Behavior inconsistent with expected utility theory:
Some examples

1. Allais Paradox
Consider the four lotteries below
Column headings are prizes; cell entries probabilities

<table>
<thead>
<tr>
<th>Lottery</th>
<th>$0</th>
<th>$1,000</th>
<th>$5,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0.1</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>C</td>
<td>0.90</td>
<td>0.10</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0.91</td>
<td>0.08</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Write $Z$ for the sure prospect of 0. Then

$$C = A \text{ with prob. } 0.1 + Z \text{ with prob. } 0.9$$
$$D = B \text{ with prob. } 0.1 + Z \text{ with prob. } 0.9$$

By the independence axiom,
an expected utility maximizer who
prefers A to B should also prefer C to D, and
if he prefers B to A, should also prefer D to C
But many people choose A over B, and D over C
2. Kinked Utility

vN-M type utility but with kink at initial wealth
Values losses from status quo much more than gains

3. Minimizing maximum regret
   When comparing choices A and B, regret of A is

   \[ \max_{i \in \text{Scenarios}} \max (U_i(B) - U_i(A), 0) \]

4. Errors in calculating probabilities
   Treating small probability events as if impossible
   Not applying correct Bayes’ rule when
   updating probabilities given some information

Expected utility approach still dominates
   in most applications – finance, game theory etc.
But alternatives being explored at research level
DEMAND FOR INSURANCE

Loss $L$ in scenario 2 (prob. $\pi_2$) Endowments $(W_0, W_0 - L)$

Each dollar of coverage requires insurance premium $p$

If insurance is actuarially (statistically) fair, $p = \pi_2$.

If buy $X$ dollars of coverage, final wealths

$$W_1 = W_0 - pX, \quad W_2 = W_0 - L - pX + X$$

Choose $X$ to maximize

$$EU(W) = \pi_1 U(W_0 - pX) + \pi_2 U(W_0 - L + (1 - p)X)$$

FONC

$$-p \pi_1 U'(W_0 - pX) + (1 - p) \pi_2 U'(W_0 - L + (1 - p)X) = 0$$

SOSC is $U'' < 0$, risk-aversion.

If fair insurance ($p = \pi_2$, $1 - p = \pi_1$) FONC becomes:

$$U'(W_0 - pX) = U'(W_0 - L + (1 - p)X), \quad W_1 = W_2$$

so risk is eliminated – full coverage $X = L$ optimum

Ins. Co.’s expected profit $E\Pi = pX - \pi_2 X$, so fair insurance will be available if:

(1) Risk-neutral insurers (by law of large nos ?)

(2) Perfect competition among insurers $\Rightarrow E\Pi = 0$

(2) No (minimal) admin. costs, no info. asymmetry
Alternative view: eliminate $X$ from $W_1, W_2$ equations:

$$(1 - p) W_1 + p W_2 = (1 - p) W_0 + p (W_0 - L)$$

Budget constraint for “contingent claims to dollars”

Prices $p, (1 - p)$. Slope $= (1 - p)/p$

Subject to this, max $EU = \pi_1 U(W_1) + \pi_2 U(W_2)$

Probabilities must be exogenous for symmetric info.

If insurance is fair, $p = \pi_2$

slope of budget line = 45-degree-line-MRS

so tangency (optimum) at 45-degree-line

If “unfair” (loaded) insurance, $p > \pi_2$

slope of budget line < 45-degree-line-MRS

Less than full insurance is optimal
TRADING RISK IN MARKETS

Markets held before uncertainty is resolved
Buy/sell “contingent claims”, like betting slips
Simplest of these – Arrow-Debreu Securities (ADS)
  Basic or elementary scenarios $i = 1, 2, \ldots, n$
  $ADS_i$ is claim to $1$ if scenario $i$, nothing otherwise
  Prices $p_i$ paid in advance
If money can be stored between now and time when
  uncertainty resolved and claims settled, $\sum_i p_i = 1$
If sure int. rate $r$ between now and then, $= 1/(1 + r)$
Equilibrium prices $p_i$ depend on probabilities $\pi_i$
  and on extent of, and attitudes toward, the risks
EXAMPLE 1 – No aggregate risk
  Individual risk can be fully insured by trade at fair prices

Two scenarios
Total $W$ same in both
Objective Probs. (1/2 each)
B more risk-averse than R
But MRS on 45-degree
  $\pi_1/\pi_2 = 1$ for both
So eqm. on that line
  $p_1/p_2 = \pi_1/\pi_2$
EXAMPLE 2 – Aggregate risk
Total $W_1 > W_2$: Scenario 1 “good”, 2 “bad”
Objective probabilities $\pi_1, \pi_2$

$E =$ initial endowment, $AB =$ core, $C =$ equilibrium

Locus of Pareto efficient allocations

Equilibrium, $MRS = p_1/p_2$

Points on 45-degree lines, $MRS = \pi_1/\pi_2$

Pareto efficiency, Core, Equil’m as in GE Theory (Wk.7)
ADS’s achieve efficient allocation of risk!
B is more risk-averse than R – so efficient points
are relatively closer to B’s 45-degree line
At any efficient risk-allocation, $p_1/p_2 < \pi_1/\pi_2$
Difference depends on risk-aversions of traders
If one is risk-neutral, $p_1/p_2 = \pi_1/\pi_2$