

Examples

1. $St(3, 1)$

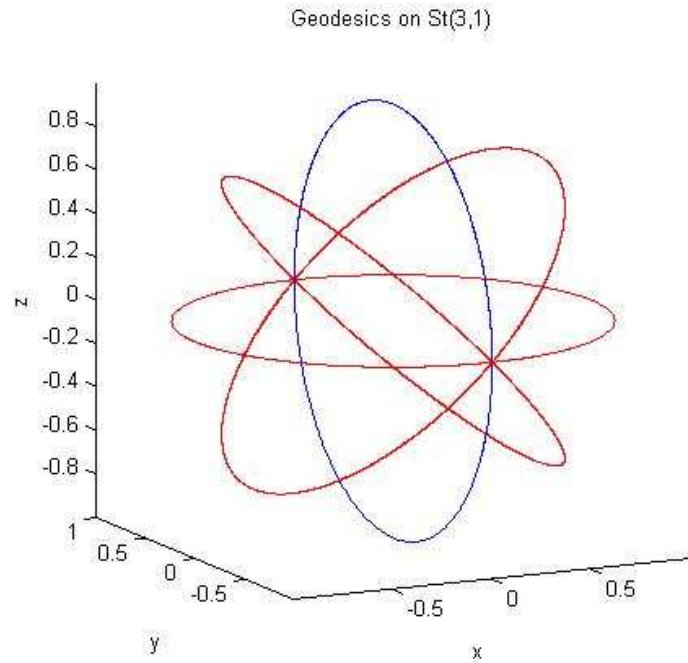
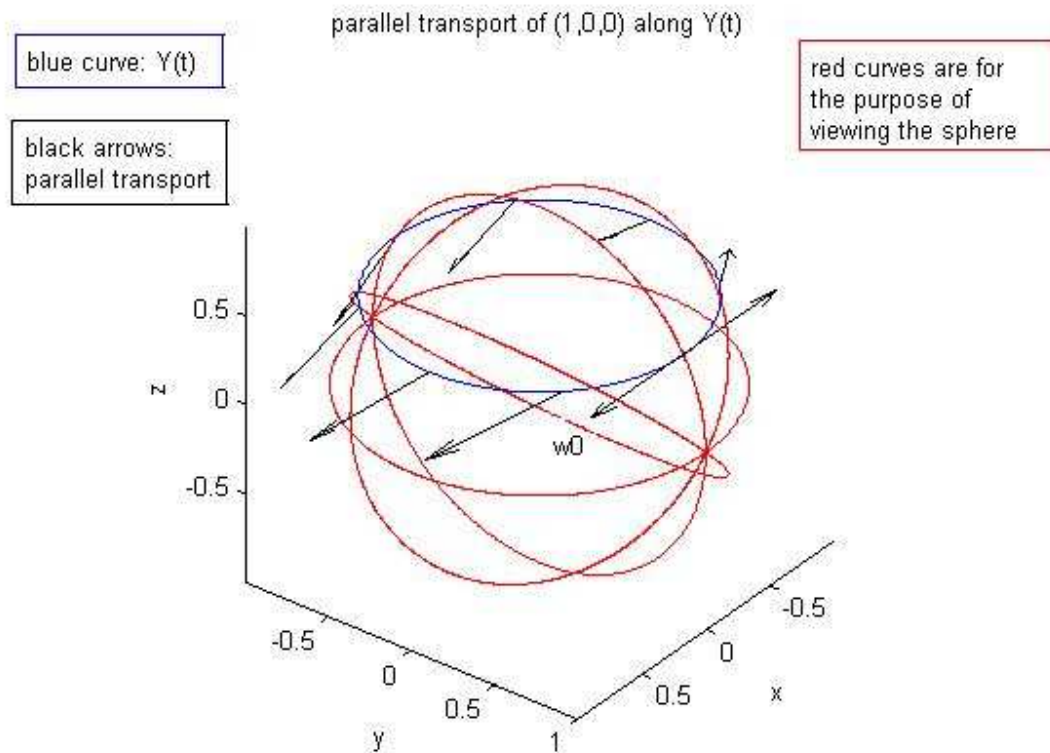


Fig.1 Four geodesics on $St(3,1)$

Matlab code for geodesic:

```
%Solving geodesics on  $St(3,1)$ 
%  $Y(t)=[Y(0) Y'(0)]*\expm(t*[A -Y'(0)'*Y'(0); I A])*I(2p,p)*\expm(-A*t)$ 
axis equal;
Y0=[0;1;0]; %the initial point on Stiefel(3,1)
Yd=[0;0;1]; %the initial tangent vector
Y=[]; %  $Y(t)$  differet columns corresponds to  $Y(t)$  at different time t
A=0;
for t=0:0.01:10
    Ynew=[Y0 Yd]*expm(t*[A -Yd'*Yd; eye(1) A])*eye(2,1)*expm(-A*t);
    Y=[Y Ynew];
end
plot3(Y(1,:),Y(2,:),Y(3,:),'b')
hold on;
```



Matlab code for parallel transport:

```

%Compute the parallel transport of  $W_0=(1,0,0)$  along curve
% $Y(t)=(\sqrt{3}/2*\sin(\sqrt{2/3}*t),\sqrt{3}/2*\cos(\sqrt{2/3}*t),0.5)$ 
% $Y'(t)=(\sqrt{2}/2)*(\cos(\sqrt{2/3}*t),-\sin(\sqrt{2/3}*t),0)$ 
dw=[];
%w=[0,0.5,-sqrt(3)/2];
w=[1,0,0];
n=1;
delta=0.001;
for t=0:delta:(2*3.1415926*sqrt(3/2))
    y(n,:)=[sqrt(3)/2*sin(sqrt(2/3)*t),sqrt(3)/2*cos(sqrt(2/3)*t),0.5];
    dy(n,:)=[cos(sqrt(2/3)*t)/sqrt(2),-sin(sqrt(2/3)*t)/sqrt(2),0];
    dw(n,:)=-dy(n,:)*(w(n,:))'*y(n,:);
    w(n+1,:)=w(n,:)+delta*dw(n,:);
    n=n+1;
end
wshow=[1,0,0];
yshow=[0,sqrt(3)/2,0.5];
for i=1:8
    wshow=[wshow;w(round(i*n/8),:)];

yshow=[yshow;[sqrt(3)/2*sin(sqrt(2/3)*delta*(round(i*n/8))),sqrt(3)/2*cos
(sqrt(2/3)*delta*(round(i*n/8))),0.5]];
end
plot3(y(:,1),y(:,2),y(:,3),'b');
quiver3(yshow(:,1),yshow(:,2),yshow(:,3),wshow(:,1),wshow(:,2),wshow(:,3)
,'k');

```

2. St(5, 2)

Matlab code for geodesic on St(5, 2):

```
%Solving geodesics on Stiefel manifold
%Y(t)=[Y(0) Y'(0)]*expm(t*[A -Y'(0)'*Y'(0); I A])*I(2p,p)*expm(-A*t)
Y0=[1 0;
    0 1;
    0 0;
    0 0;
    0 0]; %the initial point on Stiefel(5,2)
Yd=[0 0;
    1 0;
    0 1;
    0 0;
    0 0];
Y=[]; %Y(t) differet columns corresponds to Y(t) at different time t
A=Y0'*Yd;
for t=0:0.01:10
    Ynew=[Y0 Yd]*expm(t*[A -Yd'*Yd; eye(2) A])*eye(4,2)*expm(-A*t);
    Y=[Y Ynew];
end
```

The following 10 matrices are selected according to equal distance of adjacent matrices on St(5, 2):

0	0.8080	0.3153	0	-0.3043	0
0.8139	0.5891	0.9490	0.3153	0.9526	-0.3043
0.5810	-0.0000	0	0.9490	-0.0000	0.9526
0	0	0	0	0	0
0	0	0	0	0	0
-0.8071	0	-1.0000	0	0	-0.8090
0.5904	-0.8071	0.0016	-1.0000	-0.8148	-0.5879
0.0000	0.5904	0.0000	0.0016	-0.5797	0
0	0	0	0	0	0
0	0	0	0	0	0
-0.0000	-0.3073	0.0000	0.3123	0.8061	0
-0.3168	-0.9516	0.3028	-0.9500	-0.5917	0.8061
-0.9485	0	-0.9531	0.0000	-0.0000	-0.5917
0	0	0	0	0	0
0	0	0	0	0	0
1.0000	0				
-0.0032	1.0000				
-0.0000	-0.0032				
0	0				
0	0				