The distribution of grades was as follows.

<table>
<thead>
<tr>
<th>Range</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>100-</td>
<td>5</td>
</tr>
<tr>
<td>90-99</td>
<td>24</td>
</tr>
<tr>
<td>80-89</td>
<td>13</td>
</tr>
<tr>
<td>70-79</td>
<td>7</td>
</tr>
<tr>
<td>0-69</td>
<td>6</td>
</tr>
</tbody>
</table>

Nice job; try to do equally well on the exam.

**Question 1**: 10 points (3 for setting up the table, 4 for the solution for x, 3 for the last part)

COMMON ERRORS: [1] Some misunderstanding of conditional probabilities - what they are being conditioned on etc. Some mistakes in computing the probability of a defect when the test is negative (-4). [2] A couple of unsuccessful attempts to solve this from first principles without using the formulas or the matrix from the book. The matrix does make using Bayes’ Rule much easier.

For this situation, the Bayes’ Rule table of Figure 9.A1 on p. 303 becomes

<table>
<thead>
<tr>
<th>Genetic condition</th>
<th>Test</th>
<th>Sum of row</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>Defect</td>
<td>0.0099</td>
<td>0.0001</td>
</tr>
<tr>
<td>No defect</td>
<td>0.99 x</td>
<td>0.99 (1-x)</td>
</tr>
<tr>
<td>Sum of column</td>
<td>0.0099 + 0.99 x</td>
<td>0.0001 + 0.99 (1-x)</td>
</tr>
</tbody>
</table>

Therefore Probability {No Defect given that Test Positive}

\[ = \frac{0.99 x}{0.0099 + 0.99 x} = \frac{100 x}{1 + 100 x} \]

We want this to be = 0.05, or 100 x = 0.05 + 5 x, or 95 x = 0.05, or x = 0.000526.

And then Probability {Defect given that Test Negative} = \[ \frac{0.0001}{0.0001 + 0.99 \times 0.999474} = \frac{1}{1 + 99 \times 0.9995} = 0.00010105 \]

**Question 2**: 30 points

COMMON ERRORS: [1] A few students had troubles in (b) and (c) with realizing that when prices are high enough, consumers stop buying (no point given in this case). [2] Another common mistake in (b) and (c) was to consider profits rather than revenues (-5). [3] In (d), almost everyone correctly stated the incentive
compatibility constraints, but some students failed to go on to compute x and y (-3 to -5). [4] Finally, a few
students did everything correctly from (b) to (d), but failed to realize that they had to compare the three
previous results in (e).

We were not pedantic about expressing constraints as > vs ≥ etc. so long as the results were right.

(a) (5 points) To casual users, Mictel offers to sell a low-end machine for a price of 4. The firm
makes a profit of 3 by doing so; it would make only 2 by selling these users a high-end machine. To intensive
users, Mictel offers to sell a high-end machine for a price of 8. The firm makes a profit of 5 by doing so; it
would make only 4 by selling these users a low-end machine. The firm's total profit is thus 3c+(1-c)5 = 5-2c.

(b) (5 points) If producing only low-end machines, Mictel can either set x=4 and sell to everybody
(earning a profit of 3), or it can set x=5 and sell only to intensive users (earning a profit of (1-c)4). The higher
price produces a larger profit when (1-c)4 > 3. Thus, when c < 1/4, the firm sets x=5; for c > 1/4, x=4.

(c) (5 points) If producing only high-end machines, Mictel either sets y=5 and sells to everybody
(earning a profit of 2), or sets y=8 and sells only to intensive users (earning a profit of (1-c)5). The higher
price produces a larger profit when (1-c)5 > 2. Thus, when c < 3/5, the firm sets y=8; for c > 3/5, y=5.

(d) (8 points) The incentive compatibility constraints are:
   for casual users: 4 - x > 5 - y, which simplifies to y - x > 1
   for intensive users: 8 - y ≥ 5 - x, which simplifies to y - x ≤ 3
If Mictel wants to separate its customers (and sell to both types), it sets x = 4, and can't set y above 7. If y >
7, the intensive users would switch to buying the low-end machine. Mictel's profit in this case is: 3c + 4(1-c)
= 4-c.

(e) (7 points) In the absence of perfect knowledge, we have described five pricing schemes that Mictel
could use. The profit each earns is: 3, (1-c)4, 2, (1-c)5, 4-c. Clearly, only the last two might be used. Comparing those profit formulas shows that, for c < 1/4, Mictel does best by selling only high-end computers
at a price of 8. (If it sold low-end computers to casual users, it would have to drop its high-end price to 7,
which reduces its profit when c < 1/4.) When c > 1/4, Mictel does best by selling both types of computers
with low-end priced at 4 and high-end priced at 7. Note that Mictel's profit using this system is less than the
5-2c it would earn if it could costlessly identify everybody's type.

Question 3: 30 points, 10 each

COMMON ERROR: When both players had possible strategic moves, ignoring the strategic moves of one
of the players (-2).

NICE JOB: There were some nice discussions of the credibility of these moves and of the implied threat
(promise) associated with a promise (threat) (+1). I also gave +1 to a student who realized that in (a), a threat
or a commitment amounted to the same thing.

(a) (i) There are two pure-strategy equilibria (Down, Left) and (Up, Right); there is also a mixed-strategy
equilibrium in which Row uses Up with probability 2/3 and Column uses Left with probability 2/3. The
payoffs from the mixed-strategy equilibrium are 2/3 to each player. (ii) If Row commits to Up, he ensures
himself a payoff of 2. Similarly, if Column commits to Left, he ensures himself a payoff of 2.

(b) (i) Both players have dominant strategies; equilibrium: (Up, Right); payoffs (3, 4). (ii) Row can
achieve his best payoff of 4 by using the threat "Down if Right."
(c)  (i) Both players have dominant strategies; equilibrium: (Up, Right); payoffs (2, 2). (ii) Either player can make a promise that moves the game to (Down, Left) and payoffs (3, 3). Row can promise "Down if Left;" Column can promise "Left if Down."

**Question 4: 15 points**

COMMON ERRORS: [1] Failing to realize that two strategic moves were needed. [2] Some infeasible moves, like a threat from Sparta to Athens.

NICE JOB: There were some interesting discussions on which moves were the more credible; the most interesting ones (hostages, foreign country intervention) got +1.

(a) (6 points) Game tree is below, with payoffs in order: (Sparta, Athens, Small cities)

![Game Tree Diagram]

The rollback equilibrium is War (2,2,2). Everyone prefers the outcome where Sparta does not declare war, Athens relinquishes its empire, and the Small cities stay independent (3,3,3).

(b) (9 points) Strategic moves are relevant for only the later movers (Athens, and Small cities). Since each of them gets to move at only one node, conditional moves (threats and promises) are irrelevant. A commitment by just one or the other (to do something different than its natural rollback action) is not enough: If Small cities commit to stay independent, uncommitted Athens will still choose Empire, and knowing this, Sparta will choose War. If Athens commits to No empire, uncommitted small cities will choose to Join Sparta, and then Athens, knowing this, will not want to make the commitment. So commitments by both Athens and Small cities are needed. In addition to the usual problems of credibility, this raises a further problem of collective action.

**Question 5: 15 points, 5 each**

COMMON ERRORS: [1] Saying that the commitment to defend in (a) (-1) and to say that this was not credible because there are not enough troops (-1). That misses the whole point of “are our hostage to that intent.” [2] Here we have some ambiguity as to whether the move should be called a promise or a threat; see below. But you have to be clear that Sherman has created a new status quo, namely his destructive march, and will change it (stop the burning) if the South surrenders. [3] For (c), many students (between a third and a half) did not discuss how the credibility of Henry's threat came from the fact that he may lose control. Grading here depended on the overall coherence of the argument, and was probably a little generous.
NICE JOB: A student who described Kennedy's move as a delegation to the US opinion got +1. A couple of other good discussions of credibility also got bonus points.

[a] Kennedy wants to deter the Soviets from attacking West Berlin. He is using a threat: “If you attack, we will defend the city with great force.” Even though we do not currently have enough troops positioned there to achieve this, the idea is public opinion in the U.S. will insist on a forceful response if lives of American troops are at risk or actually lost. Thus the “hostage to our intent” makes the threatened response credible. (A similar event actually occurred just a few years later, when in rapid response to a supposed attack by North Vietnam on a U.S. naval ship - the Gulf of Tonkin incident - the Congress gave President Johnson an essentially free hand to expand the conflict.)

[b] Sherman wants to compel the Confederacy to stop fighting and surrender. His strategic move is a kind of promise - he will stop his barbarity and cruelty when they stop the war. If instead he threatened to start the barbarity and cruelty if the South did not surrender, they could try to defeat his purpose using salami tactics or procrastination. The reverse device, namely starting the punishment (creating a new status quo) and promising to stop it upon compliance, puts the onus on the South and gives them the incentive to act quickly. (But the promised action - stopping - is presumably automatically credible, so you may want to call this a threat - “I will continue to burn your cities until you give up.”)

[c] Henry wants to compel the men of Harfleur to surrender the town. He is threatening to ransack it and inflict all kinds of personal harm on them otherwise. He could threaten that he would order all these things in cold blood once the English have taken the city. But the citizens may not regard this to be credible - they may think that Henry would be concerned about his reputation in history and therefore would not give such an explicit order, or that the pope may excommunicate him, or whatever. If Henry loses control over his troops, they in their rage may do all the terrible things but the blame could not be assigned definitely either to Henry or to any individual soldier. Therefore the threat of losing control can be more credible.

ADDITIONAL INFORMATION:

Both Sherman and Henry are using an additional ploy. They are in effect saying to the other side: “Your fate is in your own hands. If you persist in resisting us, the bad things that happen to you will be your own fault.” Henry V actually makes this explicit just a few lines down after the quoted passage:

What say you? will you yield, and this avoid,  
Or, guilty in defence, be thus destroy’d?

This concept of “guilty in defence” is used by many attackers.