PROFIT-MAXIMIZATION – TWO-STEP APPROACH

For each level of output $Q$, produce it at minimum cost:

$$\text{min} \{ wL + rK \mid F(L, K) \geq Q \}$$

Result: conditional input demands $L^*(w, r, Q)$, $K^*(w, r, Q)$
and the “dual” or minimized cost function $C^*(w, r, Q)$

Then choose $Q$ to \( \max pQ - C^*(w, r, Q) \)

FONC: $p = \partial C^*/\partial Q$ (price = marginal cost)

SOSC: $\partial^2 C^*/\partial Q^2 > 0$ (rising marginal cost)

If fixed cost, need to compare against $Q = 0$

\[ P < P_1 : \text{no critical point; } Q = 0 \text{ optimum} \]
\[ P_1 < P < P_2 : Q = 0 \text{ global optimum, local along MC} \]
\[ P_2 < P < P_3 : \text{global optimum along MC but lossmaking} \]
\[ P_3 < P : \text{global optimum along MC and profitmaking} \]
\[ P_1 < P < P_4 : \text{local min on decreasing portion of MC} \]
PROFIT MAXIMIZATION – SINGLE-STEP APPROACH

\[
\max \quad \Pi = p \cdot F(K, L) - w \cdot L - r \cdot K
\]

FONCs – price of each input = value of its marginal product

\[ p \frac{\partial F}{\partial L} = w, \quad p \frac{\partial F}{\partial K} = r \]

SOSC – (1) diminishing marginal returns to each input, (2) diminishing returns to scale (this is not fully rigorous)

Result - (unconditional) input demand functions

\[ L^*(p, w, r), K^*(p, w, r), \text{ yielding } Q^* = F(K^*, L^*) \]

Substitute in profit expression to get “dual” profit function

\[ \Pi^*(p, w, r) = p \cdot Q^* - w \cdot L^* - r \cdot K^* \]

Properties of dual profit function:

1. Homogeneous degree 1, and
2. convex in \((p, w, r)\)
3. Hotelling’s lemma:

\[
Q^* = \frac{\partial \Pi^*}{\partial p}, \quad L^* = - \frac{\partial \Pi^*}{\partial w}, \quad K^* = - \frac{\partial \Pi^*}{\partial r}
\]

Proof of these follows same lines as those of concavity of expenditure functions - take initial \((p^a, w^a, r^a)\) and initially optimum \(L^a, K^a, Q^a\). Could go on using these when prices change, so new optimum choices should yield no less profit.
EMPIRICAL ESTIMATION

U.S. MANUFACTURING (Ernst Berndt, 1991)

\[
\ln C = \ln(\alpha_0) + \sum_i \alpha_i \ln(P_i) \\
+ \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln(P_i) \ln(P_j) \\
+ \alpha_Y \ln Y + \frac{1}{2} \gamma_{YY} (\ln Y)^2 \\
+ \sum_i \gamma_{iY} \ln(P_i) \ln Y
\]

\(i, j = \text{inputs } K, L, E, \text{ and } M\)

\[\sum_i \alpha_i = 1, \quad \gamma_{ij} = \gamma_{ji}, \quad \sum_i \gamma_{ij} = 0\]

Find factor cost share functions and estimate, e.g.

\[
\frac{P_L L}{C} = \frac{P_L C}{C} \frac{\partial C}{\partial P_L} = \frac{d \ln C}{d \ln P_L}.
\]

Results: Elasticities of substitution

\[
\sigma_{KL} = 0.97, \quad \sigma_{KE} = -3.60, \quad \sigma_{KM} = 0.35,
\]

\[
\sigma_{LM} = 0.61, \quad \sigma_{EM} = 0.83, \quad \sigma_{LE} = 0.68
\]

Own price elasticities of factor demands

\[
\epsilon_K = -0.34, \quad \epsilon_L = -0.45, \quad \epsilon_E = -0.53, \quad \epsilon_M = -0.24.
\]
\[ \ln C = a + b_1 \ln Q + b_2 (\ln Q)^2 + \sum_i c_i \ln W_i + \sum_j d_j \ln F_j + \mu, \]

where \( Q \) = size (output) of the credit union
\( W_i \) factor prices, \( F_j \) other structural variables
\( \mu \) is stochastic error term.

Results

\[ b_1 = 0.6537 \text{ with standard error } 0.0231, \]
\[ b_2 = 0.0204 \text{ with standard error } 0.0015. \]

\( b_1 < 1, b_2 > 0 \) : initial economies of scale
and eventual diseconomies

Average cost is minimized when

\[ \ln Q = \frac{1 - b_1}{2b_2} = 8.48, \text{ or } Q = 4764 \]

85% of U.S. credit unions were to the left of this.
Median \( Q = 705 \), AC penalty 7.8 %.