Interest Group Coalitions and Information Transmission

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Abstract

Though researchers of interest group behavior consistently note that groups often form coalitions when seeking to influence public policy, questions remain about when groups join forces and whether lobbying in a coalition is more or less effective than lobbying alone. In this paper, I approach these questions with a formal model of interest groups coalition formation among information-providing interest groups with non-identical preferences. I examine how the characteristics of groups and issues affect whether groups lobby alone or in coalitions and whether lobbying alone or lobbying in a coalition is most effective. Even when all groups desire the same policy, the uncertainty about other groups’ behavior can make lobbying too costly for some groups. When some groups want the policy more strongly than others, conditions that allow the most extreme groups to credibly transmit information can make lobbying too costly for more moderate groups. I show how coalitions can solve these problems and suggest that coalitions are more likely when there are multiple groups with similar, moderate ideologies, information is of low quality, and the cost of lobbying is high, relative to the benefit of the policy. I also show that coalitions make it more likely that the legislator chooses the groups’ preferred policy and that the legislator chooses the correct policy when groups can lobby in coalitions but cannot lobby alone. If groups can lobby alone or in a coalition, however, coalitions tend to make the legislator less likely to choose the groups’ preferred policy and to decrease the legislator’s welfare.
In addition to legislators and citizens, the political environment in which an interest group decides whether to lobby also includes other interest groups. If a group’s effort is not matched by lobbying by other groups with similar policy goals, it may have trouble achieving its policy goals. One strategy that interest groups use is forming coalitions with other interest groups. In this fashion, groups such as Bank of America, JPMorgan Chase & Co., the Wachovia Corporation, and other banks lobby together as the American Bankers Association instead of as individual groups. Scholars find that almost half of interest participate in coalitions (Mahoney and Baumgartner 2004) and an even greater number consider participation in a coalition to be an important strategy (Hula 1999), but questions remain about when and why interest groups join coalitions and whether and when coalitions are more or less effective than lobbying alone.

In this paper, I develop a formal model of interest group coalition formation by interest groups with non-identical preferences. In this model, all groups want the legislator to enact the same policy, but groups differ in the strength of their preferences. More extreme groups want the legislator to choose their preferred policy regardless of whether it is right for society, while moderate groups also always want the legislator to enact their preferred policy but want it much more when it is the right policy. I show that even groups that want the same thing may not all be willing to lobby on a single issue. It is especially those groups with smaller biases, or preferences more similar to those of the legislator, that find it too costly to lobby alone when they are uncertain about the behavior of other groups or because conditions that make information transmission possible for extreme groups makes lobbying unattractive for moderate groups. Coalitions can solve these problems, because groups in a coalition have less uncertainty about other groups’ lobbying behavior than groups lobbying alone. For this reason, I expect coalitions to be more likely when the information groups have is of low quality, so the legislator is only convinced when many groups lobby, and when there are multiple groups with moderate ideologies, for which the cost of lobbying is high relative to
the potential benefit from the policy. Very moderate groups can only lobby in coalitions, while more extreme groups may be able to lobby in a coalition with other extreme groups or alone.

I also examine how coalitions affect the likelihood that the legislator picks the groups’ preferred policy and the likelihood that the legislator picks the correct policy, improving her welfare and that of society. I find that the effects of coalitions depend on the types of groups that are in the coalitions. When coalitions are made up of moderate groups that find lobbying alone too costly, they increase the probability that the legislator chooses the groups’ preferred policy. In this case, there is no lobbying without coalitions, making the legislator less likely to choose the policy the groups want. More extreme groups can choose whether to lobby in coalitions or alone, and the legislator is more likely to choose their preferred policy when they choose the latter. Coalitions make lobbying less costly for these groups, so the legislator must choose their preferred policy less often so that the groups transmit information through their lobbying instead of always lobbying. These findings emphasizes the need to consider selection effects in empirical studies testing whether coalitions are more or less effective than lobbying alone.

The effect of coalitions on legislator welfare also depends on the groups that are in the coalitions. When groups are so moderate that they truthfully reveal information to the legislator when lobbying in a coalition, the coalition improves the legislator’s welfare. Groups such as this find it too costly to lobby alone, so without coalitions, the legislator receives no information about which policy is best. If groups can truthfully reveal information to the legislator when lobbying alone, then in most cases, lobbying in a coalition decreases the cost of lobbying enough that they lobby too much to try to mislead the legislator. In this case, the legislator is more likely to choose the correct policy if groups lobby alone and truthfully reveal their information. Otherwise, the legislator is equally likely to choose the correct policy when groups lobby together and when they lobby alone.
1 Interest Groups and Coalitions

Scholars of interest group behavior have long noted that interests may combine resources when trying to influence public policy (Berry 1977; Moe 1980). However, coalitions do not form for every issue and not all groups join coalitions. In a random sample of 98 federal issues between 1999 and 2003, Mahoney and Baumgartner (2004) find at least one active coalition on 55% of these issues and that 41% of major actors are active in at least one coalition. Empirical research is not clear on how interest groups decide whether to lobby in coalitions or alone. Hojnacki (1997) surveys organizations with interests in a few bills during the early 1990s and suggests that the interest groups make the decision to lobby together or alone strategically. She also finds that interest groups that habitually lobby as part of a coalition are more likely to participate in coalitions than interest groups that are infrequent coalition members. She does not explain what makes one group more likely to become a habitual coalition member than another but suggests that interest groups are more likely to work alone when they are concerned about their autonomy, that is, when they are worried about maintaining their own resources and members. Hula claims that interest groups work alone on some activities in order to maintain their reputations, or a “unique and recognized identity as a significant and legitimate voice in the policy process” (95). Mahoney and Baumgartner find fixed group characteristics, such as budget, number of affiliated PACs, and membership size are barely, if at all, significant predictors of coalition membership and that groups may act in different ways on different issue; most groups in their sample sometimes participate in coalitions and sometimes do not. Issue-specific characteristics, such as whether groups are trying to change or keep the status quo and the salience of the issue, measured by the newspaper coverage of the issue, and the amount of conflict, are better predictors of whether a coalition forms. They argue that in order to understand when interest groups join coalitions, we must examine both issue characteristics and group characteristics.
Explanations for coalition membership or lobbying alone that reference habitual membership, autonomy, reputation, and unique identity are vague and leave questions about when groups decide to join coalitions. In my model, I examine in detail how group and issue characteristics, such as the importance of an issue to the group relative to the cost of lobbying and the quality of information resources available to the group, affect coalition membership. This is in line with Mahoney’s and Baumgartner’s findings that issue characteristics are better predictors of when coalitions form. I focus on groups that are attempting to change the status quo, but I distinguish between groups that strongly wish to change policy in all situations and those that are more moderate.

Though groups do not always join coalitions, interest group leaders believe that working in coalitions is important to success. Barry (1977) reports the findings from surveys conducted between 1972 and 1973, in which 76% of public interest activists state that “joint activity with other organizations” is either important or very important (254). In more recent interviews with organizations that lobby on transportation, education, and civil rights issues, Hula (1999) finds that at least 79% of organizations in each issue area agree that, “Coalitions are the way to be effective in politics;” at least 75% of organizations agree that “Being a member of a coalition helps an organization control the outcome of an issue;” and at least 40% of organizations agree that, “If our organization does not join a coalition, we may lose our ability to shape the outcome of the issue.” Mahoney and Baumgartner empirically test whether coalitions help interest groups achieve policy success. They find that coalitions are positively related to policy success when groups are trying to maintain the status quo but negatively related to policy success when groups are trying to change policy. They suggest that in the latter case, coalitions may be a sign that a policy is in trouble.

I restrict my attention to groups wishing to change the status quo and examine when coalitions help groups achieve their policy goals. I find that coalitions are helpful to interest groups when groups find it too costly—relative to the benefits of the policy—to lobby alone.
When groups can lobby alone or in coalitions, lobbying alone is more likely to lead to policy change. My findings suggest that when measuring whether coalitions help interest groups change policy, we must consider why groups choose to join coalitions.

In this paper, I model interest groups as information providers. Describing interest groups as information providers is widespread in the political science literature. Hansen (1991) explains the rise and fall of the importance of the farm lobby partly on the competitive advantage these groups had in providing information on voter behavior. Interest groups inform Congress members about the policy positions preferred by constituents, provide legislators with rationales that they may use to convince voters that they made the right decision, and agree not to attack the legislators and sway voters against them. In exchange for this information, Congress members seriously consider the policies preferred by the interest groups. While Hansen notes that interest groups may be incorrect in their claims despite their best efforts, he argues that plentiful access to other information prevents interest groups from making arbitrary claims.

Wright (1996) also argues that interest groups are able to influence public policy because the information they provide to legislators helps them win reelection and make good public policy. Successful grassroots campaigns by interest groups show legislators that their constituents can be mobilized on an issue. Interest groups may also create public opinion by informing voters about Congress members’ votes. Lobbyists can offer information on policy positions legislators may take and the compromises they would be willing to make during the legislative process, and interest groups often employ technical experts who provide information on the potential consequences of bills. Wright suggests that interest groups may use information strategically, especially by misleading the public and influencing their support for legislation or nominations.

The decision to join a coalition may also be related to the transmission of information. In Hula’s surveys, at least 87% of interest groups in each policy area indicate that informal
information-sharing meetings are important to their activities. Hula also suggests that one benefit to being in a lobbying coalition is avoiding being left out of the loop on legislation that is important to the group.

Modeling interest groups as information providers is also common. I follow the literature that models lobbying as costly signaling. The classic work in this area is Potters and Van Winden (1992) (see also Grossman and Helpman 2001 for a summary of these models), and I make many of the same modeling decisions about interest group preferences and legislator decisions. Potters and Van Winden model lobbying by a single, perfectly-informed interest group and find that interest groups fall into four regions. Interest groups that do not receive much benefit from a particular policy never choose to lobby, because the costs of lobbying exceed the benefit. Legislators are not influenced by interest groups that receive a very high benefit from a particular policy being chosen, because they always choose to lobby, regardless of the signal they receive, so the legislators cannot learn from their lobbying. Lobbying by interest groups in the two moderate regions may influence policy. The more moderate of these interest groups find the benefit of lobbying to be more than the cost of lobbying in only one state of the world and thus reveal information truthfully to legislators. More extreme groups that can affect policy by lobbying less often in one state of the world than the other, allowing the legislator to become imperfectly informed from their actions. The situation becomes more complicated, however, when more than one interest group may lobby and interest groups are not perfectly informed.

Battaglini’s and Bénabou’s (2003) explicitly model coalition formation under this information structure, and the model I present in this paper is an extension of their work. In their paper, $n$ interest groups with identical preferences over policy receive noisy signals about the state of the world. They find two equilibria in which three interest groups lobby without coordination. In the first, which is unique when the prior probability that the interest groups’ preferred action is the correct one is sufficiently high, the lawmaker takes the
action only when all interest groups lobby. In the second, which is unique when the prior probability that the interest groups’ preferred action is the correct one is sufficiently low, lobbying by two interest groups is enough to convince the legislator to choose their preferred policy. Battaglini and Bénabou also find an equilibrium in which $n$ interest groups all play a perfectly correlated mixed strategy. They examine the welfare of the interest groups and the policymaker and find some conditions under which both prefer independent lobbying or both prefer coalitional lobbying.

In Battaglini’s and Bénabou’s coalitional equilibrium, the groups truthfully reveal their signals to each other and lobby with positive probability when their set of high signals is at or above a threshold. Each group is indifferent between the coalition lobbying and not lobbying when its signal is decisive. However, if groups have different policy preferences, the sharing of private signals is not be incentive compatible, because not all groups are indifferent between lobbying and not lobbying at the threshold. When groups have nonidentical preferences, a group with more biased preferences may attempt to mislead a group with less biased preferences about the signal it has received, if doing so will convince the latter group that lobbying is worthwhile. We should not expect all groups to truthfully reveal their signals in equilibrium.

Battaglini and Bénabou make two further assumptions that make more sense when groups have identical preferences than when their preferences may be different. First, they assume that the signal the groups receive is strong enough so that if the policymaker received a high signal herself, she would choose the interest groups’ preferred policy. If we do not focus exclusively on symmetric equilibria—a reasonable decision in the realm of groups with nonidentical preferences—we must wonder why, in this case, groups in a coalition would not coordinate to an equilibrium with only one group lobbying and bearing that cost. Making different assumptions about the strength of the signal can eliminate this concern when groups are not limited to symmetric equilibria.
Battaglini and Bénabou assume that the cost of lobbying, $c$, is lower than the benefit any type of interest group to avoid a situation in which lobbying is only beneficial when the interest group receives a particular signal, or “the rather obvious cases of an equilibrium in which types perfectly separate” (838). However, when multiple interest groups receive imperfect signals and simultaneously decide whether to lobby, the conditions for a moderate interest group to lobby when it receives one signal and not lobby when it receives another depend on the strength of the signal and the number of other interest groups.

I model how non-identical, information-providing interest groups form coalitions when lobbying is costly. I show how the presence of other groups can make lobbying too costly for moderate groups and on how coalitions solve this problem. I find conditions under which coalitions are more likely to emerge, when they help and hurt groups’ attempts to change policy, and when they increase and decrease legislator welfare.

2 Model

A single legislator wishes to choose a policy $P \in \{0, 1\}$. The legislator’s preferred policy position depends on the state of the world, $\omega \in \{0, 1\}$. The legislator wishes to match the policy to the state of the world. Her payoffs given the policy she chooses and the state of the world are given by the table below:

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$P$</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

When she believes the state of the world is more likely to be 0, she chooses policy 0. When she believes the state of the world is more likely to be 1, she chooses policy 1. When the two states of the world are equally likely, she is indifferent between policies and may
choose either policy or mix between them. Without additional information, the legislator initially believes that the state of the world is more likely to be 0, \( Pr(\omega = 0) = p_0 \geq \frac{1}{2} \).

Three biased interest groups also have preferences over the legislator’s policy decision. The interest groups always want the legislator to choose policy 1, regardless of the state of the world, but receive the highest payoff when \( \omega = 1 \) and \( P = 1 \). The interest groups’ payoffs are not identical, though they are all biased in the same direction. The payoffs to interest group \( i \) in each state of the world and each policy are given in the table below:

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>( a_1 )</td>
</tr>
</tbody>
</table>

When the order of the interest groups matters, I will refer to them as groups 1, 2, and 3, where \( a_1 < a_2 < a_3 \).

Initially, interest groups have beliefs over the state of the world that are identical to those of the legislator. However, each group receives an imperfect signal about the state of the world, \( s_i \in \{0, 1\} \). I assume that \( \delta = Pr(s_i = 0|\omega = 0) = Pr(s_i = 1|\omega = 1) > \frac{1}{2} \).

After receiving their signals, interest groups may lobby alone or in a coalition at cost \( c \). Each interest group may privately specify one coalition in which it wishes to lobby. If all groups that are supposed to be in a coalition specify that coalition, the coalition will form. If the coalition that group \( i \) has specified does not form, then group \( i \) cannot lobby. For example, if groups \( i \) and \( j \) specify that they will join a coalition of groups \( i \) and \( j \) and group \( k \) says that it would like to lobby in a coalition of groups \( i, j, \) and \( k \), a coalition of groups \( i \) and \( j \) will form and group \( k \) will not lobby. An interest group pays the cost \( c \) of lobbying whenever it lobbies alone, regardless of the policy the legislator chooses but only pays the cost \( c \) of lobbying in a coalition if the coalition is formed. For example, if group \( i \) announces that it would lobby in a coalition with groups \( j \) and \( k \) and group \( j \) decides not to lobby at
all, group \( i \) will pay no cost.

In this paper, interest groups’ strategies take the following form: group \( i \) announces that it will join a particular coalition when \( s_i = 1 \) or with probability \( l_i \) when \( s_i = 0 \). Interest group strategies are common knowledge and if group \( i \) does not join the coalition that it is supposed to join, the other groups believe that \( s_i = 0 \). The legislator’s strategy is to choose policy 1 with probability \( 0 \leq r \leq 1 \) after interest groups lobby, where \( r \) may depend on interest groups’ strategies and which groups lobby. The legislator’s strategy is also common knowledge. All actors update their beliefs using Bayes’ Rule.

We can compare group \( i \)’s expected payoff from lobbying in a grand coalition, in a coalition of two with group \( j \), and alone when \( s_i = 1 \). If group \( i \) is lobbying in a grand coalition, it knows that it will only face the cost of lobbying if groups \( j \) and \( k \) also choose to lobby. Thus, group \( i \)’s expected payoff from lobbying when \( s_i = 1 \) is:

\[
r_{gc}(Pr(\omega = 1|s_i = 1 \cap j \text{ lobby}) + Pr(\omega = 0|s_i = 1 \cap j \text{ and } k \text{ lobby})a_i) - c
\]

Where \( r_{gc} \) is the probability with which the legislator chooses policy 1 if this grand coalition lobbies. If group \( i \) is lobbying in a coalition with group \( j \), it knows that it will only pay the cost of lobbying if group \( j \) also wants to lobby, but it does not know what group \( k \) will do. Group \( i \)’s expected payoff from lobbying in a coalition of two with group \( j \) when \( s_i = 1 \) is:

\[
r_{sc}Pr(k \text{ lobbies}|s_i = 1 \cap j \text{ lobbies})(Pr(\omega = 1|s_i = 1 \cap j \text{ and } k \text{ lobby}) + Pr(\omega = 0|s_i = 1 \cap j \text{ and } k \text{ lobby})a_i) - c
\]

Where \( r_{sc} \) is the probability with which the legislator chooses policy 1 if groups \( i \) and \( j \)
lobby in a coalition and group \( k \) lobbies alone. Keep in mind that the groups may lobby with different probabilities in each configuration, so the posterior probabilities of each state of the world may be different in each case. If group \( i \) lobbies alone, it will always pay the cost of lobbying, regardless of what groups \( j \) and \( k \) decide to do. Hence, group \( i \)'s expected payoff from lobbying alone when \( s_i = 1 \) is:

\[
r_a Pr(j \text{ and } k \text{ lobby}|s_i = 1)(Pr(\omega = 1|s_i = 1 \cap j \text{ and } k \text{ lobby}) + Pr(\omega = 0|s_i = 1 \cap j \text{ and } k \text{ lobby})a_i) - c
\]

Where \( r_a \) is the probability with which the legislator chooses policy 1 when group \( i \) lobbies alone in this configuration\(^1\).

In this paper, I make a few further assumptions. First, I assume that

\[
\frac{\delta^3}{(1 - \delta)^3} > \frac{p_0}{1 - p_0} > \frac{\delta}{1 - \delta}
\]

This implies that there may be equilibria in which the legislator chooses policy 1 when two groups lobby as long as the third group does not lobby at all—so the legislator does not get any information from the third interest group. However, when one group does not lobby when its signal is 0, the legislator will not choose policy 1 when only two groups lobby. Thus, all equilibria in which all groups transmit information to the legislator through their behavior require all three groups to lobby for the legislator to choose policy 1.

I also assume that:

\[
\frac{\delta^3(1 - p_0)b + (1 - \delta)^3p_0a_i}{\delta^3(1 - p_0) + (1 - \delta)^3p_0} > c
\]

\(^1\)Groups \( j \) and \( k \) may be both lobbying alone or lobbying together.
This implies that all groups are willing to lobby under some conditions.

3 Results

I seek to explain when groups lobby in coalitions and when they lobby alone and how the ability to form coalitions affects the lobbying environment and the information interest groups transmit to the legislator. In these results, I focus on what must be true for all three groups to transmit information to the legislator through their lobbying. I mostly focus on equilibria in which all three groups lobby. I sometimes compare these to equilibria with two only two groups lobbying alone. I show that forming coalitions allows for lobbying by more groups and that in some cases, all groups can lobby in a coalition when two groups cannot lobby alone.

The first situation in which coalitions increase the potential for lobbying is when a group or groups are very moderate. Usually, the information interest groups transmit to the legislator is limited by the groups’ biases. When groups are biased, the legislator may not be able to entirely trust the information that groups provide. When the legislator and groups have common preferences, this problem is eliminated, and informative lobbying should be easy. Groups that, like the legislator, receive little benefit from policy 1 when the state of the world is 0 do not want to lobby when their signals are 0, so the legislator knows they have received 1 signals when they lobby. If we have a single interest group and the legislator chooses policy 1 whenever the group lobbies, a group with preferences similar to those of the legislator will lobby if and only if its signal is 12.

When more than one interest group exists, lobbying by a single group may not be enough for the legislator to choose the group’s preferred policy; if the other groups do not lobby, they also transmit information to the legislator, telling her that policy 0 is more likely to be

\footnote{Potters and van Winden (1992) describe this case in detail.}
the correct choice. A group lobbying alone pays the cost of lobbying whenever it lobbies but only enjoys the benefit—in the form of the legislator choosing its preferred policy—when the other groups lobby as well.

Consider the case with three interest groups that receive 0 if policy 1 is chosen when the state of the world is 0. For these groups, the cost of lobbying is larger than the benefit from policy 1 when the state of the world is 0. Intuitively, this is the situation in which lobbying should be the easiest. However, these groups may find it too costly to lobby alone when their signals are 1, because they do not know if the other groups will lobby. For group \( i \) to lobby alone when group it has signal \( s_i = 1 \), it must satisfy:

\[
Pr(\text{j and k lobby} | s_i = 1) \left( \frac{\delta^3(1 - p0)}{(\delta^3(1 - p0) + (1 - \delta)^3p0)} \right) \geq c
\]

Thus, if

\[
c > \frac{\delta^3(1 - p0)}{\delta(1 - p0) + (1 - \delta)p0}
\]

then the groups cannot lobby alone. If, instead, the groups lobby together in a grand coalition, they must satisfy:

\[
\frac{\delta^3(1 - p0)}{(\delta^3(1 - p0) + (1 - \delta)^3p0) \geq c}
\]

which may be possible even if groups cannot lobby alone. These moderate groups can inform the legislator’s decision by lobbying in a coalition but cannot lobby alone.

In this lemma, I examine lobbying behavior by groups with similar preferences to those of the legislator. I describe the conditions under which each group joins a grand coalition if and only if its signal is 1 and show that groups satisfying these conditions cannot lobby in any other configuration.
Lemma 1. If \( a_i \in [a^{**}, \bar{a}^{**}] \) for all groups \( i \) then there is a unique equilibrium with lobbying. In this equilibrium, group \( i \) joins the coalition if and only if \( s_i = 1 \) and the legislator chooses policy 1 when the coalition lobbies.

Groups that satisfy these conditions are the most moderate groups that can transmit information to the legislator through their lobbying; less biased groups find it too costly to lobby, even when they know they will not lobby alone. Though the groups satisfying these conditions all have preferences similar to those of the legislator, the uncertainty of lobbying alone means that it is too costly for them.

The above lemma does not describe the only case in which groups can only lobby in a coalition of three. Even if some groups are biased enough to lobby alone, the least biased group may be sufficiently moderate that it is unwilling to lobby, except in coalition of three. A group that satisfies the conditions described in the next theorem cannot lobby alone or in a coalition of two, regardless of the preferences of other interest groups.

Theorem 1. If \( c > \hat{c} \), then there is an \( \hat{a} > 0 \) such that group 1 can only lobby in a coalition of all three groups when \( a_1 < \hat{a} \).

Note that this theorem describes a necessary, but not sufficient, condition for the only equilibrium with all groups lobbying to have them lobbying in a grand coalition. As I show in detail below, groups must also have similar enough preferences to lobby together in a grand coalition. A group with a strong bias toward policy 1 cannot lobby in a coalition with a group that has preferences similar to the legislator’s.

Groups cannot lobby alone when uncertainty about the other groups also lobbying makes acting alone too costly. Thus, it is intuitive that the minimum bias for groups to lobby in a configuration that is not a coalition of three increases as \( c \) increases, as \( p_0 \) increases, and as \( \delta \) decreases. When \( c \) increases, lobbying becomes more expensive, and a group must expect a higher benefit from lobbying to pay the cost. When \( p_0 \) increases, the other groups are less
likely to receive 1 signals, which means that they are less likely to lobby and lobbying alone is more likely to waste money. When $\delta$ decreases, groups receive worse information about the true state of the world, so the expected payoff from the legislator choosing policy 1 is smaller, even if all three groups receive information that this is the more likely state. This leads even more biased groups to find lobbying alone too costly. If these moderate groups cannot form coalitions, then the legislator will choose policy 0 in situations when policy 1 is more likely to be the better choice.

Coalitions of all the groups are not limited to groups with preferences similar to the legislator’s. More biased interest groups can also lobby in a grand coalition, provided that they have similar enough preferences. For groups to transmit information to the legislator through their lobbying, lobbying must be sufficiently attractive for groups to wish to lobby when they receive a 1 signal but not so attractive that groups always want to lobby when they receive 0 signals. In my model, groups cannot verify the signals their coalition partners receive, so the benefit from joining a coalition cannot be so great for the most biased group that it joins regardless of its signal. At the same time, the most moderate must join the coalition when its signal is 1. When groups all lobby together in a coalition, their expected payoffs from joining the coalition depend on what they can learn about the state of the world from the other coalition members and the likelihood that the legislator will choose policy 1 when the coalition lobbies. When groups within the coalition are more biased, the legislator chooses policy 1 less often when the coalition lobbies, to make lobbying less attractive to biased groups. Since the legislator chooses a single probability, and the beliefs of groups within a single coalition about the state of the world cannot be made to be too different. Lobbying in a grand coalition decreases groups’ uncertainty about whether other groups will lobby, but it does so for all the groups, and groups must have similar preferences to lobby together.

**Lemma 2.** For each value of $a_3$, there is:
(a) a minimum value of $a_2$ such that groups 3 and 2 can lobby together in a grand coalition, and

(b) for every value of $a_2$ greater than this minimum, there is a minimum value of $a_1$ such that groups 1, 2, and 3 can lobby together in a grand coalition.

If $a_2$ is small relative to $a_3$, then $a_1$ must be closer to $a_3$ than if $a_2$ is large. The reason for this is the behavior that groups can engage in when their biases are at different levels. A more biased group 2 can sometimes lobby when its signal is 0—when it believes that the state of the world is less likely to be 1—which lowers the payoff group 3 expects to receive from lobbying. This makes it possible to create conditions under which a less biased group 1 can lobby in the same coalition with that group 3.

When groups have preferences that are similar to each other and dissimilar from those of the legislator, there will be multiple equilibria with all groups lobbying. Along with lobbying in a coalition of three, two groups may be able to lobby together with one group lobbying alone, or all three groups may be able to lobby alone. Creating situations in which lobbying is not too attractive for the most extreme group but is attractive enough for the most moderate group is easiest when groups have similar preferences. I present this as my next theorem.

**Theorem 2.** If groups have large and similar biases, then there is a multiplicity of equilibria with all groups lobbying.

The proof for this comes in three lemmas. First, as shown above in lemma 2, when groups have similar enough biases, all can lobby together in a grand coalition. This is true regardless of how biased groups are, but how close groups must be to lobby together depends on the biases. Second, in the next lemma, I show that when groups are biased enough to risk lobbying alone, groups that have similar enough biases to lobby together can also all lobby alone.
Lemma 3. For large enough $a_3$ and $a_2$, if groups 1, 2, and 3 can all lobby together in a coalition, they can also all lobby alone.

Groups all lobbying alone or all lobbying in a grand coalition face similar information environments. Either they all only lobby when they are joined by two other groups, or all are uncertain about how many other groups will join them. When two groups lobby together in a coalition while one group lobbies alone, the groups in the coalition have more information than the group lobbying alone. Each group in the coalition only faces the cost of lobbying when at least one other group lobbies; a group in a coalition of two is only uncertain about the behavior of the group lobbying alone. The group lobbying alone knows less about the other groups’ plans to lobby. In the third lemma that speaks to multiple equilibria, I consider the case when groups 1 and 3 from a coalition and group 2 lobbies alone. It is not always possible for the more biased group to lobby in a coalition—and have more information—while the less biased group lobbies alone. For environments (i.e. values of $p_0$ and $\delta$) in which this is possible, groups with similar biases can lobby in a configuration with groups 1 and 3 together and group 2 lobbying alone.

Lemma 4. For some values of $p_0$ and $\delta$, when $a_3$, $a_2$, and $a_1$ are large enough, groups 1, 2, and 3 can all lobby together in a coalition or groups 1 and 3 can lobby in a coalition of two while group 2 lobbies alone.

These results show that when groups have similar biases and are not so moderate that they are limited to lobbying together, a number of lobbying configurations are open to them. Groups may lobby all alone, all together, or two together and one alone.

Next, consider groups with dissimilar biases. In addition to allowing lobbying by groups that find lobbying alone too costly, coalitions increase the potential for lobbying when two groups have small biases and one has a large bias. Groups all lobbying together or all lobbying alone face similar information environments and uncertainty about which policy
the legislator will choose. When the behavior of other groups and the legislator that make lobbying unattractive enough for the most extreme group to not lobby when its signal is 0 makes lobbying too unattractive for the most moderate group to lobby when its signal is 1, then the groups cannot both lobby in a grand coalition or all alone\(^3\).

However, if there are two moderate groups, groups 1 and 2, they may be able to lobby together in a coalition while an extreme group, group 3, lobbies alone. The groups in the coalition know that at least one other group will lobby with them, but the group lobbying alone is more likely to get nothing from paying the cost to lobby. The group lobbying alone has less information when it lobbies than the groups lobbying in the coalition, which lowers its expected payoff from lobbying. In this manner, group 3 is made to not want to lobby when \(s_3 = 0\) and group 1 is made to want to lobby when \(s_1 = 1\) on the same issue, even though they cannot all lobby alone or all lobby in a grand coalition. As I show in the next theorem, when two groups have small biases and one has a large bias, then the unique equilibrium with all groups lobbying has the two moderate groups in a coalition. This equilibrium also allows for the greatest ideological distance between group 1 and group 3.

**Theorem 3.** When groups 1 and 2 are moderately biased and group 3 is very biased, then the only equilibrium with all groups lobbying has groups 1 and 2 lobbying together in a coalition and group 3 lobbying alone. This equilibrium allows for the greatest distance between \(a_1\) and \(a_3\).

When multiple moderate interest groups can form a coalition while extreme groups lobby alone, there can be more ideological heterogeneity among groups lobbying on an issue than if groups act alone.

In the above analysis, I have focused on situations when forming some sort of lobbying coalition, either with two or three groups, is possible. I turn now to the case when groups can lobby alone but cannot lobby in any configuration with coalitions. This occurs when

\(^3\)Group 1 and 3 also cannot lobby in a coalition of two in this situation
groups 2 and 3 have a similar bias that is greater than group 1’s bias. I show this is the following theorem.

**Theorem 4.** When groups 2 and 3 have similar large biases and group 1 is more moderate, then the only equilibrium with all groups lobbying is all groups lobbying alone.

In this situation, groups can all lobby alone but cannot lobby in a coalition together because of how one group’s behavior affects another group’s beliefs about the state of the world (and thus that group’s expected payoff) in each lobbying configuration. In both lobbying configurations, when two groups are extreme and close together and one is more moderate, then at least one of the extreme groups sometimes lobbies when its signal is 0, trying to convince the legislator that the state of the world is more likely to be 1. The moderate group, however, only wants to lobby when its signal is 1, because then it believes that it is more likely to get the higher payoff from the legislator choosing policy 1. These behaviors have different effects on the other groups’ payoffs when the groups are in a coalition and when they are alone. When all the groups are in a coalition together, the more extreme groups’ payoffs are increased by the moderate group telling the truth about the signal, and the moderate group’s payoffs are decreased by the extreme groups’ lies. This makes it more difficult to create an equilibrium in which both extreme and moderate groups can transmit information to the legislator. On the other hand, when the groups are lobbying alone their payoffs increase when other groups lobby as often as possible, because the legislator only chooses policy 1 when all three lobby. The more extreme groups’ payoffs decrease when the moderate group only lobbies when its signal is 1, and the moderate group’s payoffs increase when the extreme groups sometimes lobby when their signals are 0. This makes it possible to create an equilibrium with all groups transmitting information to the legislator when \( a_1 \) is farther from \( a_3 \).

In this equilibrium, groups 1 and 3 must have more similar biases than when group 2 is close to group 1 and they form a coalition. Because all three groups must lobby for the
legislator to choose policy 1, group 3 lobbying alone has the same uncertainty whether the other groups are lobbying alone or in a coalition of two. Group 1, however, faces more uncertainty about the outcome of its lobbying when it is alone than when it lobbies in a coalition with another group, so it must be more biased to be willing to pay the cost of lobbying.

**Empirical Implications** This analysis has several empirical implications for when groups can lobby only in coalitions, only alone, or can lobby in a number of configurations. The main implication is that moderate groups band together in a coalition. This is true when groups expect to receive little benefit from the policy—either because of the ideology of the groups or because the groups have low information about which policy is best—relative to the cost of lobbying or relative to the benefit more extreme lobbying groups receive. In the latter case, moderate groups join together in a coalition while extreme groups lobby alone. Thus, I expect groups to join coalitions when their ideologies are similar and they are moderate, when the cost of lobbying is high, and when the information groups have about the policy is of low quality. This may occur when a policy area is new and groups are less certain about the policy’s implications to their constituents.

If, instead, there are multiple groups that expect large payoffs if the legislator chooses their preferred policy and a single group that expects a small benefit, then all groups must lobby alone. In cases in which multiple groups have large amounts of resources for lobbying, are ideologically extreme, and have ample information and a single group is more moderate, I expect all groups to lobby alone.

When groups have similar ideologies and the cost of lobbying is low relative to the potential benefit to the groups from the policy, interest groups may lobby in a number of configurations. They may all lobby alone, all lobby together, or a subset may lobby together while other groups lobby alone. When groups have ample resources to lobby, extreme ide-
ologies, and information, we must seek an explanation beyond the opportunity to lobby for why groups choose to lobby together or alone.

I summarize this in the next proposition, which follows directly from the above lemmas and theorems:

**Proposition 1 (Empirical Implications).**

1. Coalitions are more likely when groups have similar, moderate ideologies, information is of low quality, and the cost of lobbying is high.

2. Groups are likely to lobby alone when there are several groups with extreme ideologies and a single moderate group, the quality of information is high, and the cost of lobbying is low.

**Outcomes and Welfare** In the above analysis, I show that coalitions allow for more lobbying by all three groups, because groups may sometimes lobby in coalitions in cases when they cannot lobby alone. If \( \frac{\delta^3}{(1-\delta)^3} > \frac{p_0}{1-p_0} > \frac{\delta^2}{(1-\delta)^2} \), then there are no equilibria with lobbying except those with all groups lobbying. In this case, the legislator’s belief that the state of the world is 0 or the groups’ information is of low enough quality that the legislator never chooses policy 1 unless all groups lobby, even if one group never lobbies and thus conveys no information by not lobbying. If \( \frac{\delta^2}{(1-\delta)^2} > \frac{p_0}{1-p_0} > \frac{\delta}{1-\delta} \), then some groups may be able to lobby alone in equilibria with only two groups lobbying, even in cases in which all three groups cannot lobby alone. However, even then, some groups find it too costly to lobby alone when they are one of two groups lobbying alone.

If \( \frac{\delta^3}{(1-\delta)^3} > \frac{p_0}{1-p_0} > \frac{\delta^2}{(1-\delta)^2} \) and there is no possibility of an equilibrium with only two groups lobbying, then if groups cannot form coalitions, the legislator always chooses policy 0 when all three groups cannot lobby alone. Obviously, in all of the cases in which all three groups cannot lobby alone but can lobby in coalitions, the possibility of creating coalitions makes it
more likely that the legislator chooses policy 1. However, if all three groups can lobby alone or all lobby in coalitions, then the legislator chooses policy 1 less often when the groups lobby in a coalition. In a coalition, groups face less uncertainty and potentially higher payoffs, so the legislator chooses policy 1 less often so that lobbying is not too attractive.

If \( \frac{\delta^2}{(1-\delta)^2} > \frac{p_0}{1-p_0} > \frac{\delta}{1-\delta} \), then in addition to comparing the policy outcomes in cases in which all three groups may lobby alone or in coalitions, it is useful to compare the policy outcomes when two groups lobby alone and when all three lobby in coalitions. Again, the legislator generally chooses policy 1 less often when all groups lobby in an equilibrium with a coalition than when two groups lobby alone in an equilibrium. In the next proposition, I discuss this and describe the case in which all three groups lobbying in a configuration with a coalition leads the legislator to choose policy 1 more often than when two of the groups lobby alone.

**Proposition 2** (Policy Outcomes).

1. If groups can lobby in a coalition in equilibrium but cannot lobby alone, then coalitions make it more likely that the legislator chooses policy 1.

2. If all three groups can lobby alone in any equilibrium, then in any equilibrium with those groups lobbying in coalitions, the legislator chooses policy 1 less often.

3. There are cases in which two groups can lobby alone in equilibrium or all three groups can lobby in an equilibrium with a coalition, and the legislator chooses policy 1 more often in the latter equilibrium.

Interest group coalitions affect policy outcomes in two ways. First, when coalitions allow for lobbying by groups that would find it too costly to lobby alone, they make it more likely that the legislator chooses the groups’ preferred policy. If, instead, groups may lobby alone or in a coalition, then except for in the case described in the above proposition, the legislator is
less likely to choose the groups’ preferred policy if they lobby together. This result speaks to Mahoney’s and Baumgartner’s findings that coalitions decrease the probability that groups attempting to change achieve their policy goals, but improve the success of groups attempting to maintain the status quo. The effect of coalitions on policy success depends on the groups within the coalitions. These results suggest that it is important to consider the preferences of groups within a coalition when comparing outcomes to groups lobbying alone; if groups desire the policy enough to lobby alone, coalitions are less successful than lobbying alone, but if groups cannot lobby alone, coalitions are more successful than no lobbying. Perhaps groups that attempt to maintain the status quo have different preferences relative to policy benefits than groups attempting to change the status quo. In any case, this is an area for future research.

Though interest groups transmit information to the legislator through their lobbying, lobbying does not always lead to the correct policy choice. When all groups are unbiased enough to lobby only when their signals are 1, either all alone, all in a coalition together, or two together and one alone, their lobbying makes it more likely that the legislator chooses the correct policy. Otherwise, the legislator is as likely to match the policy to the state of the world when there is lobbying as when there is no lobbying and she always chooses policy 0.

Lobbying in a coalition decreases a group’s uncertainty and increases its expected payoffs from lobbying. A group that only wishes to lobby alone when its signal is 1 often finds lobbying in a coalition attractive enough to also wish to do it when its signal is 0. In fact, if groups can all lobby alone if and only if their signals are 1—whether in an equilibrium with all groups lobbying or in one with two groups lobbying—then they find lobbying in a grand coalition too attractive to only join when their signals are 0. Thus, if there is an equilibrium with all groups separating alone and an equilibrium in which all those groups lobby together in a coalition of three, the legislator is more likely to choose the correct policy.
in the former. Coalitions make the legislator more likely to choose the correct policy when
the groups cannot all lobby alone but can join a coalition if and only if their signals are 1 or
two groups can join a coalition when their signals are 1 while the third group lobbies alone
when its signal is 1. Otherwise, coalitions do not change the legislator’s welfare.

In the next proposition, I summarize the cases in which coalitions make the legislator bet-
ter off, when they make the legislator worse off, and when they do not change the legislator’s
welfare.

Proposition 3 (Legislator Welfare).

1. Coalitions strictly improve legislator welfare when groups lobby if and only if their
signals are 1 in a coalition, or when:

   (a) \( a_i \in [a^{**}, \bar{a}^{**}] \) for all \( i \) or

   (b) \( a_{1,2} \in [a^{*}, \bar{a}^{*}] \) and \( a_3 \in [a^{*}, \bar{a}^{*}] \)

   where \( \bar{a}^{***} < a^{**} \) and \( \bar{a}^{**} < a^{*} \).

2. Coalitions worsen legislator welfare when groups can lobby alone if an only if their
signals are 1 but cannot do so in a coalition, or when:

   (a) \( a_i \in [a^{*}, \bar{a}^{*}] \) for all \( i \) or

   (b) \( a_j \in [a^{*}, \bar{a}^{*}] \) for any two groups \( j \) and groups do not satisfy condition 1(b).

3. Otherwise, the ability to form coalitions does not affect the legislator’s welfare.

Generally speaking, coalition increase the legislator’s welfare in the same cases in which
they make it more likely that the legislator chooses policy 1: when coalitions allow unbiased
groups to lobby.

An additional consideration is the welfare of the interest groups. As I show above, when
interest groups have similar preferences, they are able to lobby in a variety of considerations.

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Unlike the legislator, who only cares about matching the policy to the state of the world, interest groups care about how often they pay the cost of lobbying. Discovering when groups prefer different configuration is beyond the scope of this paper and a large project that I will tackle in a future paper.

4 Discussion

In the results lemmas and theorems above, I describe when groups can all lobby alone, all lobby in a single coalition, or when two groups can lobby together while the third lobbies alone. This is best illustrated in a picture, shown in Figure 1.

In this picture, values for $a_3$ are on the $x-axis$, while values for $a_1 = a_2$ are on the $y-axis$. At the top and left of the picture, two groups are extreme and one is moderate. At the bottom and the right, two groups are moderate and one is extreme. The red region shows when groups can all lobby in a single coalition. In the blue region, groups can all lobby alone. In the yellow region, groups 1 and 2 form a coalition together and group 3 lobbies alone. In the green region, groups 1 and 3 lobby together, and group 2 lobbies alone.

While this figure shows when groups may lobby in various configurations under specific values of $p_0, \delta$, and $c^4$, the general areas remain the same. Groups must have sufficiently extreme preferences to lobby alone, but when two groups are extreme and one is more moderate, all lobbying alone is the only equilibrium. When all groups are very moderate, a grand coalition offers them the only opportunity lobby. As long as groups all have similar preferences, they are able to lobby in a grand coalition, but as their preferences become more extreme, this is no longer the only configuration in which they can lobby. More extreme

$^4p_0 = .75, \delta = .63, \text{and } c = .2$
groups with similar preferences can also lobby all alone or groups 1 and 3 can lobby together while group 2 lobbies alone. When there are two moderate groups and a single extreme group, the only lobbying occurs when the two moderate groups join together in a coalition and the extreme group lobbies alone. When there are multiple extreme groups and a single moderate groups, however, coalitions do not enhance groups’ ability to lobby. This figure illustrates many of the empirical implications of my results, as well.

In the above analysis, I focus on an environment with strong status quo bias or low information for the interest groups. We may have equilibria with only two groups lobbying as long as the third group does not transmit any information by not lobbying, but if one group’s decision not to lobby reveals that its signal was 0, the legislator does not choose policy 1. In an extensions section available upon request, I briefly consider the case when $\frac{\delta}{1-\delta} > \frac{p_0}{1-p_0}$. In this case, there may be equilibria in which all three groups transmit information to the legislator through lobbying or not lobbying and the legislator chooses policy 1 when only two of the groups lobby. I find that many of the general results are similar in this case with higher information. A coalition that only lobbies when at least two groups join may allow very moderate groups to lobby when they cannot lobby alone. Coalitions of two groups may also allow for differentiation of payoffs from lobbying for the moderate groups in the coalition and the extreme group lobbying alone. My preliminary findings also suggest that in this case, additional research is needed to examine how coalitions change if some groups can commit to lobbying within them, perhaps by organizing the coalition and paying costs up front.

In this paper, I focus on a world with only three interest groups. While this is enough interest groups for lobbying in a number of configurations—all groups together, a coalition of two groups with one group lobbying alone, and all lobbying alone—it is not enough groups to have lobbying by multiple coalitions. Even if adding more groups did not complicate the calculations even further, it is unclear that it would add to our understanding. Regardless
of whether there are 3 groups or 100, the larger the coalition that a group belongs to is, the less uncertainty, and groups in coalitions of different sizes or lobbying alone will have different expected payoffs from lobbying.

5 Conclusion

The findings in this paper emphasize the importance of other interest group to the lobbying environment. Intuitively, groups with small biases and preferences most similar to those of the legislator should have no incentive problems when it comes to lobbying. However, the presence of other groups leads these moderate groups to worry that if other groups do not lobby, they may waste money with nothing to show for it. This is especially likely when information is of low quality and the cost of lobbying is high relative to the benefits the groups expect to receive from the policy. Though I assume that lobbying as part of a coalition has the same cost as lobbying alone, coalitions change the information environment for groups lobbying on a particular issue. Groups in a coalition know that they do not lobby alone and have more information about the correct policy when they decide to lobby. Belonging to a coalition can make lobbying attractive to groups too moderate to lobby alone. In the cases when coalitions allow for lobbying by groups too moderate to lobby alone, coalitions can also improve the legislator’s welfare. These groups with small biases and preferences similar to those of the legislator truthfully reveal their information to the legislator when lobbying in a coalition. With this information, the legislator is better able to choose the correct policy.

My future research on this topic will include testing the empirical implications of this paper. My findings suggest that coalitions are more likely when groups have similar, moderate ideologies, information is of low quality, and the cost of lobbying is high, relative to benefit that the groups receive from the policy. While group ideology and group resources may be somewhat fixed group characteristics, the way they are presented in this model is specific
to the issue at hand. Ideology here means the benefit a group receives from the policy in question, and the cost of lobbying is important relative to that benefit. These empirical implications suggest that coalitions may be most useful at the beginning stages of discussion over a policy. When groups are less certain about whether a policy will benefit them and have less information about the policy, coalitions may help make lobbying more attractive.

My findings also emphasize the importance of considering how groups select into coalitions when comparing the effectiveness of lobbying in coalitions and alone. When the benefit groups receive from a policy is high enough that the groups can lobby alone, then I expect coalitions to be less effective at getting groups the policy they desire; when the benefit from lobbying is low enough that groups find it too costly to lobby alone, I expect coalitions to be more effective. In the latter case, we cannot simply compare outcomes to those of groups lobbying alone, because without coalitions, those groups would not be able to lobby alone.

My results also suggest areas for additional research in modeling interest group coalitions. Since I examine how interest group coalitions affect legislator welfare and the likelihood that the legislator chooses the groups’ preferred policy, the obvious next step for my research is to examine how coalitions affect interest group welfare. In this model, interest groups care about policy, but they want to achieve their policy goals at the lowest cost. Even when lobbying alone makes it more likely that the legislator chooses the groups’ preferred policy, it may make it more likely that paying the cost to lobby results in no policy change. In ideological regions where groups may lobby together or alone, interest group welfare may help explain why groups choose one configuration or another.

A second area of future research is the process by which coalitions form. Empirical descriptions of interest group coalitions note that coalitions do not appear out of nowhere. Interest group coalitions have founders and brokers who invite other groups to lobby with them. From a modeling perspective, the group that creates the coalition may be able to commit to upfront costs that other groups do not face. This flexibility may allow for a wider
range of equilibria. Research in this area could also examine which groups are most effective in founding coalitions and which groups they will invite to work with them.
Figure 1: Lobbying in Coalitions and Alone
A Proofs of Results

Proof of Lemma 1

Proof. First, let group $i$ join the coalition when $s_i = 1$ and the coalition lobby when all groups join it. Thus, when the coalition lobbies, the legislator assumes that $s_i = 1$ for all groups $i$. She updates her beliefs about the probability that $\omega = 1$ using Bayes’ Rule. Since $rac{\delta^3}{(1-\delta)^3} > \frac{p_0}{1-p_0}$ by assumption,

$$Pr(\omega = 1|\text{coalition lobbies}) = \frac{\delta^3(1 - p_0)}{\delta^3(1 - p_0) + (1 - \delta)^3p_0} > \frac{1}{2}$$

and the legislator chooses policy 1 with certainty.

For group $i$ to join the coalition if and only if $s_i = 1$, each group must satisfy two conditions. First, group $i$ must wish to lobby as part of the coalition when $s_i = 1$, given that the other two members also have 1 signals and the legislator chooses policy 1 when the coalition lobbies:

$$\frac{\delta^3(1 - p_0) + (1 - \delta)^3a_i}{\delta^3(1 - p_0) + (1 - \delta)^3p_0} \geq c$$

$$a^{***} = \frac{\delta^3(1 - p_0)(c - 1) + (1 - \delta)^3p_0c}{(1 - \delta)^3p_0} \leq a_i$$

Second, group $i$ must not wish to join the coalition when $s_i = 0$, even when it is pivotal and its participation in the coalition is certain to lead the legislator to choose policy 1:

$$\frac{\delta(1 - p_0) + (1 - \delta)p_0a_i}{\delta(1 - p_0) + (1 - \delta)p_0} \leq c$$

$$\bar{a}^{***} = \frac{\delta(1 - p_0)(c - 1) + (1 - \delta)p_0c}{(1 - \delta)p_0} \geq a_i$$

Thus, when $a_i \in [a^{***}, \bar{a}^{***}]$, there is an equilibrium with all groups separating on their signals.
when deciding whether to join a coalition and the legislator choosing policy 1 with certainty when the coalition lobbies.

Group $i$’s payoff from lobbying in a grand coalition is decreasing in the probability Since group $i$ does not wish to lobby when $s_i = 0$ and the other two groups received 1 signals, it also does not wish to lobby when $s_i = 0$ in any other case. Thus, no group is willing to mix in a grand coalition, and there is no other equilibrium with a coalition of all three groups.

Consider what must be true for all groups to be willing to lobby alone. In an equilibrium with all groups separating alone, each group must be willing to lobby alone when $s_i = 1$ and not wish to lobby alone when $s_i = 0$, or:

$$\frac{\delta^3(1 - p_0) + (1 - \delta)p_0a_3}{\delta(1 - p_0) + (1 - \delta)p_0} \geq c$$

$$a^* = \frac{\delta(1 - p_0)(c - \delta^2) + \delta p_0 c}{\delta(1 - \delta)^2 p_0} \leq a_i$$ and

$$\frac{\delta(1 - \delta)(\delta(1 - p_0) + (1 - \delta)p_0a_3)}{(1 - \delta)(1 - p_0) + \delta p_0} \leq c$$

$$\bar{a}^* = \frac{(1 - \delta)(1 - p_0)(c - \delta^2) + \delta p_0 c}{\delta(1 - \delta)^2 p_0} \geq a_i$$

Thus, we must have $a_i \in [a^*, \bar{a}^*]$ for all $i$ for groups to separate alone. Algebraic manipulation shows that $\bar{a}^{***} < a^*$, so groups that can separate in a grand coalition find separating alone too costly. When groups lobby alone, their payoffs increase in the probability that other groups lobby when their signals are 0. However, for any group to mix alone when its signal is 0, $a_i > \bar{a}^*$ for some group $i$. That is not possible, here. Thus, if groups can separate in a grand coalition, they cannot all lobby alone.

Next, consider an equilibrium with one group lobbying alone while two groups lobby in a coalition. The above shows that no group $i$ can separate alone when the other two groups also separate. However, since payoffs to a group lobbying alone increase in the probability that other groups lobby when their signals are 0. For a group within the coalition of two to
mix, at least one group \( i \) must have \( a_i \) greater than the range for separating in a coalition of two when the third group is separating alone. For group \( i \) to separate in a coalition of two, it must wish to join the coalition when \( s_i = 1 \) and not join the coalition when \( s_i = 0 \):

\[
\delta^3(1 - p_0) + (1 - \delta)^3 p_0 a_{1,2} \geq c
\]

\[
\tilde{a}^{**} = \frac{\delta^2(1 - p_0)(c - \delta) + (1 - \delta)^2 p_0 c}{(1 - \delta)^3} \leq a_i \text{ and }
\]

\[
\delta(1 - p_0) + (1 - \delta)p_0 a_{1,2} \leq c
\]

\[
\tilde{a}^{**} = \frac{c - \delta(1 - p_0)}{(1 - \delta)p_0} \geq a_i
\]

Thus, for group \( i \) to separate in a coalition of two when one group separates alone, we must have \( a_i \in [\tilde{a}^{**}, \bar{a}^{**}] \). Algebra shows that \( \tilde{a}^{***} < \tilde{a}^{**} \). Thus, no group can separate in a coalition of two, much less mix in a coalition of two. Hence, when \( a_i \in [a^*, \bar{a}^*] \), there is no equilibrium with two groups lobbying together in a coalition while the third lobbies alone.

The last possibility that remains is that there may be an equilibrium with two groups lobbying alone if \( \frac{\delta^2}{(1 - \delta)^2} > \frac{p_0}{1 - p_0} \). Consider what must be true for two groups \( i \) to separate alone in equilibrium, while the third group never lobbies:

\[
\frac{\delta^2(1 - p_0) + (1 - \delta)^2 p_0 a_j}{\delta(1 - p_0) + (1 - \delta)p_0} \geq c
\]

\[
\frac{\delta(1 - \delta)((1 - p_0) + p_0 a_j)}{(1 - \delta)(1 - p_0) + \delta p_0} \leq c
\]

or:

\[
a_i \geq \frac{\delta(1 - p_0)(c - \delta) + (1 - \delta)p_0 c}{(1 - \delta)^2 p_0} = \bar{a}^*
\]

\[
a_i \leq \frac{(1 - \delta)(1 - p_0)(c - \delta) + \delta p_0 c}{\delta(1 - \delta)p_0} = \tilde{a}^*
\]
Again, since $\bar{a}^* < a^*$, if $a_i \in [\bar{a}^*, a^*]$, two groups cannot separate in a coalition with only two groups lobbying. Thus, if $a_i \in [a^*, \bar{a}^*]$ for all $i$, this is the unique equilibrium with lobbying.

Proof of Theorem 1

Proof. Group 1, the most moderate group, can lobby only in a grand coalition if the cost of lobbying is too high for it to lobby alone or in a coalition of two, even when its payoffs from such lobbying are at their maximum. When groups lobby alone, their payoffs increase in the probability that other groups lobby when their signals are 0. Their payoffs are largest when exactly one other group mixes at the maximum probability with which a group can mix when the legislator is willing to pick policy 1. When a group is lobbying in a coalition of two, its payoff also increases in the probability that the group lobbying alone lobbies when its signal is 0. The legislator is willing to pick policy 1 as long as the posterior probability that $\omega = 1$ is at least $\frac{1}{2}$. Let $l$ be the maximum probability with which group $i$ lobbies when $s_i = 0$ and groups $j$ and $k$ are separating, such that the legislator is indifferent between the two policies:

$$\frac{\delta^2(\delta + l(1 - \delta))(1 - p_0)}{\delta^2(\delta + l(1 - \delta))(1 - p_0) + (1 - \delta)^2((1 - \delta) + l\delta)p_0} = \frac{1}{2}$$

$$\frac{\delta^3(1 - p_0) - (1 - \delta)^3p_0}{\delta(1 - \delta)^2p_0 - \delta^2(1 - \delta)(1 - p_0)} = l$$

Let $\bar{l}$ be the maximum probability with which group $i$ lobbies when $s_i = 0$ and group $j$ is separating, in an equilibrium with only two groups lobbying:

$$\frac{\delta(\delta + \bar{l}(1 - \delta))(1 - p_0)}{\delta(\delta + \bar{l}(1 - \delta))(1 - p_0) + (1 - \delta)((1 - \delta) + \bar{l}\delta)p_0} = \frac{1}{2}$$

$$\frac{\delta^2(1 - p_0) - (1 - \delta)^2p_0}{\delta(1 - \delta)(p_0 - (1 - p_0))} = \bar{l}$$
Group 1 cannot lobby alone when all three groups are lobbying alone, its maximum expected payoff from the legislator choosing policy 1 when it does so is smaller than the cost of lobbying. Or if:

\[
\frac{\delta^2(\delta + l(1 - \delta))(1 - p_0) + (1 - \delta)^2((1 - \delta) + l\delta)p_0a_1}{\delta(1 - p_0) + (1 - \delta)p_0} = \frac{\delta(1 - \delta)p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)(1 + a_1)}{((1 - \delta)p_0 - \delta(1 - p_0))(\delta(1 - p_0) + (1 - \delta)p_0)} < c
\]

Likewise, group 1 cannot lobby in a coalition of two when its maximum expected payoff from the legislator choosing policy 1 when it does so is smaller than the cost of lobbying:

\[
\frac{\delta^2(\delta + l(1 - \delta))(1 - p_0) + (1 - \delta)^2((1 - \delta) + l\delta)p_0a_1}{\delta^2(1 - p_0) + (1 - \delta)^2p_0} = \frac{\delta(1 - \delta)p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)(1 + a_1)}{((1 - \delta)p_0 - \delta(1 - p_0))(\delta^2(1 - p_0) + (1 - \delta)^2p_0)} < c
\]

Group 1 cannot lobby alone when two groups are lobbying if its maximum expected payoff from the legislator choosing policy 1 when it does so is smaller than the cost of lobbying:

\[
\frac{\delta(\delta + \bar{l}(1 - \delta))(1 - p_0) + (1 - \delta)((1 - \delta) + \bar{l}\delta)p_0a_1}{\delta(1 - p_0) + (1 - \delta)p_0} = \frac{p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)(1 + a_1)}{(p_0 - (1 - p_0))(\delta(1 - p_0) + (1 - \delta)p_0)} < c
\]

Since

\[
\frac{\delta(1 - \delta)p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)(1 + a_1)}{((1 - \delta)p_0 - \delta(1 - p_0))(\delta^2(1 - p_0) + (1 - \delta)^2p_0)} > \frac{\delta(1 - \delta)p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)(1 + a_1)}{((1 - \delta)p_0 - \delta(1 - p_0))(\delta(1 - p_0) + (1 - \delta)p_0)}
\]

and

\[
\frac{\delta(1 - \delta)p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)(1 + a_1)}{((1 - \delta)p_0 - \delta(1 - p_0))(\delta^2(1 - p_0) + (1 - \delta)^2p_0)} > \frac{p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)(1 + a_1)}{(p_0 - (1 - p_0))(\delta(1 - p_0) + (1 - \delta)p_0)}
\]

a group that always finds lobbying in a coalition of two too costly also finds lobbying alone.
too costly. Thus, if:
\[
a_1 < \frac{c(\delta^2(1 - p_0) + (1 - \delta)^2p_0)((1 - \delta)p_0 - \delta(1 - p_0))}{\delta(1 - \delta)p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)} - 1 = \hat{a}
\]
then group 1 can only lobby in a grand coalition.

If the cost of lobbying is sufficiently low, all groups with \(a_1 > 0\) are sometimes willing to lobby alone or in a coalition of two. However, if:
\[
\frac{c(\delta^2(1 - p_0) + (1 - \delta)^2p_0)((1 - \delta)p_0 - \delta(1 - p_0))}{\delta(1 - \delta)p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)} > 1
\]
\[
\hat{c} = \frac{\delta(1 - \delta)p_0((1 - \delta)p_0 - \delta(1 - p_0))}{(\delta^2(1 - p_0) + (1 - \delta)^2p_0)((1 - \delta)p_0 - \delta(1 - p_0))} < c
\]
then there are some values of \(a_1 > 0\) such that group 1 never lobbies alone or in a coalition of two.

\[\square\]

**Proof of Lemma 2**

*Proof.* First, consider the case when \(a_3 \leq \bar{a}^{***}\), described in lemma 1. Then, by lemma 1, groups 1 and 2 must have \(a_{1,2} \in [\underline{a}^{***}, \bar{a}^{***}]\) for groups to lobby in a grand coalition. Thus, the claim is satisfied.

Next, assume that \(a_3 > \bar{a}^{***}\). Then, there is no equilibrium with all groups separating in a grand coalition; in any equilibrium with a grand coalition, at least one group mixes. Since groups’s payoffs from lobbying in a coalition decrease in the probability that other coalition members lobby when their signals are 0, group 2 only mixes in equilibrium if group 3 can separate when group 2 is the only group mixing. If only one group mixes in equilibrium, the legislator chooses policy 1 with probability to make that group indifferent between lobbying and not lobbying when its signal is 0. Thus, for group 3 to be willing to separate when group
2 mixes, it must satisfy:

$$a_2 \geq \frac{\delta^2(\delta + l(1 - \delta))(1 - p_0) + (1 - \delta)^2((1 - \delta) + l\delta)p_0a_{1,2}}{\delta^2(\delta + l(1 - \delta))(1 - p_0) + (1 - \delta)^2((1 - \delta) + l\delta)p_0} = \frac{\delta(1 - p_0) + (1 - \delta)p_0a_2}{\delta(1 - p_0) + (1 - \delta)p_0}$$

where \( l \) is the value described in the proof of theorem 1. Thus, if \( a_2 < \tilde{a}_2 \), group 3 is the only group mixing in a grand coalition. Thus, for groups to lobby together, groups 1 and 2 must satisfy:

$$a_{1,2} \geq \frac{2(\delta(1 - p_0) + (1 - \delta)p_0a_3)}{\delta(1 - p_0) + (1 - \delta)p_0} - 1$$

Hence, there are minimum values of \( a_1 \) and \( a_2 \) such that groups 1 and 2 can lobby in a coalition with group 3.

Next, consider the case when \( a_2 \geq \tilde{a}_2 \). Then, there exist \( l \geq l_2 \geq 0 \) and \( l \geq l_3 \geq 0 \) such that:

$$\frac{(\delta + l_3(1 - \delta))(1 - p_0) + ((1 - \delta) + l_3\delta)p_0a_2}{(\delta + l_3(1 - \delta))(1 - p_0) + ((1 - \delta) + l_3\delta)p_0} = \frac{(\delta + l_2(1 - \delta))(1 - p_0) + ((1 - \delta) + l_2\delta)p_0a_3}{(\delta + l_2(1 - \delta))(1 - p_0) + ((1 - \delta) + l_2\delta)p_0}$$

and \( \delta(\delta + l_2(1 - \delta))(\delta + l_3(1 - \delta))(1 - p_0) = (1 - \delta)((1 - \delta) + l_2\delta)((1 - \delta) + l_3\delta)p_0 \)

If

$$\frac{(\delta + l_3(1 - \delta))(1 - p_0) + ((1 - \delta) + l_3\delta)p_0a_2}{(\delta + l_3(1 - \delta))(1 - p_0) + ((1 - \delta) + l_3\delta)p_0} = \frac{(\delta + l_2(1 - \delta))(1 - p_0) + ((1 - \delta) + l_2\delta)p_0a_3}{(\delta + l_2(1 - \delta))(1 - p_0) + ((1 - \delta) + l_2\delta)p_0} < c$$

then group 2 cannot mix in a grand coalition with group 3, so we are back in the situation
above, with only group 3 mixing. Otherwise, if

\[
\frac{(\delta + l_3(1-\delta))(1-p_0) + ((1-\delta) + l_3\delta)p_0a_2}{(\delta + l_3(1-\delta))(1-p_0) + ((1-\delta) + l_3\delta)p_0} = \frac{(\delta + l_2(1-\delta))(1-p_0) + ((1-\delta) + l_2\delta)p_0a_3}{(\delta + l_2(1-\delta))(1-p_0) + ((1-\delta) + l_2\delta)p_0} \geq c
\]

the smallest value of \(a_1\) such that group 1 can lobby in a coalition with groups 2 and 3 is the \(a_1\) such that:

\[
\frac{1 + a_1}{2} = \frac{(\delta + l_3(1-\delta))(1-p_0) + ((1-\delta) + l_3\delta)p_0a_2}{(\delta + l_3(1-\delta))(1-p_0) + ((1-\delta) + l_3\delta)p_0}
\]

Hence, in this case, the claim is also satisfied. \(\square\)

**Proof of Lemma 3**

*Proof.* Let \(a_3\) be large enough so that group 3 can mix alone or in a grand coalition. As shown in the proof of lemma 2, when \(a_2\) is close to \(a_3\), groups 2 and 3 can mix in a grand coalition and having them mix allows for the smallest value of \(a_1\) such that group 1 can lobby with groups 2 and 3 in equilibrium. Since a group’s payoff to lobbying alone increases in the probability that the other groups lobby when their signals are 0, when \(a_2\) is close to \(a_3\), both group 2 and group 3 must mix in an equilibrium in which both lobby alone. Thus, \(a_1\) must be large enough so that group 1 can separate alone when groups 2 and 3 mix.

Consider the case of the largest possible \(a_2\), \(a_2 = a_3\). Then, in both configurations, groups 2 and 3 mix with the same probability:

\[
\hat{l} = \frac{\delta(1-\delta)(\delta(1-p_0) - (1-\delta)p_0) + (2\delta - 1)\sqrt{\delta(1-\delta)p_0(1-p_0)}}{\delta(1-\delta)(\delta p_0 - (1-\delta)(1-p_0))}
\]

For group 1 to lobby in a grand coalition with groups 2 and 3 in this case, we must have:

\[
\frac{1}{2} + \frac{1}{2}a_1 \geq \frac{(\delta + \hat{l}(1-\delta))(1-p_0) + ((1-\delta) + \hat{l}\delta)p_0a_3}{(\delta + \hat{l}(1-\delta))(1-p_0) + ((1-\delta) + \hat{l}\delta)p_0}
\]
For group 1 to wish to lobby alone when groups 2 and 3 lobby alone in this case, we must have:

\[
\left( \frac{1}{2} + \frac{1}{2}a_1 \right) \left( \frac{\delta(\delta + \hat{l}(1 - \delta))(1 - p_0) + (1 - \delta)((1 - \delta) + \hat{l}\delta)^2p_0}{\delta(1 - p_0) + (1 - \delta)p_0} \right) \geq \frac{\delta(\delta + \hat{l}(1 - \delta))((1 - \delta) + \hat{l}\delta)p_0}{(1 - \delta)(1 - p_0) + \delta p_0}
\]

For all values of \( p_0 \) and \( \delta \) that satisfy the assumptions and all values of \( a_3 \):

\[
\frac{\delta(1 - \delta((\delta + \hat{l}(1 - \delta))(1 - p_0) + ((1 - \delta) + \hat{l}\delta)p_0)\delta(1 - p_0) + (1 - \delta)p_0}{((1 - \delta)(1 - p_0) + \delta p_0)(\delta(\delta + \hat{l}(1 - \delta))^2(1 - p_0) + (1 - \delta)((1 - \delta) + \hat{l}\delta)^2p_0} < 1
\]

Thus, when \( a_3 \) and \( a_2 \) are large enough, the minimum \( a_1 \) for lobbying in a grand coalition is larger than the minimum \( a_1 \) for lobbying alone; if groups can lobby in a grand coalition, they can also lobby alone.

\[\Box\]

**Proof of Lemma 4**

*Proof.* Let \( a_3 > \bar{a}^{**} \). Thus, group 3 finds lobbying too attractive to separate in a coalition of two when its coalition partner and the group lobbying alone are separating. Since \( \bar{a}^{**} > \bar{a}^{**} \), group 3 is too extreme for all groups to separate in a grand coalition. Group 3’s smallest payoff from the legislator choosing policy 1 when it lobbies in a coalition of two and \( s_3 = 0 \) is larger than group 2’s largest payoff from the legislator choosing policy 1 when it lobbies alone when \( s_2 = 0 \). Hence, if groups 1 and 3 are in a coalition and group 2 lobbies alone, group 2 must separate. Group 1 must satisfy the following for groups 1 and 3 to lobby.
together in a coalition:

\[
\frac{\delta^2(\delta + l(1 - \delta))(1 - p_0) + (1 - \delta)^2((1 - \delta) + l\delta)p_0a_1}{\delta(\delta + l(1 - \delta))(1 - p_0) + (1 - \delta)((1 - \delta) + l\delta)p_0} \geq \frac{\delta(1 - p_0) + (1 - \delta)p_0a_3}{\delta(1 - \delta)}
\]

\[
a_1 \geq \frac{\delta(1 - p_0) + (1 - \delta)p_0a_3}{\delta(1 - \delta)} - 1
\]

In addition to this, for there to be an equilibrium with groups 1 and 3 lobbying together and group 2 lobbying alone, group 2 must be willing to separate when group 3 mixes and group 1 separates:

\[
\frac{\delta^2(\delta + l(1 - \delta))(1 - p_0) + (1 - \delta)^2((1 - \delta) + l\delta)p_0a_2}{\delta(1 - p_0) + (1 - \delta)p_0} \geq \frac{\delta(1 - p_0) + (1 - \delta)p_0a_3}{\delta(1 - \delta)}
\]

\[
\frac{\delta(1 - \delta)p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)(1 + a_2)}{(1 - \delta)p_0 - \delta(1 - p_0))(\delta(1 - p_0) + (1 - \delta)p_0)} \geq \frac{\delta(1 - p_0) + (1 - \delta)p_0a_3}{\delta(1 - \delta)}
\]

This is true when \(a_2 = a_3\), and thus for sufficiently large values of \(a_2\), for some values of \(p_0\) and \(\delta\).

If \(a_1\) is sufficiently close to \(a_3\), then group 1 can separate when group 3 is mixing in a coalition of all three groups. This is the case when:

\[
\frac{1 + a_1}{2} \geq \frac{\delta(1 - p_0) + (1 - \delta)p_0a_3}{\delta(1 - p_0) + (1 - \delta)p_0}
\]

\[
a_1 \geq \frac{2(\delta(1 - p_0) + (1 - \delta)p_0a_3)}{\delta(1 - p_0) + (1 - \delta)p_0} - 1
\]

Since \(\delta > (1 - \delta)\),

\[
\frac{2(\delta(1 - p_0) + (1 - \delta)p_0a_3)}{\delta(1 - p_0) + (1 - \delta)p_0} - 1 > \frac{\delta(1 - p_0) + (1 - \delta)p_0a_3}{\delta(1 - \delta)} - 1
\]

Thus, for appropriate values of \(p_0\) and \(\delta\), when \(a_1\) is sufficiently close to \(a_3\) that group 1 can separate when group 3 mixes in a grand coalition, groups 1 and 3 can also lobby in a
cohesion together while group 2 lobbies alone.

Proof of Theorem 3

*Proof.* Let $a_3$ be sufficiently large such that group 3 can mix alone when groups 1 and 2 separate. Let $a_2$ be such that:

$$\frac{\delta^2(\delta + l(1 - \delta))(1 - p_0) + (1 - \delta)^2((1 - \delta) + l\delta)p_0 a_2}{\delta(1 - p_0) + (1 - \delta)p_0} < \frac{\delta(1 - \delta)(\delta(1 - p_0) + (1 - \delta)p_0 a_3)}{(1 - \delta)(1 - p_0) + \delta p_0}$$

$$a_2 < \frac{((1 - \delta)^2 p_0^2 - \delta^2(1 - p_0)^2)(\delta(1 - p_0) + (1 - \delta)p_0 a_3)}{p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)((1 - \delta)(1 - p_0) + \delta p_0)} - 1$$

and

$$\frac{\delta^2(\delta + l(1 - \delta))(1 - p_0) + (1 - \delta)^2((1 - \delta) + l\delta)p_0 a_2}{\delta^2(\delta + l(1 - \delta))(1 - p_0) + (1 - \delta)^2((1 - \delta) + l\delta)p_0} < \frac{\delta(1 - p_0) + (1 - \delta)p_0 a_3}{\delta(1 - p_0) + (1 - \delta)p_0}$$

$$a_2 < \frac{2(\delta(1 - p_0) + (1 - \delta)p_0 a_3)}{\delta(1 - p_0) + (1 - \delta)p_0} - 1$$

Then, group 2 cannot lobby with group 3 in a grand coalition and both groups cannot lobby alone in an equilibrium with all groups lobbying. Consequently, groups 2 and 3 also cannot lobby in a coalition when group 1 lobbies alone and groups 1 and 3 cannot lobby in a coalition when group 2 lobbies alone.

However, if groups 1 and 2 satisfy:

$$\frac{\delta^2(\delta + l(1 - \delta))(1 - p_0) + (1 - \delta)^2((1 - \delta) + l\delta)p_0 a_{1.2}}{\delta^2(1 - p_0) + (1 - \delta)^2 p_0} \geq \frac{\delta(1 - \delta)(\delta(1 - p_0) + (1 - \delta)p_0 a_3)}{(1 - \delta)(1 - p_0) + \delta p_0}$$

$$\frac{(\delta + l(1 - \delta))(1 - p_0) + ((1 - \delta) + l\delta)p_0 a_{1.2}}{(\delta + l(1 - \delta))(1 - p_0) + ((1 - \delta) + l\delta)p_0 a_{1.2}} \leq \frac{\delta(1 - \delta)(\delta(1 - p_0) + (1 - \delta)p_0 a_3)}{(1 - \delta)(1 - p_0) + \delta p_0}$$

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or

\[
\begin{align*}
\alpha_{1,2} & \geq \frac{((1-\delta)p_0 - \delta(1-p_0))((1-\delta)^2p_0)((1-\delta)p_0 + (1-\delta)p_0a_3)}{p_0(1-p_0)(\delta^2 - (1-\delta)^2)((1-\delta)(1-p_0) + \delta p_0)} - 1 \\
\alpha_{1,2} & \leq \frac{\delta^2((1-\delta)p_0 - \delta(1-p_0))((1-\delta)p_0 + (1-\delta)p_0a_3)}{\delta^2p_0(1-p_0)(\delta^2 - (1-\delta)^2)((1-\delta) + \delta p_0)} - \frac{(1-\delta)^2}{\delta^2}
\end{align*}
\]

Then groups 1 and 2 can separate in a coalition of two while group 3 lobbies alone. Since:

\[
\frac{((1-\delta)p_0 - \delta(1-p_0))((1-\delta)^2p_0)((1-\delta)p_0 + (1-\delta)p_0a_3)}{p_0(1-p_0)(\delta^2 - (1-\delta)^2)((1-\delta)(1-p_0) + \delta p_0)} < \frac{((1-\delta)^2p_0^2 - \delta^2(1-p_0)^2)((1-\delta)p_0 + (1-\delta)p_0a_3)}{p_0(1-p_0)(\delta^2 - (1-\delta)^2)((1-\delta)(1-p_0) + \delta p_0)}
\]

and

\[
\frac{((1-\delta)p_0 - \delta(1-p_0))((1-\delta)^2p_0)((1-\delta)p_0 + (1-\delta)p_0a_3)}{p_0(1-p_0)(\delta^2 - (1-\delta)^2)((1-\delta)(1-p_0) + \delta p_0)} < \frac{2(1-p_0) + (1-\delta)p_0a_3)}{\delta(1-p_0) + (1-\delta)p_0}
\]

all of these conditions can be satisfied simultaneously. Thus, when groups 1 and 2 are moderate and group 3 is extreme, the only equilibrium with all groups lobbying has groups 1 and 2 in a coalition while group 3 lobbies alone.

The greatest distance between \(a_1\) and \(a_3\) when both groups are lobbying alone is when group 3 is the only group mixing. Thus, having groups 1 and 2 separating in a coalition together when group 3 mixes alone allows for a greater distance between groups 1 and 3. However, greatest distance between groups 1 and 3 in a grand coalition exists when \(a_2 = \dot{a}_2\) such that:

\[
\frac{(\delta + l(1-\delta))(1-p_0) + ((1-\delta) + l\delta)p_0a_3}{(\delta + l(1-\delta))(1-p_0) + ((1-\delta) + l\delta)p_0} = \frac{\delta(1-p_0) + (1-\delta)p_0\dot{a}_2}{\delta(1-p_0) + (1-\delta)p_0}
\]

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and group 2 is the only group mixing in the coalition. Then, group 1 must satisfy:

\[
\frac{1 + a_1}{2} \geq \frac{(1 - \delta)^2 + \delta^2 a_3}{(1 - \delta)^2 + \delta^2} \\
1 \geq \frac{2((1 - \delta)^2 + \delta^2 a_3)}{(1 - \delta)^2 + \delta^2} - 1
\]

Since,

\[
\frac{((1 - \delta)p_0 - \delta(1 - p_0))(\delta^2(1 - p_0) + (1 - \delta)^2 p_0)(\delta(1 - p_0) + (1 - \delta)p_0a_3)}{p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)((1 - \delta)(1 - p_0) + \delta p_0)} < \frac{2((1 - \delta)^2 + \delta^2 a_3)}{(1 - \delta)^2 + \delta^2}
\]

having group 1 and group 2 separate in a coalition together when group 3 lobbies alone allows for a greater distance between groups 1 and 3 than the most favorable grand coalition. Thus, this configuration allows for the greatest distance between the most moderate and most extreme group lobbying on a single issue.

**Proof of Theorem 4**

*Proof.* Let \( a_3 \) be sufficiently large such that group 3 can mix alone in an equilibrium with the other groups separating alone, and let \( a_2 \) be close to \( a_3 \). By lemma 3, for large enough \( a_2, a_1 \) must be larger if group 1 is to lobby in a single coalition with groups 2 and 3 than if all groups are to lobby alone. Clearly, \( a_1 \) would need to be larger for group 1 to lobby alone when groups 2 and 3 form a coalition than when groups 2 and 3 lobby alone, because groups 2 and 3 have less uncertainty and thus higher payoffs in the former case.

Consider what must be true for group 1 to form a coalition with either group 2 or group 3, while the third group lobbies alone. When \( a_2 \) and \( a_3 \) are sufficiently similar, the group lobbying alone must separate, by the logic presented in the proof for lemma 4. For group 1
to lobby in a coalition with group 3, it must satisfy the condition found in that proof:

\[
a_1 \geq \frac{\delta (1 - p_0) + (1 - \delta) p_0 a_3}{\delta (1 - \delta)} - 1
\]

However, as described in the proof for lemma 3, for group 1 to lobby alone when groups 2 and 3 are lobbying alone and \( a_3 = a_2 \), \( a_1 \) must satisfy:

\[
a_1 \geq \frac{\delta (\delta + \hat{l}(1 - \delta))^2 (1 - p_0) + (1 - \delta) ((1 - \delta) + \hat{l}\delta)^2 p_0 a_1}{\delta (1 - p_0) + (1 - \delta) p_0}
\]

\[
\frac{\delta (1 - \delta)((\delta + \hat{l}(1 - \delta))(1 - p_0) + ((1 - \delta) + \hat{l}\delta) p_0 a_3)}{(1 - \delta)(1 - p_0) + \delta p_0}
\]

\[
\frac{\delta (\delta + \hat{l}(1 - \delta))^2 (1 - p_0)}{(1 - \delta)((1 - \delta) + \hat{l}\delta)^2 p_0)
\]

For all applicable values of \( p_0 \) and \( \delta \),

\[
\frac{\delta (1 - \delta)(\delta (1 - p_0) + (1 - \delta) p_0)((\delta + \hat{l}(1 - \delta))(1 - p_0) + ((1 - \delta) + \hat{l}\delta) p_0 a_3)}{(1 - \delta)((1 - \delta) + \hat{l}\delta)^2 p_0)(1 - p_0) + \delta p_0}
\]

\[
> \frac{\delta (\delta + \hat{l}(1 - \delta))^2 (1 - p_0)}{\delta (1 - \delta)} - 1
\]

Thus, the minimum value of \( a_1 \) such that group 1 can lobby when all groups are lobbying alone is smaller than the minimum value of \( a_1 \) such that group 1 is willing to lobby in a coalition with group 3 (or group 2) when group 2 (or group 3) is lobbying alone.

Hence, when \( a_3 \) and \( a_2 \) are large, the minimum \( a_1 \) such that group 1 is willing to lobby is when all groups lobby alone. Thus, when groups 2 and 3 have large biases and group 1 is less biased, the only equilibrium with all groups lobbying has all groups lobbying alone.  

Proof of Proposition 2
Proof. If groups do not lobby, the legislator chooses policy 0 with certainty, but in any equilibrium with lobbying, the legislator chooses policy 1 with probability greater than 0. Thus, if there is no equilibrium with groups lobbying alone and there is an equilibrium with groups lobbying in a coalition, the legislator is more likely to choose policy 1 in the latter case.

The more interesting cases are when there is are multiple equilibria, some with all groups lobbying alone and some with groups lobbying in coalitions.

The legislator chooses policy 1 with certainty when all groups lobbying alone lobby only when their signals are 1. Thus, when there is an equilibrium three groups lobbying alone only when their signals are 1 or two groups lobbying alone when their signals are 1, the legislator cannot choose policy 1 more often if those same groups lobby in a coalition.

Consider the case in which there is an equilibrium with all three groups lobbying alone and at least one group sometimes lobbies when its signal is 0. In such an equilibrium, the legislator chooses policy 1 with probability \( r \) when all groups lobby, picking \( r \) such that the mixing group or groups are indifferent between lobbying and not lobbying when their signals are 0 and any separating groups do not wish to lobby when their signals are 0; when groups’ expected payoffs from the legislator choosing policy 1 are higher, \( r \) is smaller.

When all groups are lobbying alone, group \( i \)’s greatest possible payoff from the legislator choosing policy 1 when \( s_i = 0 \) is:

\[
\frac{\delta(1 - \delta)((\delta + l(1 - \delta))(1 - p_0) + ((1 - \delta) + l\delta)p_0a_i)}{(1 - \delta)(1 - p_0) + \delta p_0}
\]

If, instead, group \( i \) is part of a coalition of three groups, its smallest expected payoff from the legislator choosing policy 1 when \( s_i = 0 \) is:

\[
\frac{\delta(1 - \delta)((\delta + l(1 - \delta))(1 - p_0) + ((1 - \delta) + l\delta)p_0a_i)}{\delta(1 - \delta)((\delta + l(1 - \delta))(1 - p_0) + ((1 - \delta) + l\delta)p_0)}
\]

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The numerators of these two expressions are the same, but the denominator of the latter is smaller. Thus, the smallest payoff group $i$ receives in a grand coalition is larger than the largest payoff group $i$ receives from lobbying alone. Hence, the legislator chooses policy 1 less often when group $i$ lobbies in a grand coalition than when group $i$ mixes alone.

If group $i$ lobbies in a coalition of two, its smallest expected payoff from the legislator choosing policy 1 when $s_i = 0$ is:

$$\frac{\delta(1 - \delta)((\delta + l(1 - \delta))(1 - p_0) + ((1 - \delta) + l\delta)p_0 a_i)}{(1 - \delta)(\delta + l(1 - \delta))p_0 + \delta((1 - \delta) + l\delta)p_0}$$

Again, the numerator is the same as in the largest expected payoff group $i$ can receive from lobbying alone when $s_i = 0$, but the denominator is smaller. Thus, the legislator chooses policy 1 less often when group $i$ lobbies in a coalition of two groups than when group $i$ mixes alone.

When groups lobby alone, if any group sometimes lobbies when its signal is 0, then group 3 sometimes lobbies when its signal is 0. Therefore, the above shows that the legislator chooses policy 1 less often when group lobbies in a grand coalition and when group 3 lobbies in a coalition of two than when group 3 mixes alone. The case that remains is when group 3 lobbies alone and groups 1 and 2 lobby together in a coalition. We may worry that group 3 has a higher expected payoff when all groups lobby alone than when groups 1 and 2 form a coalition. However, group 3’s payoff from the legislator choosing policy 1 when $s_3 = 0$ and group 3 lobbies alone is increasing with the probability that the other groups lobby when they receive 0 signals. Thus, for group 3 to receive anything but its smallest possible payoff when $s_3 = 0$ and all groups lobby alone, group 2 must mix as well. By the above, this implies that group 2 receives a greater payoff from lobbying in a coalition of two than from mixing alone, and the legislator chooses policy 1 less often when groups 1 and 2 form a coalition than when all groups lobby alone.

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Hence, when there is an equilibrium with all three groups lobbying alone in which at least one group sometimes lobbies when its signal is 0 and there is an equilibrium in which these same groups lobby in a coalition, the legislator is more likely to choose the groups’ preferred policy in the former.

Next, consider cases in which two groups can lobby alone in equilibrium. If both groups are lobbying only when their signals are 1, the legislator chooses policy 1 with certainty. Thus, coalitions cannot make the legislator more likely to choose policy 1 if there is an equilibrium with two of the groups separating alone.

Consider the more interesting cases with two groups lobbying alone: those with at least one group mixing in each equilibrium. When one group mixes and the other group separates in an equilibrium with two groups lobbying, the mixing group lobbies with probability $\bar{l}$ when its signal is 0, where $\bar{l}$ is chosen to make the legislator indifferent between policies 0 and 1 when both groups lobby. Since the legislator updates her beliefs using Bayes’ Rule:

$$\delta(\delta + \bar{l}(1 - \delta))(1 - p0) = (1 - \delta)((1 - \delta) + \bar{l} \delta)p_0$$

$$\bar{l} = \frac{\delta^2(1 - p0) - (1 - \delta)^2p_0}{\delta(1 - \delta)(p_0 - (1 - p_0))}$$

Since the payoff to a group lobbying alone increases in the probability that the other group lobbies when its signal is 0, the largest expected payoff from the legislator choosing policy 1 to group $i$ when $s_i = 0$ is$^5$:

$$\frac{(1 - \delta)(\delta + \bar{l}(1 - \delta))(1 - p0) + \delta((1 - \delta) + \bar{l} \delta)p_0a_i}{(1 - \delta)(1 - p0) + \delta p_0} = \frac{p_0(1 - p_0)(\delta^2 - (1 - \delta)^2)((1 - \delta)^2 + \delta a_i)}{\delta(1 - \delta)(p_0 - (1 - p_0))((1 - \delta)(1 - p_0) + \delta p_0)}$$

If that same group $i$ lobbies in a coalition of all three groups, its expected payoff decreases in $^5$If group $i$ is mixing, its maximum payoff is actually smaller, since the other group must mix with probability smaller than $\bar{l}$ for the legislator to be willing to choose policy 1, but this value is easier to work with.
the probability that the other groups join the coalition when their signals are 0. If another group joins the grand coalition with probability $l$ when its signal is 0, then group $i$’s payoff from the legislator choosing policy 1 when $s_i = 0$ is:

$$\frac{(\delta + l(1 - \delta))(1 - p_0) + ((1 - \delta) + l\delta)p_0a_i}{(\delta + l(1 - \delta))(1 - p_0) + ((1 - \delta) + l\delta)p_0a_i} = \frac{(1 - \delta)^2 + \delta^2a_i}{(1 - \delta)^2 + \delta^2}$$

Since, $p_0 > \delta$ by assumption:

$$\frac{1}{(1 - \delta)^2 + \delta^2} > \frac{p_0(1 - p_0)((\delta^2 - (1 - \delta)^2)}{\delta(1 - \delta)(p_0 - (1 - p_0))((1 - \delta)(1 - p_0) + \delta p_0)}$$

Hence, this is larger than the largest payoff group $i$ expects from the legislator choosing policy 1 when $s_i$. Thus, the legislator chooses policy 1 less often if group $i$ lobbies in a coalition of all groups than if group $i$ mixes when it is one of two groups lobbying alone.

There are, however, cases where groups receive lower expected payoffs in a configuration with two groups lobbying together and one group lobbying alone than in any equilibrium in which two of these groups lobby alone. Group $i$ mixing alone in an equilibrium with two groups lobbying has a smallest payoff from the legislator picking policy 1 when $s_i = 0$ of:

$$\frac{\delta(1 - \delta)((1 - p_0) + p_0a_i)}{(1 - \delta)(1 - p_0) + \delta p_0}$$

If, instead, group $i$ is lobbying alone when all three groups are lobbying, its greatest expected payoff from the legislator choosing policy 1 when $s_i = 0$ is:

$$\frac{\delta(1 - \delta)((\delta + l(1 - \delta))(1 - p_0) + ((1 - \delta) + l\delta)p_0a_i)}{(1 - \delta)(1 - p_0) + \delta p_0}$$
The denominators of the payoffs are the same, and:

\[ \delta(1 - \delta)((1 - p_0) + p_0a_i) > \delta(1 - \delta)((\delta + l(1 - \delta))(1 - p_0) + ((1 - \delta) + l\delta)p_0a_i) \]

since \( l < 1 \). Thus, if group \( i \) has a higher expected payoff—and the legislator chooses policy 1 less often in equilibrium—when it mixes alone when one other group lobbies than when it mixes alone and two other groups lobby. This implies that if group 1 finds lobbying alone too costly, even when only two groups lobby, but can lobby in a coalition with group 2 when group 3 mixes alone, the legislator chooses policy 1 more often than when groups 2 and 3 both lobby alone in an equilibrium. Cases in which the groups can lobby in both of these equilibria exist. Also, there are cases in which group 3 does not mix in an equilibrium in which it lobbies alone when groups 1 and 2 form a coalition, but the groups in the coalition have smaller expected payoffs than in equilibria in which two groups lobby alone.  

\[ 6 \]

Proof of Proposition 3

Proof. The legislator’s welfare depends on her choosing the correct policy for the state of the world. She receives a payoff of 1 when the policy matches the state of the world and a payoff of 0 when the policy and the state of the world do not match. Her welfare is given by:

\[
Pr(P = 1 \cap \omega = 1) + Pr(P = 0 \cap \omega = 0) = \\
Pr(P = 1|\omega = 1)Pr(\omega = 1) + Pr(P = 0|\omega = 0)Pr(\omega = 0) = \\
rPr(\text{groups lobby}|\omega = 1)(1 - p_0) + ((1 - r)Pr(\text{groups lobby}|\omega = 0) + \\
Pr(\text{groups do not lobby}|\omega = 0))p_0
\]

\[ 6 \]

Further details of these cases are available upon request.
where $r$ is the probability with which the legislator chooses policy 1 when the groups lobby. If the legislator is indifferent between policies when the groups lobby, then:

$$Pr(\text{groups lobby}|\omega = 1)(1 - p_0) = Pr(\text{groups lobby}|\omega = 0)p_0$$

This implies that the legislators welfare is equal to:

$$rPr(\text{groups lobby}|\omega = 0)p_0 + ((1 - r)Pr(\text{groups lobby}|\omega = 0)Pr(\text{groups do not lobby}|\omega = 0))p_0 = Pr(\text{groups lobby}|\omega = 0)p_0 + Pr(\text{groups do not lobby}|\omega = 0)p_0 = p_0$$

Thus, in any equilibrium in which the legislator is indifferent between policies, her welfare is $p_0$. This is also the same welfare the legislator receives if she always chooses policy 0, regardless of the groups’ lobbying activities. Hence, the legislator’s welfare is improved by lobbying only when groups separate, or lobby if and only if their signals are 1.

There are three types of equilibria in which all three groups separate. In the first, all groups join a coalition only when their signals are 1. In the second, two groups lobby together only when both have 1 signals and the third group lobbies alone when its signal is 1. In the third, all groups lobby alone when their signals are 1. By the proof of lemma ??, all groups separate in a grand coalition when $a_i \in [\underline{a}^{**}, \bar{a}^{**}]$ for all $i$, groups 1 and 2 separate in a coalition while group 3 separates alone when $a_{1,2} \in [\underline{a}^{**}, \bar{a}^{**}]$ and $a_3 \in [\underline{a}^*, \bar{a}^*]$, and all groups separate alone when $a_i \in [\underline{a}^*, \bar{a}^*]$ for all $i$.

Thus, coalitions improve the legislator’s welfare when $a_i \in [\underline{a}^{***}, \bar{a}^{***}]$ for all $i$ or $a_{1,2} \in [\underline{a}^{**}, \bar{a}^{**}]$ and $a_3 \in [\underline{a}^*, \bar{a}^*]$. In these situations, at least some groups find lobbying alone to be too costly. If, however, $a_i \in [\underline{a}^*, \bar{a}^*]$ for all $i$, then the groups can all separate alone, but any equilibrium with a coalition includes at least one group mixing. When all groups can
separate alone in equilibrium, coalitions decrease the legislator’s welfare.

If two groups separate alone in equilibrium, the legislator is more likely to pick the correct policy than in an equilibrium with no lobbying or in an equilibrium with lobbying and at least one group mixing. However, the legislator is less likely to pick the correct policy if two groups separate than if three groups separate. By the proof for lemma ??, two groups $j$ can separate in an equilibrium with only two groups lobbying when they satisfy $a_j \in [\alpha^*, \bar{\alpha}^*]$. By the same proof, there is no overlap between this region and the region where groups all three groups can separate in a single coalition. For large enough values of $c$, however, $\alpha^* < \bar{\alpha}^{**}$. Thus, it is possible for a group to separate alone in an equilibrium with one other group and to separate in a coalition of two groups. Hence, there are values of $a_1$, $a_2$, and $a_3$ such that two of the groups can separate alone in an equilibrium and such that groups 1 and 2 can separate together in an equilibrium while group 3 separates alone. In these cases, coalitions increase the legislator’s welfare. Otherwise, when $a_j \in [\alpha^*, \bar{\alpha}^*]$ for any two $j$, the legislator is better off if two of the groups lobby alone than if all three groups lobby in an equilibrium with a coalition. 

\[\square\]
References


