Problem Set #5

1. **Discrete-time convolution (3pts).**
   Compute and plot \( y[n] = x[n] * h[n] \), where
   \[
   x[n] = \delta[n-2] + 2\delta[n-3] - 2\delta[n-4] - \delta[n-5]
   \]
   \[
   h[n] = \begin{cases} 
   1 & \text{if } 3 \leq n \leq 7, \\
   0 & \text{otherwise}. 
   \end{cases}
   \]

2. **Continuous-time convolution (6 pts).**
   Define the function \( x(t) \) by
   \[
   x(t) = \begin{cases} 
   2 & \text{if } 0 \leq t < 1 \\
   -1 & \text{if } 1 \leq t < 2 \\
   0 & \text{otherwise} 
   \end{cases}
   \]
   Please recall the definition of the two continuous-time functions: unit step function \( u(t) \), and Rect function \( \text{rect}(t) \), and then sketch each of the following convolved signals:
   - (a) \( u(t) * u(t) \)
   - (b) \( x(t) * u(t) \)
   - (c) \( x(t) * \text{rect}(t) \)

3. **Convolution and the Fourier transform (3 pts).**
   What is the Fourier transform of \( \text{rect}(t) * \text{sinc}(t) \)? The convolution integral will not be the easiest way to do this.

4. **Averaging system (6 pts).**
   Suppose \( x[n] \) denotes the closing price of a stock on day \( n \). To smooth out fluctuations, a tool often used by technical analysts is the 30-day moving average of the stock price. Let \( y[n] \) denote this 30-day moving average, where the average at time \( n \) uses the closing price on day \( n \) together with the previous 29 days.
   - (a) Write an expression for \( y[n] \) in terms of \( x[\cdot] \).
   - (b) \( y[n] \) can be thought of as the output of an LTI system when the input is \( x[n] \). What is the impulse response of this system?
   - (c) How does the impulse response change if instead of the “lagging” average above we use \( x[n] \) together with 15 days in the past and 14 days in the future?
   - (d) What is a practical problem of using the average of part (c)?
5. System response (3 pts).
A continuous-time LTI system has impulse response $h(t)$ with Fourier transform $H(f)$. What is the output of the system when the input is $\sin(t)$?