ECO 199 – GAMES OF STRATEGY  
Spring Term 2004 – April 15 
COLLECTIVE ACTION GAMES

TWO GENERAL CONCEPTS

1. SPILOVERS (EXTERNALITIES)

One person’s action affects another’s payoff
Simplest case: Two players A and B,
each has two choices: inaction (0) and action (1)
Payoffs A(a,b), B(a,b) where a denotes A’s choice, b denotes B’s
A’s action has negative spillover (external effect)
on B if B(1,b) < B(0,b) for b = 0 or 1
(in general could depend on what B is doing)
Need not be symmetric,

Similarly with many players.
In many examples, such effects often depend only on the total numbers taking action, not on individual identities
Examples – (1) traffic congestion, smoking or other pollution
(2) price cut or sales increase by one of competing sellers

Similarly positive spillover or externality
Examples – (1) from beekeepers to fruit growers
(2) getting vaccinated

2. POSITIVE FEEDBACK

A’s action increases B’s incentive to act also:
B(1,1) – B(1,0) > B(0,1) – B(0,0)
Example of assurance game:
B(1,1) > B(1,0) but B(0,1) < B(0,0)
Then multiple equilibria can arise
Group can get stuck in worse of these equilibria
Example with many players: Windows vs. Unix?
CAUSES OF AND REMEDIES FOR INEFFICIENT OUTCOMES

A’s choice: action if \( A(1,b) - A(0,b) > 0 \), inaction if \( < 0 \)

Ignores addition to group payoff \( B(1,b) - B(0,b) \)

Therefore we should expect to see

- too little of activities that have positive spillovers on others
- too much of activities that have negative spillovers on others

To achieve group optimality, must create extra incentive

(+ or – as appropriate) for person conferring spillover

Within the group –

1. Some players do partly “internalize” effects on others
   This can reduce inefficiency
2. Enforceable binding contracts with transfer payments
   to create the correct incentives for socially optimal actions
3. Repeated interaction, social norms
   with detection and enforcement mechanisms

From outside (usually government) – fines, rewards, compulsion
   That also create correct incentives

In fact government itself can be thought of as
an overall social contract

Numerous case studies of group attempts to generate
socially optimal outcomes from collective action games:
(Provision of public goods, protection of property rights
controlling depletion of common property resources, ... )

Important features of successful collective action

[1] Stable and small group of players
[2] System designed and monitored by
   local people, using better local information
[3] Rules of the game (what is permitted, what sanctions
   on violaters, ... ) clear and well understood

The first three conform to general ideas of repeated games
Fourth goes against idea of “grim trigger strategies”
but fits with the “forgiving” aspect of tit-for-tat,
avoids locking in too severe a response to mistakes
AVERAGE AND MARGINAL PAYOFFS

When \( n \) drivers on the road, each takes time \( t(n) \)
Total time \( T(n) = n \cdot t(n) \)

Example – Total time with 15 drivers = gray rectangle
Time taken by 16th driver = blue rectangle
Increase in time for original 15 drivers = red rectangle
Total time with 16 drivers = sum of the three rectangles
If social total net payoff is maximized when 15 drivers, need congestion charge to deter extra 16th driver = But must charge each because everyone in principle is 16th
General formula: \( T(n+1) - T(n) = (n+1) \cdot t(n+1) - n \cdot t(n) \)
\[ = t(n+1) + n \cdot [t(n+1) - t(n)] \]
Marginal cost to group (increase in total time)
\[ = \text{average cost of each member} \]
\[ + \text{external effect inflicted by one player on all the others} \]
For optimality, charge tax to each player = neg. spillover he creates

Calculus: treating \( n \) as continuous variable:
\[ T(n) = \frac{d[n \cdot t(n)]}{dn} = t(n) + n \cdot T(n) \]
\[ \text{If } t(n) = a + b \cdot n, \text{ then } T(n) = a \cdot n + b \cdot n^2, \text{ and } T(n) = a + 2 \cdot b \cdot n \]
\[ \text{external effect} = T(n) - t(n) = b \cdot n \]
\[ \text{If } t(n) = c \cdot n^k, \text{ then } T(n) = c \cdot n^{k+1}, \text{ and } T(n) = (k+1) \cdot c \cdot n^k, \text{ so} \]
\[ \text{external effect} = T(n) - t(n) = k \cdot c \cdot n^k = k \cdot t(n) \]
For traffic congestion case, can find \( k \mu 4 \)
MANY-MALE VERSION OF BEAUTIFUL BLONDE GAME

Active players: n men
Background: 1 blonde, n or more brunettes
Each man chooses Blonde or Brunette
Payoffs:
- 4 if you are the only man choosing Blonde
- 3 if you choose Brunette and no one wins the blonde
- 2 if you choose Brunette and someone else wins the blonde
- 1 if you choose Blonde and so does at least one other

Asymmetric pure strategy equilibria:
- exactly one chooses Blonde, the other (n-1) choose Brunette

Symmetric mixed strategy equilibrium:
- each mixes with probabilities p for Blonde, (1-p) for Brunette
Each must be indifferent between the two pure strategies
given the mixed strategies of all the others
Condition for equal expected payoffs:

\[
4 \cdot (1-p)^{n-1} + 1 \cdot [1-(1-p)^{n-1}] = \\
2 \cdot [(n-1) \cdot p \cdot (1-p)^{n-2}] + 3 \cdot [1 - (n-1) \cdot p \cdot (1-p)^{n-2}]
\]
or
\[
(1-p)^{n-2} \cdot [3 + n \cdot p - 4 \cdot p] = 2
\]

This can be solved numerically to find p for any given n
Then we can also find probability that
the blonde is not chosen at all, call it q = (1-p)^n
Table of solutions:

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<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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As n increases, p goes down so fast that q goes up!