GAME THEORY CONCEPTS

Players \{1, 2, \ldots, n\}
Strategies \(s_1, s_2 \ldots s_n\)
Payoff functions \(\Pi_1(s_1, s_2, \ldots s_n), \Pi_2(s_1, s_2, \ldots s_n)\), \ldots

Simultaneous moves: Nash equilibrium

Definition 1 – Each chooses own best strategy given the others’ strategy.

Two players: \((s_1^*, s_2^*)\) is NE if for any other \(s_1, s_2\)

\[
\Pi_1(s_1^*, s_2^*) \geq \Pi_1(s_1, s_2^*), \quad \Pi_2(s_1^*, s_2^*) \geq \Pi_2(s_1^*, s_2)
\]

“Best responses” – given \(s_2, s_1 = BR_1(s_2)\) maxes \(\Pi_1\). Nash equilibrium is intersection of best responses. But what does “response” mean when moves simultaneous? So

Definition 2 – Each chooses own best strategy given his belief about others’ strategy; AND these beliefs are correct.

Sequential moves: Backward induction or rollback reasoning, leading to subgame perfect equilibrium:
For simple two-player, two-stage game, this means
For any \(s_1\), response \(R_2(s_1)\) maxes \(\Pi_2(s_1, s_2)\)
\(s_1\) maxes \(\Pi_1(s_1, R_2(s_1))\)
Example of simultaneous-move game

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Example of sequential-move game

Our economic application will have continuously variable strategies (price etc.)
QUANTITY-SETTING (COURNOT) DUOPOLY

\[ \Pi_1(x_1, x_2) = (p_1 - c_1) x_1 = [(a_1 - c_1) - b_1 x_1 - k x_2] x_1 \]

Firm 1’s best response

\[ (a_1 - c_1) - 2 b_1 x_1 - k x_2 = 0 \]

Similarly firm 2’s. Solve jointly for Cournot-Nash eqm:

\[ x_1^* = \left[ 2 b_2 (a_1 - c_1) - k (a_2 - c_2) \right] / (4 b_1 b_2 - k^2) \]
\[ x_2^* = \left[ 2 b_1 (a_2 - c_2) - k (a_1 - c_1) \right] / (4 b_1 b_2 - k^2) \]
STACKELBERG LEADERSHIP
Sequential: firm 1 chooses $x_1$; then firm 2 chooses $x_2$
COURNOT OLIGOPOLY

Homog. product, $n$ identical firms
Constant marg. cost $c$, fixed cost $f$ for each
Linear industry demand : $p = a - b X$
Firm $i$ profit:

$$\Pi_i = [a - b (x_1 + x_2 + \ldots + x_n)] x_i - c x_i - f$$

FONC : $a - b (x_1 + x_2 + \ldots + x_n) - c - b x_i = 0$
Adding FONCs : $n [a - b X - c] - b X = 0$.
Solution for eqm.

$$X = \frac{n}{n+1} \frac{a-c}{b}, \quad x = \frac{1}{n+1} \frac{a-c}{b}, \quad p = \frac{a + n c}{n+1}$$

As $n \uparrow \infty$, $p \downarrow c$ (competitive limit).

But max $n$ compatible with $\Pi > 0$

$$\bar{n} \equiv \frac{a - c}{\sqrt{b f}} - 1$$
PRICE-SETTING (BERTRAND) DUOPOLY

Profit $\Pi_1(p_1, p_2) = (p_1 - c_1) \left( \alpha_1 - \beta_1 p_1 + \kappa p_2 \right)$

Best response $p_1 = \left[ (\alpha_1 + \beta_1 c_1) + \kappa p_2 \right] / (2 \beta_1)$

COMPARISONS

For substitute products, ranked by Prices ↑, Quantities ↓,
Firms’ profits ↑, Cons. surplus and Social efficiency ↓

1. Marginal cost pricing
2. Bertrand
3. Cournot
4. Cartel (Joint profit max)