CLASSIFICATION

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<th>Sequential</th>
<th>Simultaneous</th>
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<td>Equilibrium concept</td>
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<td>Nash Pure Mixed</td>
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COMBINATIONS – SCHEMATIC EXAMPLES

(1) Tree with simultaneous moves at some nodes:
Equilibrium payoffs from matrix go at the node that leads to the matrix game

(2) Simultaneous move game, but one (or more) action combination leads to another simultaneous-move game
Equilibrium payoffs from second matrix to into the cell of the first matrix which leads to the second matrix
RULE CHANGES

Cold war example

Original simultaneous-move game

<table>
<thead>
<tr>
<th>USSR</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Restrained</td>
</tr>
<tr>
<td>Restrained</td>
<td>3, 4</td>
</tr>
<tr>
<td>Aggressive</td>
<td>4, 1</td>
</tr>
</tbody>
</table>

Sequential moves – US moves first

USSR responds A irrespective of whether US chooses R or A – A is USSR’s dominant strategy.
In view of the different US responses, USSR chooses R, even though A is its dominant strategy.

Concept of dominance is relevant for simultaneous moves, or for later mover in sequential-move game.

The concept of dominance may not be relevant for an earlier mover in a sequential-move game. Dominance says that one strategy is better for you than another for each given strategy of the other. But for an earlier mover, the later movers’ strategies are not given; the choice of the early move by one player can change these responses of the others. In fact the very purpose of the early move may be to bring about such changes; that is what a leadership or first-mover advantage is all about. This will be the essential idea of the "strategic move" called Commitment that we will study later.
Prisoners’ dilemma example

An information set encompasses the nodes between which the player scheduled to move there is unable to distinguish.
Necessary to have the same player(s) acting, and the same number and labels of actions available, at all points of an info. set.
The same action must be chosen at all nodes in an info. set.

Thus new definition of strategy – a complete plan of action, specifying the choice to be made at every information set (replacing the previous at every node) where it is that player’s turn to move, whether or not that information set is going to be reached on the actual path of play.

Reminder – For a game to be simultaneous-move, the physical timing of moves is not important. It is important that the later mover does not observe the earlier mover’s choice.
Conversely, for a game to have sequential moves, it is important that earlier actions are (1) observable, (2) irreversible.
ANALYZING A SEQUENTIAL-MOVE GAME
IN MATRIX (OR "STRATEGIC") FORM

Regard the complete plans of action as strategies, and show
them and the resulting payoffs in a matrix.

In the sequential version of the cold war example, with the
USSR moving first, the US has four strategies:
1. R if R, A if A (Match)  2. R if A, A if R (Reverse)

<table>
<thead>
<tr>
<th>USSR</th>
<th>Match</th>
<th>Reverse</th>
<th>R always</th>
<th>A always</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>3 , 4</td>
<td>1 , 3</td>
<td>3 , 4</td>
<td>1 , 3</td>
</tr>
<tr>
<td>A</td>
<td>2 , 2</td>
<td>4 , 1</td>
<td>4 , 1</td>
<td>2 , 2</td>
</tr>
</tbody>
</table>

This has two Nash equilibria shown with gray fill in the cells.

Bottom-right is the Nash equilibrium of the simultaneous-move
game. It is a Nash equilibrium of the sequential move game written in
its matrix or "strategic" form, because given that if US plays "A
always" (or the USSR believes that it doe), it is best for the USSR to
play A, and given that the USSR plays A (or the US believes it does),
it is best (although tied with "Match") for the US to play "A always". So
the beliefs and actions can be mutually reinforcing.

But top-left is the rollback equilibrium of the sequential-move
game. Here the US can observe what the USSR has done, so it need
not rely on any previous belief about the USSR’s move. Then the
USSR can correctly predict that the US will follow the "Match"
strategy, and that makes it optimal for the USSR to choose R.

Rollback is also "subgame perfect" – after any sequence of
earlier moves, the continuation of the strategies to the remaining part
of a full game remains an equilibrium of that part.