CRITICISMS OF ROLLBACK THEORY, AND RESPONSES

1. Too simplistic to do justice to complex trees in real world

   Simple textbook theory is only to give basic understanding
   actual use needs building on this to more complex reality
   This is no different than most science textbooks
   Logic of rollback in simplest games possible without drawing tree
   Opposite extreme of complexity – Chess. Full rollback infeasible
   People (and computers) use partial rollback
   combined with “intermediate valuation function”
   A good function of this kind requires “knowledge” and “art”
   can combine it with “science” of rollback
   Typical application in analysis of economics, politics, business etc.
   is intermediate in size and complexity
   Needs explicit tree. Solution by inspection, or
   use computer programs like Gambit

2. Rollback not observed even in simple games

   Classic examples – Waiting in “Centipede” game
   Sharing in “Ultimatum” and “Dictator” games

   Explanation – Rollback theory is not (necessarily) wrong
   theorists often misuse it by assuming wrong payoffs
   (1) dividers motivated by fairness – social norm or genetic
   (2) choosers dislike unfairness or insult, or dividers fear this
   (3) actual game is an ongoing interaction –
       exploiting short-run advantage may hurt you later
   Such “nicer, fairer” behavior can have long run social value

   But inexperienced players can make errors, try experiments
   So use equilibrium concept as starting point of analysis
When studying a strategic interaction with sequential moves:
1. Do the rollback analysis
2. See if actual outcomes conform to this
3. If not, double-check the following
   (a) have you have specified the payoffs etc. correctly?
   (b) are the players novices; do they learn by playing?
   (c) is the game too complex?
4. In cases (a) or (b), opportunity for advice / consulting
   With (c), look for heuristics, art

DISTINCTION BETWEEN GAMES WITH SEQUENTIAL AND SIMULTANEOUS MOVES

Timing is not crucial, what matters is
(1) information when you move
(2) irreversibility of moves

Bidders in a sealed-bid auction may make their choices and mail their envelopes with bids at different times.
Even if A has mailed his bid when B is making his choice, B does not know the content of A’s envelope.
So moves are simultaneous.

In each play in football, the offensive and defensive coaches send in their play and alignment simultaneously.
But upon observing each other’s squad and alignment, quarterback can change play at line of scrimmage, defense can realign, either can call a timeout so moves are closer to sequential.

Moves sequential if
(1) later mover can observe earlier moves
   (this is one aspect of “perfect information”)
(2) the earlier moves cannot be changed secretly or simultaneously
Otherwise moves are simultaneous.


**PAYOFF MATRIX (table or spreadsheet) Examples**

<table>
<thead>
<tr>
<th>“Princess Bride”</th>
<th>Sicilian</th>
<th>Features to note:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1) Zero-sum</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2) Needs mixing</td>
</tr>
<tr>
<td><strong>Westley</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1, 0</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0, 1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prisoners’ Dilemma (smaller numbers better)</th>
<th>Pianist</th>
<th>Features to note:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1) One strategy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>uniformly best</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2) Outcome bad</td>
</tr>
<tr>
<td><strong>Tchaikovsky</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Confess</td>
<td>10, 10</td>
<td></td>
</tr>
<tr>
<td>Hold out</td>
<td>25, 1</td>
<td></td>
</tr>
</tbody>
</table>

**GENERAL SCHEMATIC “PAYOFF MATRIX”**

For two players, **Row** and **Column**
Row has available 4 strategies R1, R2, R3, R4
Column has available 3 strategies C1, C2, C3
SOLUTION CONCEPT – NASH EQUILIBRIUM

a list of strategies, one for each player, such that each player’s strategy is his best response to those of the others,

OR no player can gain from a unilateral deviation

Natural concept of equilibrium for non-cooperative game
In general, games may have no, 1, or many Nash equilibria
So need other concepts or restrictions – later

To illustrate, suppose strategy choices (R₃,C₂) with payoffs (R₃₂, C₃₂) are a Nash equilibrium.
This requires that

(1) R₃₂ is the largest row payoff in its column
and
(2) C₃₂ is the largest column payoff in its row

We interpret "largest" in weak sense, so only need

R₃₂ $ R₁₂,  R₃₂ $ R₂₂,  R₃₂ $ R₄₂
and
C₃₂ $ C₃₁,  C₃₂ $ C₃₃

ALTERNATIVE INTERPRETATION – CONSISTENT BELIEFS AND CHOICES

Players move simultaneously,
so neither observes other’s choice.
Instead, each forms "belief" (or "theory") about what the other is choosing.
Each makes his own choice rationally (maximize own payoff) given his belief about others’ choices.
Nash equilibrium is where all these beliefs happen to be correct