate (2)
\( a_N^{\text{eff}} \times \delta_{th}/S_L: \text{effective normalised normal strain rate (3)} \)

- The magnitude of \( a_N^{\text{eff}} \) remains small in comparison to the magnitudes of \( a_N \) and \( N_j \partial S_d/\partial x_j \).

- The mean contributions of \( a_N \) and \( N_j \partial S_d/\partial x_j \) almost cancel each other.

- The above behaviour does not seem to be affected by the variation of \( u'/S_L \) and \( L_{11}/\delta_{th} \) within the current parameter range.
The standard deviations are shown as bars. Clearly the uncertainty in the difference in the mean value is not significant to draw any meaningful conclusion.

No difference in effective normal strain rate is reflected in no difference in $|\nabla c| \times \delta_{th}$.

No major $u'/S_L$ and $L_{11}/\delta_{th}$ dependences on the mean normalised effective normal strain rate.
\( a_T^{\text{eff}} \times \delta_{th}/S_L \): effective normalised stretch rate (1)

- The magnitude of \( a_T^{\text{eff}} \) remains small in comparison to the magnitudes of \( a_T \) and \( 2S_d \kappa_m \).

- Although there are differences in the magnitudes of mean values of \( a_T \) and \( 2S_d \kappa_m \), the mean contributions of \( a_T \) and \( 2S_d \kappa_m \) almost cancel each other.

- Thus, \( a_T^{\text{eff}} \) does not show a significant pressure dependence for the current parameter range.
The standard deviations are shown as bars. Clearly the uncertainly in the difference in the mean value is not significant to draw any meaningful conclusion.

No major pressure dependence on the mean normalised effective tangential strain rate within this parameter range.
The magnitude of $a_T^{\text{eff}}$ remains small in comparison to the magnitudes of $a_T$ and $2S_d \kappa_m$.

Although there are differences in the magnitudes of mean values of $a_T$ and $2S_d \kappa_m$, the mean contributions of $a_T$ and $2S_d \kappa_m$ almost cancel each other.

Thus, the behaviour of $a_T^{\text{eff}}$ does not show significant dependences of $u'/S_L$ and $L_{11}/\delta_{th}$ for the current parameter range.
The standard deviations are shown as bars. Clearly the uncertainty in the difference in the mean value is not significant to draw any meaningful conclusion.

No major $u'/S_L$ and $L_{11}/\delta_{th}$ dependences on the mean normalised effective tangential strain rate.
Burning rate and flame area (1)

The burning rate can be estimated as: \( \int_V \dot{\omega}dV \) and it can be presented in the non-dimensional form as:

\[
\Omega = \frac{\int_V \dot{\omega}dV}{\rho_0 S_L D^2}
\]

The flame surface area can be estimated as: \( \int_V |\nabla c|dV \) and it can be presented in the non-dimensional form as:

\[
A = \frac{\int_V |\nabla c|dV}{D^2}
\]
## Burning rate and flame area (2)

<table>
<thead>
<tr>
<th>Case</th>
<th>$P/P_0$</th>
<th>$\langle \Omega \rangle$</th>
<th>$SD(\Omega)$</th>
<th>$\langle A \rangle$</th>
<th>$SD(A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.0</td>
<td>4.49</td>
<td>0.39</td>
<td>4.15</td>
<td>0.31</td>
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<tr>
<td>B</td>
<td>5.0</td>
<td>4.60</td>
<td>0.42</td>
<td>4.46</td>
<td>0.38</td>
</tr>
<tr>
<td>C</td>
<td>10.0</td>
<td>4.60</td>
<td>0.15</td>
<td>4.40</td>
<td>0.14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>$P/P_0$</th>
<th>$u'_{inlet}/S_L$</th>
<th>$L_{11}/\delta_{th}$</th>
<th>$\langle \Omega \rangle$</th>
<th>$SD(\Omega)$</th>
<th>$\langle A \rangle$</th>
<th>$SD(A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>10.0</td>
<td>1.0</td>
<td>15.48</td>
<td>4.60</td>
<td>0.15</td>
<td>4.40</td>
<td>0.14</td>
</tr>
<tr>
<td>D</td>
<td>1.0</td>
<td>3.1</td>
<td>4.88</td>
<td>4.53</td>
<td>1.91</td>
<td>4.13</td>
<td>0.43</td>
</tr>
<tr>
<td>E</td>
<td>1.0</td>
<td>1.0</td>
<td>15.79</td>
<td>4.10</td>
<td>0.21</td>
<td>3.85</td>
<td>1.63</td>
</tr>
</tbody>
</table>

Here $\langle \ldots \rangle$ is the mean of realisations and $SD(\ldots)$ is the standard deviation between the realisations.

- Appropriate normalisation and pressure scaling lead to collapse of data for different pressure conditions for a given set of values of $U_{inlet}/S_L, u'_{inlet}/S_L$ and $L_{11}/D$.
- D-L instability at high pressure condition leads to increased flame surface area and burning rate for a given set of values of $U_{inlet}/S_L, u'_{inlet}/S_L$ and $L_{11}/\delta_{th}$.
Conclusions

- Pressure does not seem to have any major impact on the normalised flame thickness and displacement speed at least in the context of simple chemistry. Thus, the models which have been derived and validated for atmospheric pressure are also likely to work for elevated pressures provided correct pressure scalings are used.

- The possibility of having D-L instability is high at elevated pressure conditions. The effects of this instability need to be modelled separately.

- Appropriate normalisation and pressure scaling give rise to collapse of burning rate and flame area for different pressure conditions for a given set of values of $U_{inlet}/S_L, u'_{inlet}/S_L$ and $L_{11}/D$.

- D-L instability at high pressure condition leads to increased flame area and burning rate for a given set of values of $U_{inlet}/S_L, u'_{inlet}/S_L$ and $L_{11}/\delta_{th}$. 
THANK YOU for your attention!