6.8 a) A point estimate of the proportion of all such components that are not defective can be \( \hat{p} = \frac{80 - 12}{80} = 0.85 \)

b) \( P(\text{system work}) = \hat{p}^2 = 0.72 \)

6.15 a) Show that \( E(X^2) = 2\theta \):

\[
E(X^2) = \int_0^\infty x^2 \cdot \frac{1}{\theta} e^{-x^2/2\theta} dx = 2\theta \cdot \int_0^\infty \frac{x^2}{2\theta} - \frac{x^2}{2\theta} \frac{d}{dx} \frac{x^2}{2\theta} dx = 2\theta \cdot \int_0^\infty ye^{-y} dy \quad \text{(replace} \frac{x^2}{2\theta} \text{by} \ y) \]

So a estimator of \( \theta \) can be:

\[
\hat{\theta} = \frac{\sum X_i^2}{2n}
\]

To show it is unbiased:

\[
E(\hat{\theta}) = E\left(\frac{\sum X_i^2}{2n}\right) = \frac{1}{2n} \sum E(X_i^2) = \frac{1}{2n} (n) E(X^2) = 1 \cdot \frac{2\theta}{2} = \theta
\]

b) \( \hat{\theta} = 74.505 \)

6.20 a) \( f(X; n; p) = \binom{n}{X} p^X (1 - p)^{n-X} \)

so \( \ln[f(X; n; p)] = \ln C + X \ln(p) + (n - X) \ln(1 - p) \)

so mle of \( p \) (given \( n \)) satisfies: \( \frac{d \ln(f)}{dp} = 0 \), i.e.,

\[
\frac{X}{p} - \frac{n - X}{1 - p} = 0
\]

\( \hat{p} = p = \frac{X}{n} \)

b) \( E(\hat{p}) = E\left(\frac{X}{n}\right) = \frac{E(X)}{n} = \frac{np}{n} = p \),

so it is unbiased.

c) The mle of \( (1 - p)^5 \) is \( (1 - \hat{p})^5 = (1 - \frac{X}{n})^5 = 0.44 \).
6.24 The condition shown here is the same as: there are r-1 flaws in X-1 samples
and the Xth sample has flaw. So: 
\[
f(X; r, p) = \binom{X-1}{r-1} p^{r-1}(1-p)^{X-1-(r-1)},
\]
and
\[
ln[f(X; r, p)] = lnC + rln(p) + (X - r)ln(1 - p)
\]
We can see it is the same as what we found in 6.20 except the constant term. So the estimate will be the same.

6.28 a) 
\[
f(x_1, x_2, \ldots, x_n; \theta) = \prod_{i=1}^{n} \frac{e^{-x_i^2/2\theta}}{\theta^n} = \frac{1}{\theta^n} \exp\left(-\frac{1}{2\theta} \sum_{i=1}^{n} x_i^2\right)
\]
The mle of \(\theta\) satisfies: 
\[
\frac{dln(f)}{d\theta} = 0, \text{ i.e.,}
\]
\[
-\frac{1}{\theta} - \left(\frac{\sum x_i^2}{\theta}\right) = 0 
\]
\[
\theta = \frac{\sum x_i^2}{2n}
\]
b) Note: the median \(\hat{x}\) satisfies:
\[
\int_{-\infty}^{\hat{x}} f(x)dx = 0.5
\]
so, \(\hat{x} = \sqrt{2\theta/n}\)

7.8 a) Use the formula: 
\[
P \left( -z_{a/2} < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < z_{a/2} \right) = 1 - \alpha
\]
b) The interval will be expanded.
c) Same as (a) since for an exponential random variable, \(\mu = \sigma\).

7.10 Same as 7.8 a

7.14 Same as 7.8 a

7.18 Same as 7.8 a

Note: For problem 7.8-7.18, do not forget dividing the \(\sqrt{n}\) term.

7.20 Search the table on pg. 707 of Devore’s book.

7.22 Use the formula: 
\[
CI = \left( \bar{x} - t_{a/2, n-1} \cdot \frac{s}{\sqrt{n}}, \bar{x} + t_{a/2, n-1} \cdot \frac{s}{\sqrt{n}} \right)
\]
Note: How to decide when to use the method of 7.22 or of 7.10? The criteria are the same as of the validity of CLT, i.e., if \(n\) is ‘large enough’.