1. a) \( N_1 \) is binomial \( \sim B(n, p) \)
so, \( E(N_1) = np \), \( \text{Var}(N_1) = np(1 - p) \)
thus: \( E(\hat{p}) = np/n = p \)
\( \text{Var}(\hat{p}) = np(1 - p)/n^2 = p(1 - p)/n \)
According to Central Limit Theorem, as \( n \) is large enough, the probability distribution of \( \hat{p} \) will be normal.
b) \( E(\hat{p}) = 70/100 = .7 \)
The CI is \( .7 \pm z_{.025} \sigma_{\hat{p}} = (.61, .79) \)
c) \( L = 0.04 \)
\( n \geq \frac{4\sigma^2_{\hat{p}}}{L^2} \)
\( p = q = .5 \) maximizes the right hand side, so
\( n = 2400 \)

Note: For those considered \( L = 0.04p, n = 9600. \)

2. a) \( z = s - 6 = \sum_{i=1}^{12} u_i - 6 \) where \( u_i \sim U(0, 1) \) and independent.
so,
\( \mu_z = \mu_s - 6 = 12 \mu_U - 6 = 12 \times 1/2 - 6 = 6 - 6 = 0 \)
\( \sigma^2_z = \sigma^2_s = 12 \times \sigma^2_U = 12 \times 1/12 = 1 \)
b) Step 1: Generate 12 Us, Calc. Z, repeat m times
Step 2: Calc. Y from the m Zs
Step 3: If not enough Ys, go to step 1 else stop
c) Since \( Z_i \) is symmetrically distributed around a zero mean, \( E(Z_i^3) = 0 \).
\( E(Y) = E(\sum Z_i^3) = \sum E(Z_i^3) = 0 = 0 \).
Note: For those who considered \( E(Z^3) = E(Z)^3 \), you were wrong and could not get credits from this part.

The standard deviation of \( Y \) will be \( \sqrt{m} \sqrt{\text{Var}(Z_i^3)} \). So it is positive relative to \( \sqrt{m} \).

3. a) Since \( X \) is normal distributed with unknown variance, we should use \( t \) statistic.

<table>
<thead>
<tr>
<th>Accept ( H_0 )</th>
<th>( H_0 ) true</th>
<th>( H_0 ) false</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject ( H_0 )</td>
<td>type I error ( \alpha )</td>
<td>type II error ( \beta )</td>
</tr>
</tbody>
</table>

b) \( H_a: \mu < 100,000 \)
Reject \( H_0 \) when \( |t| \leq t_{a,n-1} \)
\( t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = -2.17 \)
\( t_{0.01,16} = -2.583 \)
So, cannot reject \( H_0 \).
c) There are not enough statistical evidence to accuse the manufacturers of false advertising.
d) $P = t(-2.17) = .023$

P-Value means the value of $\alpha$ at which level we would just reject the hypothesis.

When we increase the sample size, the P-value will be decreased.