COST-MINIMIZATION

\[ C(w, r, Q) = \min \{ wL + rK \mid F(K, L) \geq Q \} \]

FONCs

\[ w = \lambda \frac{\partial Q}{\partial L}, \quad r = \lambda \frac{\partial Q}{\partial K} \]

Interpretation: \( \lambda \) = marginal cost

\[ MRTS_{KL} = - \left. \frac{dK}{dL} \right|_{Q \text{ const.}} = \frac{\partial Q/\partial L}{\partial Q/\partial K} = \frac{w}{r} \]

Expansion path: Increase \( Q \) holding \((w, r)\) fixed:

Production function *homothetic* if every expansion path is a ray through origin

If \( F \) is homogeneous (of any degree \( a \)):

\[ F(sK, sL) = s^a F(K, L), \text{ then it is homothetic} \]

Converse not true
PROPERTIES OF COST FUNCTION

Vary $Q$ holding $(w, r)$ fixed – (ECO 102 material)

- Fixed cost: $C(0) = 0$, but as $Q \downarrow 0$, $\lim C(Q) > 0$
- Sunk cost: $C(0) > 0$

$AC(Q) = C(Q)/Q$, $MC(Q) = C'(Q)$

Returns to scale ↑ at margin: $AC \downarrow$, $MC < AC$
Returns to scale ↓ at margin: $AC \uparrow$, $MC > AC$

If rets to scale first ↑, then ↓, U-shaped cost curves

Vary $(w, r)$ holding $Q$ constant – (new material)

Properties similar to consumer’s expenditure function
(1) Homogeneous degree 1. (2) Concave, and
(3) Hotelling’s (Shepherd’s) Lemma, cost-minimizing input choices are given by

$$L^* = \frac{\partial C}{\partial w}, \quad K^* = \frac{\partial C}{\partial r}$$

\[\text{Diagram: Graph of cost function with isoquants.} \]

\[\text{2} \]
SHORT- AND LONG-RUN COST CURVES

- Short-Run Input Expansion Path
- Long-Run Input Expansion Path
- SRT \(_2\) = SRT \(_1\)
- LRTC \(_2\) = SRT \(_2\)
- LRTC \(_1\)
- Sunk cost \(rK2\)
- \(Q_1\), \(Q_2\), \(Q_3\)
- K2
- L
- K
Q = quantity, TC = total cost, AC = average cost, MC = marginal cost
LR = long run, SR = short run, SC = sunk cost
Subscript 2 denotes that short run in which K is optimal to produce Q₂
Subscript M stands for the AC-minimizing quantity
SRQₘ > Q₂; even in SR, there may be some scale economies beyond Q₂